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To the problem of a number of creaks in composite materials with periodically curved layers

Alizade I. Seyfullayev

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Abstract. *Based on piecewise homogeneous body model, using exact equations of linear theory of elasticity, a method admitting to calculate the stress intensity factor in composites with antiphasally periodically curved layer that contain infinite series of free finite length cracks parallel to the direction of action of external normal forces, is developed.*

Keywords. composites · laminated · curving · crack · stern intensity stresses.

Mathematics Subject Classification (2010): yoxdur

1 Introduction

At present composite materials are widely used in shipbuilding, engineering industry and in other key industries of economics and the volume of their production in the world is intensively growing. The solution of different problems of engineering and productions requires reliable information about capacities of structural elements made of composite materials. In connection with the stated one, recently there is a great interest to problems of mechanics of composite materials, including fracture of these materials with cracks. In mechanics of composite materials, the issues associated with peculiarities of their structure one of which is the curving of reinforcing elements, rank high. It should be noted that successful use of artificially created composite materials in practice is considerably related with investigation of problems on determination of load bearing capacity, including the problems of mechanics of failure of these materials with regard to peculiarities of their structure, in particular curving of reinforcing elements.

Today a lot of problems of cracks in sandwich materials are studied. Note that the results of these studies may serve as a mechanism of failure of unidirectional sandwich composites in planes and surfaces perpendicular to the reinforcement direction under uniaxial tension of these materials along the layers and also as a mechanism of these materials in planes parallel to the reinforcement direction under uniaxial tension in lamination direction. The indicated

A.I.Seyfullayev
Institute of Mathematics and
Mechanics, Baku, AZ 1143, str.
B. Vahabzadeh 9.
E-mail: a.seyfullayev@yahoo.com

mechanisms can in no way explain failure of sandwich composite materials, for example in the form of lamination under uniaxial slopes along reinforcement of these materials.

In the papers [6], [5], as a result of experimental investigations on failure of composite materials under tension-compression along reinforcing elements (mainly for unidirectional composites with obviously expressed primary reinforcement in one of directions) it was revealed a failure consisting of separation of material into separate parts along the direction of the action of external load.

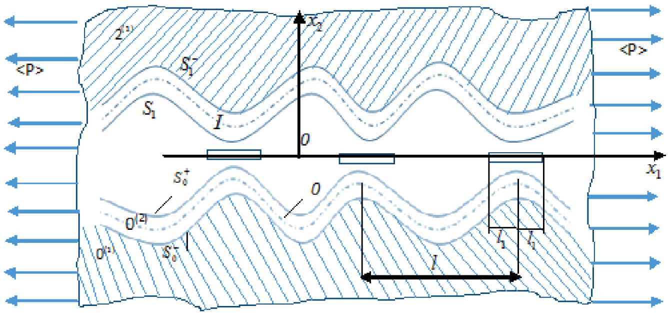

Fig.1. The scheme of a composite with antiphasally periodically curved layers.

Total failure along planes and surfaces that are arranged along reinforcing elements is typical for the stated ones; therefore it is logical to expect that the indicated kind of failure occurs as a result of action of forces directed perpendicularly to reinforcing elements. Nevertheless under uniaxial tension –compression along reinforcing elements, the external load is applied only along reinforcing elements; consequently, the considered kind of failure may occur only at the expense of internal forces (stresses) that arise under loading along reinforcing elements and are directed perpendicularly to reinforcing elements. In a number of papers such a failure or phenomenon was called "fraying" of a composite material. Thus, for describing fracture mechanism called linting of a composite material, it is necessary to clarify the causes of appearance of normal stresses in perpendicular direction or of tangential stresses; one of these causes may be negligible curving of reinforcing elements.

