Transactions of NAS of Azerbaijan, Issue Mechanics, **36** (7), 81–90 (2016).
Series of Physical-Technical and Mathematical Sciences.

Determination of the damping coefficients of the orthotropic solid based on the combination finite element method and genetic algorithm

Arkadiy N. Soloviev · Mikhail Y. Shevtsov

Received: 12.07.2016 / Revised: 06.09.2016 / Accepted: 18.10.2016

Abstract. *Mechanical properties of composite materials are characterized by a considerable dissipation. In modern CAE packages account of these properties is done with the help of some damping coefficients. This paper considers the elastic orthotropic material properties of which correspond to some effective elastic constant. A method is proposed for identifying dissipative properties of such material in which there are damping coefficients. We consider the forced oscillations of a plate with a cantilevered in the low frequency domain and built an amplitude-frequency characteristics of the transverse displacement of some of its points, which are additional information for solving inverse problems. The inverse coefficient problem of determining the dissipative constants decided on the basis of minimizing the functional discrepancy between measured and calculated values of the transverse displacement of the selected points in a discrete set of frequencies. As a result of numerical experiments to answer the question of the adequacy of the additional information to uniquely identify the damping coefficients. And it found that for the determination of the two parameters of the information in the vicinity of the resonance is not enough, because the surface of the target, in this case takes the form of "ravine". .*

Keywords. genetic algorithm · finite element method · non-destructive testing · the elastic constants · anisotropic material · damping coefficients

Mathematics Subject Classification (2010): 74B05 · 74R10

1 Introduction

Mechanical properties of composite materials are characterized by dissipation of mechanical energy associated with the viscoelastic properties of their constituents (polymers)

Arkadiy N. Soloviev
Don State Technical University, Rostov-on-Don, Russian Federation,
E-mail: solovievarc@gmail.com

Mikhail Y. Shevtsov
Southern Federal University, Rostov-on-Don, Russian Federation
E-mail: mesouug@gmail.com

or with the microstructure (porous composites). For the adequate calculation of structures using such composites within the effective properties, in addition to the elastic constants and need to know the parameters of dissipation. For steady-state vibrations, this problem is solved by using a complex of modules which exhibit significant dependence on the oscillation frequency. In [1, 2] for a three-layer panels with an internal "soft" layer solved the inverse problem of determining the viscoelastic properties depending on the frequency in integrated modules, and a comparison was made of the dynamic characteristics of the calculation model with experimental data, which showed a good consistency of the results. Number of functions to be determined increases considerably in the case where the composite material modeled using orthotropic material with effective properties. In modern finite-element packages ACELAN, ANSYS etc., dissipation is described using a set of parameters that can be determined by the resonant frequencies and quality factor concept to the respective resonance [3, 4]. In general, the problem of determining these parameters is important. One way to solve this inverse coefficient problem is the combination of the finite element method (FEM) with optimization algorithms, including genetic algorithm (GA). Thus, in [5] GA is used to minimize residual experimental and calculated values of displacement to determine the Young's modulus and Poisson's ratio. Article [6] describes a procedure for determining the elastic constants of an orthotropic body, using GA together with the ultrasonic wave propagation velocity data. A study [7] describes a numerical method for the determination of elastic constants, based on the use of the hybrid GA. Scientific work [8] describes the combination of GA with the least squares method to solve the problem of determining the properties of the composite material. The purpose of [9] is to describe the concept of using GA to optimize composite structures layout. The article [10] propose a method for determining the properties of functionally graded material of the cylinder using GA. In [11] reconstruction of rigidity of the material properties is described of composite materials reinforced with unidirectional fiber from the data of the ultrasonic wave directed at an angle using the inverse method based on GA. In [12] studied the mechanical properties of the material polymer-composite helicopter blade spar. A complete set of elastic constants is based on a series of experiments and adjusted by means of the GA, which minimizes the discrepancy between the measured and calculated frequencies representative volume reinforced composite in the form box. In turn, [13] and [14] describe the determination of elastic and dissipative properties of materials using a combination of the finite element method and of complex artificial neural networks. The important is to mention the publications covering the viscoelastic properties of the materials. Thus, in [15] presents an analysis of the properties of viscoelastic damping composite plates according to the thickness of the layers. A study [16] describes the determination of the viscoelastic properties of polymer composites with carbon nanotubes inclusions. Publication [17] describes the developed anisotropic viscoelastic-plastic solid model to simulate the mechanical properties of high-density celluloid materials which are subjected to static and dynamic loads.