Proceeding from the above stated one, in the paper [1], proceeding from the results of investigations on determination of stress-strain state in composite materials with curved layers, within the framework of the model of piecewise-homogeneous body, by using exact three-dimensional equations of linear theory of elasticity, qualitative and quantitative explanation of the "fraying" effect in mechanics of fracture of composite materials, was suggested. However, therewith a macrospic criterion of fracture (in maximum normal and tangential stresses) was used as a crition of fracture of material.

By the well known principle, any macroscopic criterian of fracture of inhomogeneous materials may give only crude estimate of fracture and is not able to describe rather strict fracture mechanism. The indicated fracture mechanism may be set up within the frames of mechanics of fracture of crack in the considered composites. Thus, there arises a necessity for solving the problems of a cracks in composite materials with curved layers whose investigations are very urgent.

In [3] a problem of mechanism of fracture of composite materials with curved layers when one (collinear) crack is in the matrix is studied. In this paper a problem on determination of stress intensity factors with infinitely many longitudinal cracks in a composite with curved layers under plane deformation is studied.

2 Problem statement

Let us consider an infinite elastic body reinforced with two antiphasally periodically curved layers of the filler. Accept that in the direction of the OX_1 axis at infinity this body is under the action of uniformaly distributed normal forces of intensity (Fig.1).

Note that under $< P >$ we will understand the stress averaged on all the area of the considered body on which acts normal external force in the direction of the axis OX_1. The quantities referring to the matrix and the filler are designated by the superscripts (2.1) and (2.2), respectively. To each layer we refer the rectangular Cartesion system of coordinates $O_m^{(k)}\, X_{1m}^{(k)}\, X_{2m}^{(k)}\, X_{3m}^{(k)}$ $(k = 1, 2;\ m = 1, 2)$, that is obtained from the system of coordinates $Ox_1x_2x_3$ by parallel translation along the Ox_2 axis and is associated with median surface of appropriate layers. Accept that the curving of the considered layers is independent of x_3.

We consider that the reinforcing layers are located in the $X_{1m}^{1.2}\, X_{2m}^{1.2} X_{3m}^{1.2}$ planes and the thickness of each filler layer is constant. We take the matrix and filler materials homogeneous isotropic and linear elastic. Within every layer the equilibrium equations, Hooke's law and Cauchy relations

$$\frac{\partial \sigma_{ij}^{(k)m}}{\partial x_{ij}^{(k)}} = 0; \quad \sigma_{ij}^{(k)m} = \theta^{(k)m} \lambda^{(k)m} \delta_{ij} + \mu^{(k)m} \varepsilon_{ij}^{(k)m} \tag{2.1}$$

$$\varepsilon_{ij}^{(k)m} = \frac{1}{2}\left(\frac{\partial u_i^{(k)m}}{\partial x_{jm}^{(k)}} + \frac{\partial u_j^{(k)m}}{\partial x_{im}^{(k)}}\right); \quad \theta^{(k)m} = \frac{\partial u_i^{(k)m}}{\partial x_{im}^{(k)}} \quad i, j = 1, 2$$

where standard designations are used.

Let us assume that the conditions of complete adhesion are satisfied at the interface of the matrix filler materials. Considering the above stated designations adopted in Fig 1, we write these conditions in the form

$$\begin{aligned}
&\sigma_{ij}^{(1)1}\Big|_{S_1^+} n_j^{1,+} = \sigma_{ij}^{(2)1}\Big|_{S_1^+} n_j^{1,+}; && u_i^{(1)1}\Big|_{S_1^+} = u_i^{(2)1}\Big|_{S_1^+};\\
&\sigma_{ij}^{(1)2}\Big|_{S_1^-} n_j^{1,-} = \sigma_{ij}^{(2)1}\Big|_{S_1^-} n_j^{1,-}; && u_i^{(1)2}\Big|_{S_1^-} = u_i^{(2)1}\Big|_{S_1^-};\\
&\sigma_{ij}^{(1)2}\Big|_{S_2^+} n_j^{2,+} = \sigma_{ij}^{(2)2}\Big|_{S_2^+} n_j^{2,+}; && u_i^{(1)2}\Big|_{S_2^+} = u_i^{(2)2}\Big|_{S_2^+};\\
&\sigma_{ij}^{(1)3}\Big|_{S_2^-} n_j^{2,-} = \sigma_{ij}^{(2)2}\Big|_{S_2^-} n_j^{2,-}; && u_i^{(1)3}\Big|_{S_2^-} = u_i^{(2)2}\Big|_{S_1^-};
\end{aligned} \tag{2.2}$$

where $S_1^+\left(S_1^-\right), S_2^+\left(S_2^-\right)$ are upper (lower) surfaces of the filler layers $1^{(1.1)}$ and $2^{(1.2)}$, respectively; $n_j^{1,\pm}, n_j^{2,\pm}$- is the ortho of normal vector to the surface $S_1^{\pm}$ and $S_2^{\pm}$, respectively.

Assume that the curving of each filler layer in the direction of $OX_{1m}^{(1.2)}$ axis is periodic with equal periods l. We take the equations of median surface of the filler – layer $1^{(1.2)}$ in the form $x_{21}^{(1.2)} = L\sin(2\pi x_{11}^{(2)}/L)$, the equations of the median surface of the filler – layer $2^{(1.2)}$in the form $x_{22}^{(1.2)} = -L\sin(2\pi x_{12}^{(2)}/l)$, where $L$is the length of the rise; l is the length of the wave of distortion form.

Accept that $L < l$ and introduce a non-dimensional small parameter $\varepsilon = L/l$. Besides above stated ones, accept that the layer of the matrix has an infinite series of free cracks of finite length $2l_1$, that are on the plane $x_{21}^{(1.1)} = 0$.

The conditions on the cracks faces are as follows:

$\sigma_{12}^{(2.1)2}\left(x_{11,}+0\right)=\sigma_{12}^{(2.1)2}\left(x_{11,}-0\right)=0;$ for $x_{11}^{(1.1)}$ $\left(\ln -l_1, \ln +l_1\,, -\infty<n<\infty\right)$
Thus the statement of the problem considered is completed. We divide the solution of the problem into two stages. In the first state the stress-strain state is determined in the material without a crack in the given form of action of external forces and there by the stresses operating in the region containing cracks are found.

In the second state the stress state is determined in the composite with cracks whore faces are subjected to the stresses found at the previous stage.

On stage I according to [2] , we will look for the quantities characterizing the stress strain state of any $m^{(k)}$-th layer in the form of the series in parameter ε in the form.

$$\left\{\sigma_{ij}^{(k)m}, \varepsilon_{ij}^{(k)m}, u_i^{(k)m}\right\}=\sum_{q=o}^{\infty}\varepsilon^q\left\{\sigma_{ij}^{(k)m,q}, \varepsilon_{ij}^{(k)m,q}, u_i^{(k)m,q}\right\} \tag{2.3}$$

By linearity relation (2.1) will be satisfied for each approximation of (2.3) separately. The methods for determining the values of each approximation of (2.3) are given in detail in [3].

From the solution on stage I if follows that the function $\sigma^{(1.1)m}(x_{1m}^{(1)})$ has the form

$$\sigma^{(1)m}(x_{1m}^{(1)})=\sum_{q=1}^{\infty}\varepsilon^q\sigma^{(1)m,q}(x_{1m}^{(1)}) \tag{2.4}$$

We find the value of $\sigma^{(1.1)m,q}(x_{1m}^{(1)})$ when solving boundary value problems corresponding to q-th approximation on stage I. Taking into account (2.4), we represent the variables characterizing the stress-strain state in any layer, in the form of series (2.3) in small parameter ε.