This paper deals with the development of methods for determining the damping constant, used in packages that implement FEM and continues a series of works of authors [18, 19]. One of the issues addressed in the work is a numerical definition of a sufficient amount of additional information for unambiguous finding these characteristics. The paper deals with the forced oscillations bracket-attached composite plate, effective orthotropic elastic properties are assumed to be known. As additional information to solve the inverse problem the amplitude-frequency characteristic (AFC) for set of points shifts is used. In this embodiment, the discrete points on the AFC in the set in the vicinity of the resonance frequency. The inverse coefficient problem of determining the dissipative characteristics is achieved by minimizing discrepancy GA functional between measured and calculated values ??in ANSYS transverse displacements of the points in the set of frequencies.

2 Formulation of the direct problem.

We consider the steady-state oscillations with angular frequency ω of orthotropic body in cartesian coordinate system $Oxyz$ ($\bar{x} = (x, y, z)$), the displacement vector is given $\bar{u}(\bar{x}, t) = \bar{U}(\bar{x}) \cdot e^{i\omega t}$. Motion vector amplitude values $\bar{U}(\bar{x})$ with dissipation satisfies the system of equations [3].

$$-\rho(\omega^2 - i\omega\alpha)\bar{U} - \nabla \cdot \sigma = \bar{f} \tag{2.1}$$

$$\sigma = c \cdot \cdot (1 + \beta i\omega)\varepsilon \tag{2.2}$$

$$\varepsilon = (\nabla\bar{U} + \nabla\bar{U}^T)/2 \tag{2.3}$$

where $\rho(\bar{x})$ - material density; σ - stress tensor; $\bar{f}$- vector density of mass forces; c - fourth-rank tensor of elastic moduli; ε - strain tensor; α β- non-negative damping factors such consideration of damping adopted in modern finite element packages such as ANSYS, ACELAN and others. Equations (2.2) can be written in matrix form using the complex values of elastic coefficients:

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix} = \begin{pmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{xy} \end{pmatrix}, \tag{2.4}$$

where $C_{ij} = c_{ij}(1 + \beta i\omega)$, c_{ij} - components of corresponding matrix of elastic coefficients excluding damping.

When using a dynamic testing for determination of the mechanical properties of composite materials samples of special shape can be used, for example thin plates cut from structural elements under investigation. Therefore further considered harmonic vibrations of a rectangular plate fixed cantilever ($a \times b$ - dimensions in plane) thickness h, which plane coincides with one of the planes orthotropy elastic properties, for example Oxy (Figure 1.).

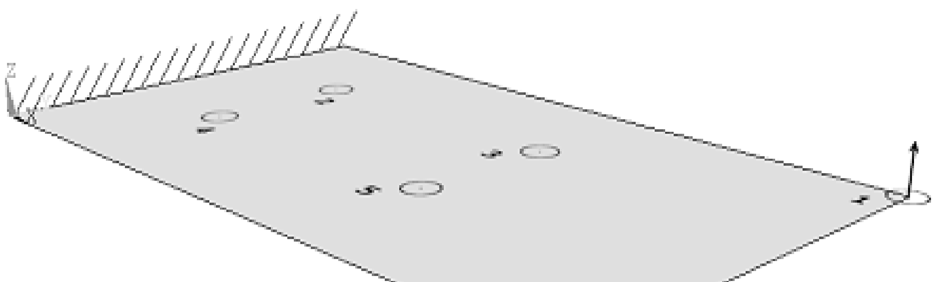

Fig. 1. Schematic representation of the fixed plate

We introduce the notation:

$$\begin{array}{l} \alpha_{x1} = C_{11} - \frac{C_{13}^2}{C_{33}}; \quad \alpha_{xy1} = C_{12} - \frac{C_{13} \cdot C_{23}}{C_{33}}; \quad \alpha_{y1} = C_{22} - \frac{C_{23}^2}{C_{33}}; \\ \alpha_{x2} = C_{11} - \frac{C_{12}^2}{C_{22}}; \ \alpha_{xz2} = -\frac{C_{12} \cdot C_{23}}{C_{22}} + C_{13}; \ \alpha_{z2} = -\frac{C_{23}^2}{C_{22}} + C_{33}; \\ \alpha_{y3} = -\frac{C_{12}^2}{C_{11}} + C_{22}; \ \alpha_{yz3} = -\frac{C_{13} \cdot C_{12}}{C_{11}} + C_{23}; \quad \alpha_{z3} = C_{33} - \frac{C_{13}^2}{C_{11}}; \end{array} \tag{2.5}$$

Displacement vector $\bar{U}$ consists of three components:

$$\bar{U} = (U_x, U_y, U_z) \tag{2.6}$$

The equation of harmonic oscillations of an anisotropic plate parallel to the plane Oxy has the form [20]:

$$D_{x1}\frac{\partial^4 w}{\partial x^4} + 2H_{b1}\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{y1}\frac{\partial^4 w}{\partial y^4} = -\rho \Omega^2 w, \tag{2.7}$$

where $w = U_z$- transverse displacement, $\Omega^2 = \omega^2 - i\omega\alpha$;

$$D_{x1} = \frac{\alpha_x \cdot h^3}{12}; D_{y1} = \frac{\alpha_y \cdot h^3}{12}; D_{xy1} = \frac{C_{44} \cdot h^3}{12}; H_{b1} = \frac{1}{12}\cdot\alpha_{xy1}\cdot h^3 + \frac{1}{6}\cdot C_{44}\cdot h^3; \tag{2.8}$$

Border conditions:
transverse forces $P_1,\ P_2$ defined on the sides of the plate and no moments
at $x = a$:

$$D_{x1}\frac{\partial^2 w}{\partial x^2} + \frac{\alpha_{xy1}h^3}{12}\cdot\frac{\partial^2 w}{\partial y^2} = 0; \frac{\partial}{\partial x}\left[D_{x1}\frac{\partial^2 w}{\partial x^2} + (H_{b1} - 2D_{xy1})\frac{\partial^2 w}{\partial y^2}\right] = P_1 \tag{2.9}$$

at $y = 0,\ b$:

$$D_{y1}\frac{\partial^2 w}{\partial y^2} + \frac{\alpha_{xy1}h^3}{12}\cdot\frac{\partial^2 w}{\partial x^2} = 0\frac{\partial}{\partial y}\left[D_{y1}\frac{\partial^2 w}{\partial y^2} + (H_{b1} - 2D_{xy1})\frac{\partial^2 w}{\partial x^2}\right] = P_2. \tag{2.10}$$

on the edge of the rigid clamping plate $w = 0$, $\partial w/\partial n = 0$
at $x = 0$:

$$w = 0, \quad \frac{\partial w}{\partial x} = 0, \tag{2.11}$$

Direct problem involves the solution of (2.7) with the boundary conditions (9-11) with known elastic constants and damping coefficients α and β.

3 The solution of the inverse problem.

Formulation of inverse coefficient problem is assumed that the elastic constants c_{ij} are known and the damping coefficients α, β are unknown and to be determined.