It is easy to establish that on stage II, the zero order quantities are identically equal to zero. For the quantity of first approximation note that contact conditions (2.2) remain valid in this case as well for $\sigma_{ij}^{(k)m,0}\equiv 0, \quad u_i^{(k)m,0}\equiv 0.$

Thus, with regard to the quantity of first approximation we reduce the solution of the considered problem to the solution of a problem for a composite made of ideally arranged (not curved) layers of the same material and the same thicken (Fig. 2) that were shown in Fig.1.

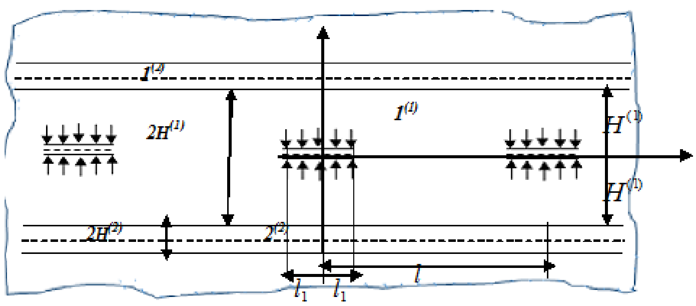

Fig.2. The schema of a composite with ideally arranged layers.

Based on the method of [5], in the paper [3] the first stage of the stated problem was studied in detail, the stresses were determined in the regions containing cracks. Using the one stated in [3], we study the second stage of the problem considered.

According to the problem statement, for $x_2 = 0$, in the layer with cracks there hold the following conditions for $x_2 = 0$.

$$\sigma_{12}^{(1)2}(x_{1,}+0) = \sigma_{12}^{(1)2}(x_{1,}-0) = 0; \quad (-\infty < x_1 < \infty) \tag{2.5}$$

$$\sigma_{22}^{(1)2}(x_1, 0) = -p(x_1); \quad (l \cdot n - l_1 < x_1 < l \cdot n + l_1, \quad -\infty < n < \infty) \tag{2.6}$$

$$u_2^{(1)2}(x_{1,} 0) = 0; \quad (l \cdot n + l_1 \leq x \leq l \cdot n + l - l_1, \quad -\infty < n < \infty) \tag{2.7}$$

$$u_1^{(1)2}(x_{1,}+0) = u_1^{(1),2}$$

where $p(x_1)$ are determined from the first stage of the solution of the problem. As in [3], representing the sought for quantities in the form of series in small parameter of $\varepsilon$and restricting with zero and first approximation, from (2.2) we get the following contact conditions for the first approximation:

$$\sigma_{i2}^{(1.1)1} = \sigma_{i2}^{(1.2)1}, \quad u_i^{(1.1)1} = \sigma_i^{(1.2)1} \quad i = 1, 2 \text{for} x_2 = H^{(1.2)}, \, -\infty < x_1 < \infty \tag{2.8}$$

$$\sigma_{i2}^{(2)1}\left(x_{1,} - H^{(2)}\right) = \sigma_{i2}^{(1)2}\left(x_{1,} \; H^{(1)}\right), \quad u_i^{(2)1}\left(x_{1,} - H^{(2)}\right) = u_i^{(1)1}\left(x_{1,} H^{(2)}\right)$$

for $-\infty < x_1 < \infty$, $i = 1, 2$

Using the Papkovich – Neibier representation, we look for the function $\Phi_0^{(k)m,1}$, $\Phi_2^{(k)m,1}$in the form.

$$\Phi_n^{(2)i,1} = \int\limits_{-\infty}^{+\infty} \left[B_{n1}^{(2)i,1}(s) ch(s x_{2i}^{(2)}) + B_{n2}^{(2)i,1}(s) sh(s x_{2i}^{(2)})\right] e^{-isx_1} dx_1$$

$$\Phi_n^{(1)i,1} = \Phi_{n1}^{(1)i,1} + \Phi_{n2}^{(1)i,1}, \tag{2.9}$$

$$\Phi_n^{(1)i,1} = \int\limits_{-\infty}^{+\infty} \left[B_{n1}^{(1)i,1}(s) ch(s x_{2i}^{(1)}) + B_{n2}^{(1)i,1}(s) sh(s x_{2i}^{(1)})\right] e^{-isx_1} dx_1.$$

$$\Phi_{n2}^{(1)i,1} = \begin{cases} \int\limits_{-\infty}^{+\infty} D_{n1}^{(1)i,1}(s) e^{-|s| x_{2i}^{(1)}} e^{-isx_1} dx_1 & \text{for} \quad x_{2i}^{(1)} > 0, \\ \int\limits_{-\infty}^{+\infty} D_{n2}^{(1)i,1}(s) e^{|s| x_{2i}^{(1)}} e^{-isx_1} dx_1 & \text{for,} \quad x_{2i}^{(1)} < 0 \end{cases} \tag{2.10}$$

By solving this problem (2.1), (2.3), (2.8) we introduce a new unknown function:

$$Q(x_1) = \frac{\partial}{\partial x_1}\left[u_2^{(1)2}(x_{1,}+0) - u_2^{(1)2}(x_{1,}-0)\right] \tag{2.11}$$

Taking into account $Q(x_1) = 0$, $l \cdot n + l_1 \leq x \leq l \cdot n + l - l_1$the condition of uniqueness of displacements of crack faces has the following form $\sum\limits_{l \cdot n - l_1}^{l \cdot n + l_1} Q(x_1) dx_1 = 0$.

After some transformations and calculations we get the following singular integral equation with the Hilbert kernel for the function $Q\left(x_1\right)$

$$\frac{1}{l_1}\frac{\pi}{\left(1-2\nu_1\right)}\int\limits_{-l_1}^{l_1}Q(t)ctg\pi\left(\frac{t-x_1}{l_1}\right)dt+$$

$$+\frac{\pi}{l_1}\int\limits_{-l_1}^{l_1}Q(t)\left\{\sum_{n=-\infty}^{\infty}2\sin\left(\frac{t-x_1}{l}2\pi n\right)\cdot\mathrm{K}\left(\frac{2\pi}{e}n\right)\right\}dt=-p(x_1) \tag{2.12}$$

Because of its bulky form we don't cite here the expression for K.

When obtaining this equation the known sums were used

$$\sum_{n=-\infty}^{\infty}\frac{1}{b-n}=\pi ctg\pi b\,,\qquad \frac{1}{2}+\sum_{n=1}^{\infty}\cos nt=\sum_{n=-\infty}^{\infty}\delta\left(t-2\pi n\right)$$

where $\delta(t)$ is the Dirac function.

As $l\to\infty$, in equation (2.12) the Hilbert kernel turns into the Cauchy kernel, and the sum of infinite series in the Fredholm kernel into the Riemann integral sum, as $l\to\infty$, equation (2.8) takes the form:

$$\frac{1}{\pi}\int_{-l_1}^{l_1}\frac{Q(t)dt}{t-x}+\frac{1}{\pi}\int_{-l_1}^{l_1}Q(t)\mathrm{K}\left(t,x\right)dt=-\frac{2\left(1-\nu_1\right)}{G_1}p(x_1) \tag{2.13}$$

This equation corresponds to one crack in the composite that was obtained earlier in the paper [6].

Use the following expansion:

$$xctgx=1-\sum_{n=1}^{\infty}\frac{2^{2n}B_n}{(2n)!}x^{2n}\qquad |x|<\pi \tag{2.14}$$

here $B_n$is the Bernoulli number.