Additional information for the task of identifying the damping coefficients α, β are the amplitude-frequency characteristics of the displacement $w^*(x_k, y_k,\ \omega)$ measured in a set of points ($k = 1, 2, ..5$ - positional scanning), in a discrete set of frequencies in the vicinity of resonances (frequency scan). Figure 2 shows the frequency dependence of the real part of the transverse displacement (calculation in ANSYS). The inverse problem solution is constructed on the basis of minimizing the functional residual $F(\alpha, \beta)$ between the measured w^* and numerically obtained values of the transverse displacement at specified α, β:

$$F(\alpha, \beta) = \sum_{k=1}^{K}\sum_{n=1}^{N}\left(\frac{w^*(x_k, y_k,\ \omega_n) - w(x_k, y_k,\ \omega_n)}{\max(|w^*|)}\right)^2 \tag{3.1}$$

where $\max(|w^*|)$ - the maximum value of the module of measured transverse displacement for a given point, K - number of positional scanning points, N - number of frequency scanning frequencies.

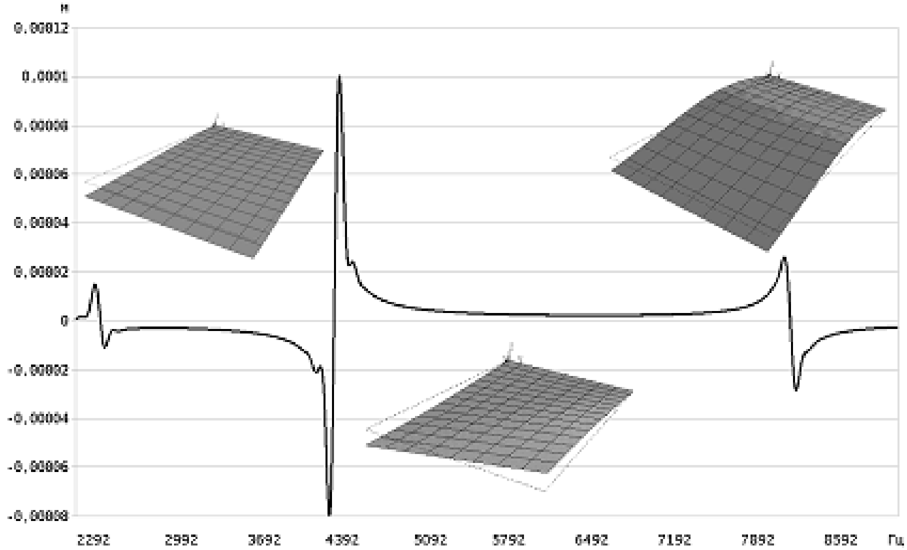

Fig. 2 AFC transverse displacement of point 1

Direct problems were solved by the finite element method in ANSYS package. The genetic algorithm (software implementation code [18]) has been used as a tool to minimize the functional (3.1). Sufficiency of the additional information was investigated in numerical realization of the proposed method (number of points of the position and the frequency scanning) for the sustainable recovery of damping factors.

4 Numerical implementation.

In the numerical implementation of the approach considered harmonic oscillations of the plate, the size of which $a = 0,025m$, $b = 0,0145m$, $h = 0,0055 <$, density $\rho = 7970kg/m^3$. Oscillations are excited by force $P = 10N$ applied at the point number 1 (Figure 1). The elastic properties of the plate material are shown in Table 1 the damping coefficients $\alpha = 110,144$ $\beta = 1,542 \cdot 10^{-07}$.

Table 1. The elastic properties of the plate ($AgInS_2$):

	C_{11}	C_{12}	C_{13}	C_{22}	C_{23}
N/m^2	$83,783 \times 10^9$	$39,397 \times 10^9$	$33,577 \times 10^9$	$33,487 \times 10^9$	$18,995 \times 10^9$
	C_{33}	C_{44}	C_{55}	C_{66}	
N/m^2	$24,143 \times 10^9$	5×10^9	$1,3 \times 10^9$	$1,5 \times 10^{10}$	

Positional coordinates of scanning points (in meters): $x_1 = 0,025$ $y_1 = 0,0145$; $x_2 = 0,015$ $y_2 = 0,01$; $x_3 = 0,005$ $y_3 = 0,01$; $x_4 = 0,015$ $y_4 = 0,005$; $x_5 = 0,005$ $y_5 = 0,005$.