Taking into account (2.14) in equation (2.12) and passing to non-dimensional quantities, we get the following equation:

$$\frac{\pi}{l_1\left(1-2\nu_1\right)}\int\limits_{-l}^{l}\frac{Q(t)dt}{t-x}+\frac{l_1}{l}\pi\int\limits_{-l}^{l}Q(t)\left\{\sum_{n=-\infty}^{\infty}2\sin\left(2\pi n\frac{l_1\left(t-x\right)}{n}\right)\times\right.$$

$$\left.\times K\left(\frac{2\pi n}{l}\right)-\frac{1}{\left(1-2\nu_1\right)}\sum_{n=1}^{\infty}\frac{2^{2n}B_n\left(t-x\right)^{2n-1}}{(2n)!}\left(\frac{l_1}{l}\right)^{2n-1}\right\}dt=-p(x_1) \tag{2.15}$$

Thus, the solution of the problem stated is reduced to the solution of singular integral equation (2.11). For solving the last one, we can apply the algorithm well developed and used in [6] and in other papers for numerical determination of the function $Q(t)$.

Therewith the SIF is determined by the function $Q_0(t)$in the from

$$K_I=\frac{\varepsilon\overline{K_I}\sqrt{\pi}}{2}\ \text{or}\ \overline{K_I}\left(-l_1\right)=\frac{Q_0\left(x-l_1\right)}{\sqrt{l_1}},\quad \overline{K_I}\left(l_1\right)=-\frac{Q_0\left(l_1\right)}{\sqrt{l_1}};Q(t)=\frac{Q_0\left(t\right)}{\sqrt{1-t^2}}$$

Let us consider some numerical results obtained within the stated technique, using only the first approximation. Accept that $E^{(1.2)}/E^{(1.1)} = 50,\ \nu^{(1.1)} = \nu^{(1.2)} = 0.3$

The values of SIF for some cases where $H^{(1.1)}/l$is the thickness of the matrix, $H^{(1.2)}/l$is the thickness of the filler, are given in Table 1.

The value's of $K_1/\sqrt{\pi\, l_1 \sigma_{11}^{(1.1),0}}$

Table 1.

$H^{1.1}/l$	$H^{1.2}/l$		$2\pi l_1/l$		
		$\pi/8$	$\pi/6$	$\pi/4$	π
0,1	0,1	81,98	86,37	88,25	10,62
	0,2	90,07	91,21	89,96	9,87
	1	71,98	71,22	68,19	6,23
0,2	0,1	115,04	119,11	124,02	21,15
	0,2	133,11	131,1	130,08	18,6
	1	112,61	109,4	101,44	10,07
0,3	0,1	138,23	135,34	135,27	29,54
	0,2	149,71	147,75	140,68	24,54
	1	112,67	108,89	97,35	10,45

It follows from the obtained results that in the cases under consideration, growth of a crack, i.e. growth of the length l_1/L, reduces to monotone increase of the value of K_1. Thus, we deduce that presence of a crack in the form represented in fig.1, in a composite with locally antiphasally curved layers when it is loaded at "infinity" by uniformly distributed normal forces in the reinforcement direction, may reduce to failure of the composite in the form of "fraying".

Thus, in the paper, based on a piece vise-homogeneous body model, using exact equation of linear theory of elasticity, a statement was given and an approach for solving the problem of mechanics of cracks in composite materials with antiphasally periodically curved layers of filler was suggested.

When investigating the problem, the sought-for quantities are represented in the form of series in the indicated small parameter. Therewith, zero and first approximation were used. It was shown that zero approximation doesn't satisfy the influence of the existence of crack on stress distribution in composites, i.e. it was shown that in a composite with ideally parallel arranged layers of filler, in the above stated form of loading, the presence of the considered cracks does influence of stern distribution in the composite. The indicated influence satisfies in the first approximation.

Therewith, at first we solve the problem in the case of absence of any cracks and determine the stress acting as the areas with cracks, and then study approximate problems of crack of normal breaking-off in composite materials with ideally arranged (uncurved) layers and with ideal contacts between layers. Distinction of the letters from the appropriate classical problems, in addition to other specifying factors, is that the conditions acting on the crack faces depend on structural parameters of the considered composites and on the length of a crack.

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