Table 2. Resonant frequency of the plate:

f_1, Hz	f_2, Hz	f_3, Hz	f_4, Hz
2239,12	4256,48	5132,57	8086,41

Table 2 lists the first four own frequency oscillations of plate with desired characteristics.

The number of points and scanning the frequency ranges are shown in Table 4 in column 2 and 3, respectively, wherein the selected ranges were divided by scan points evenly.

The inverse problem was solved through a combination of FEM and the GA, settings of the last listed below:

Table 3. The parameters of the genetic algorithm

Parameter	Value
Population size	256
Bit depth of genome	16 bit
Number of generations	300
Probability of crossover	0,95
Probability of mutation	0,05
Number of parents	2
Selection strategy	Roulet
Crossover strategy	Point, 2 points.
Use elitism strategy	Yes

The initial population of the GA was formed at random, and as the experience of the application of the GA has a significant impact on the result for the complex structure of the target surface. In this work, this influence is not found in numerical experiments.

The numerical experiment was performed with 4 frequencies in the region of the first resonance (2200-2400Hz). The obtained values $\alpha = 103,125$ and $\beta = 1,895 \cdot 10^{-07}$ correspond to the error of 7% and 19%. In view of the fact that in constructing the general damping matrix, and stiffness matrix mass multiplied by the parameters α and β accordingly, there is no problem of the search related to the fact that their values differ by 9 orders of magnitude. Figure 3 shows the target surface in the form of a "ravine", indicating a lack of additional information for a reliable determination of two coefficients immediately. This assumption is confirmed by the following experiment, in which the search was made only for β coefficient. Target function has one extremum that allows you to determine the desired ratio with an error of less than 1% which indicates about sufficiency of additional information.

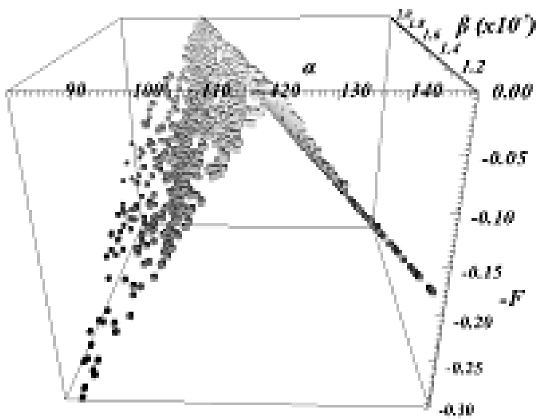

Fig. 3. Points of target surface of the first experiment

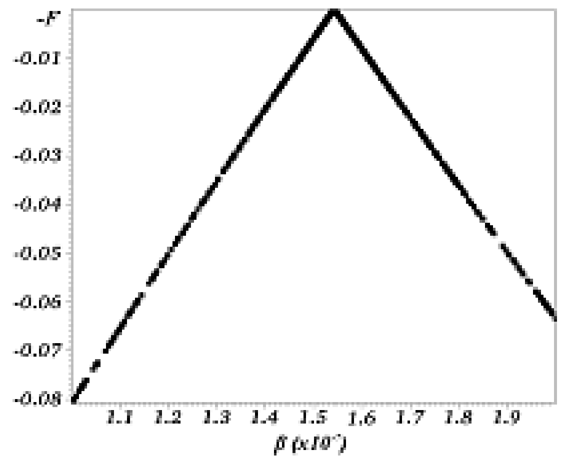

Fig. 4. Points of target surface of the second experiment

Under the assumption that the coefficients α and β are constant and on an interval, we can extend the range of the frequency scanning and numerically investigate the minimum amount of additional information for a reliable determination of the damping coefficients. The results of these numerical experiments are presented in table 4. First column indicates the number of numerical experiment, in the fourth position indicates the number of scan points, and the fifth and sixth of the desired identification error relative damping coefficient.

Table 4. The results of numerical experiments

No.	The number of used frequencies	Frequency range, Hz	The number measuring points	α relative deviation, %	β relative deviation, %
1	30	2000-9000	1	0,0029	0,0000
2	3	2000-2300	1	-1,9599	7,0676
3	4	2000-2400	1	2,0287	-7,2171
4	5	2000-5000	1	-0,0873	0,1574
5	3	2150-2300	1	3,4504	-12,4496
6	3	2150-2300	4	-0,1397	0,5652
7	5	8150-9000	4	-1,7204	0,3088
8	5	4000-9000	4	0,0513	-0,0258
9	7	1000-9000	4	0,2996	-1,1899
10	10	2000-2400 4000-4400	1	-0,0020	0,0039

Analysis of the results presented in table 4 allows to divide numerical experiments into two groups, 1, 8, 10, 2-7, 9. The target surface for the first group of experiments has a pronounced minimum (Figure 5 for 1 experiment of table 4) and the surface is characterized by a long "ravine" (similar to Figure 3) for the second group. Error recovery of α, β coefficients for the first group is less than 0.1%, whereas for the second group of experiments, this error reaches 12%. The first group is characterized by a more informative frequency range (present in two or three resonance). The second group is characterized by a small number of points of frequency scanning or by using range with one resonance. The number of points of positional scanning in this case is not critical to restore accuracy.

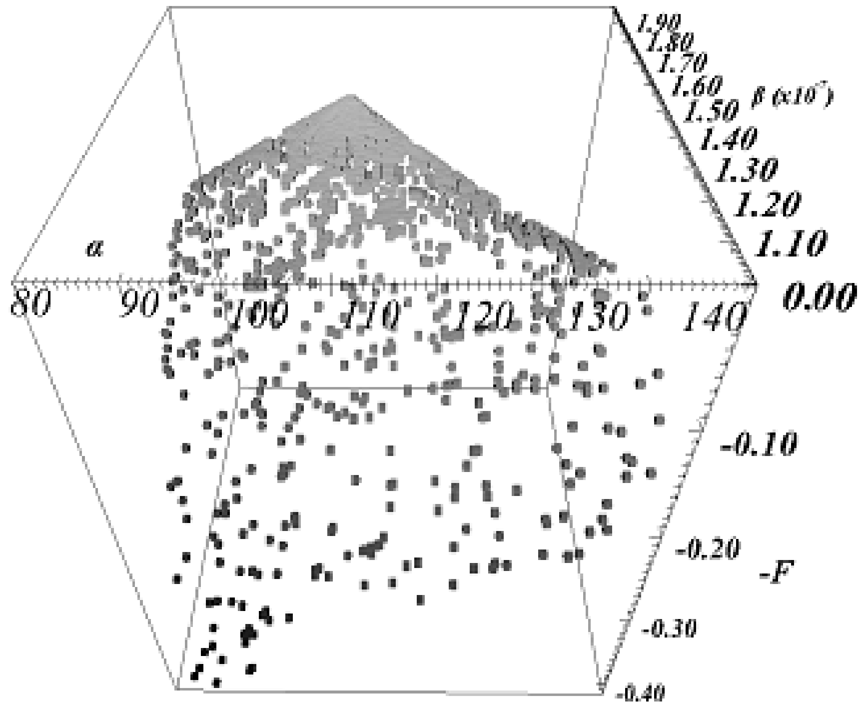

Fig. 5. Points of target surface in experiment 1 of table 3

5 Conclusions.

The paper presents a method for determining the damping coefficients for orthotropic material on the basis of data of the frequency response of displacement cantilevered plate. A feature of this method is the combination of FEM and GA. In the accomplished numerical experiments frequency ranges were set for measurement of wave fields amplitudes for an unambiguous determination of the two coefficients in particular two resonances should be presented in a range, in this case the error recovery for a given search time does not exceed one percent.

References

1. Barkanov, E., Skukis, E., Petitjean, B.: *Characterisation of viscoelastic layers in sandwich panels via an inverse technique*. Journal of Sound and Vibration. **327**, 402–412 (2009).
2. Barkanov, E.: *Dynamic Analysis of Composite Structures made from Viscoelastic Frequency and Temperature Dependent Materials* . Proceedings of the Twelfth International Conference on Civil, Structural and Environmental Engineering Computing, B.H.V. Topping, L.F. Costa Neves and R.C. Barros, (Editors), Civil-Comp Press, Stirlingshire. - Scotland, Paper 198. 13p. (2009)
3. Belokon A.V., Yeremeyev V.A., Nasedkin A.V., Soloviev A.N.: *Blochnyye skhemy metoda konechnykh elementov dlya dinamicheskikh zadach akustoelektrouprugosti [Block scheme of the method of finite elements for dynamic problems of acousto electroelasticity].* Prikladnaya matematika i mekhanika - Applied Mathematics and Mechanics **64**, Rel.3, 381–393 (2000).

4. Belokon, A.V., Nasedkin, A.V., Soloviev, A.N.: *Novyye skhemy konechno-elementnogo dinamicheskogo analiza p'yezoelektricheskikh ustroystv [New schemes of the finite element dynamic analysis of piezoelectric devices]*. Prikladnaya matematika i mekhanika - Applied Mathematics and Mechanics, **66** (3), 491–501 (2002).
5. Baranov, I.V., Vatulyan, A.O., Soloviev, A.N.: *Ob odnom geneticheskom algoritme i yego primenenii v obratnykh zadachakh identifikatsii uprugikh sred [A genetic algorithm for solving the inverse identification problem for elastic media]*. Vychislitelnyye tekhnologii- Computational technologies, **11** (3), 14-26 (2006).
6. Vishnuvardhan, J., Krishnamurthy, C.V., Krishnan, Balasubramaniam: *Genetic algorithm reconstruction of orthotropic composite plate elastic constants from a single non-symmetric plane ultrasonic velocity data*. Composites Part B. Engineering, **38**, 216–227 (2007).
7. Shun-Fa, Hwang, Jen-Chih, Wu, Rong-Song, He.: *Identification of effective elastic constants of composite plates based on a hybrid genetic algorithm*. Composite Structures, **90**, 217-224 (2009).
8. Liu, G.R., Han, X., Lam, K.Y.: *A combined genetic algorithm and nonlinear least squares method for material characterization using elastic waves*. Comput. Methods Appl. Mech. Engrg. **191**, 1909-1921 (2002).
9. Muc, A., Gurba, W.: *Genetic algorithms and finite element analysis in optimization of composite structures*. Composite Sturctures, **54**, 275–281 (2001).
10. Liu, G.R., Han, X.: *Determination of material property of functionally graded cylinder using genetic algorithm*. Inverse problems in engineering mechanics III. 117–124 (2002).
11. Balasubramaniam, K., Navin, S.R.: *Inversion of composite material elastic constants from ultrasonic bulk wave phase velocity data using genetic algorithms*. Composites Part B: Engineering, **29** (2), 171–180 (1998).
12. Akopyan, V.A., Bykov, A.A., Rozhkov, E.V., Soloviev, A.N., Shevtsov, S.N.: *K opredeleniyu effektivnykh svoystv polimerkompozitnogo materiala na osnove garmonicheskogo i modal'nogo analiza [Determination of the effective properties of polymer composite material based on harmonic and modal analysis]*. Mekhanika kompozitsionnykh materialov i konstruktsiy - Mechanics of Composite Materials and Structures, **14** (1), 35–48 (2008).
13. Soloviev, A.N., Giang, N.D.T., Chang, S. H.: *Determination of elastic and dissipative properties of material using combination of FEM and complex artificial neural networks*. Springer Proceedings in Physics. **152**, 137–148 (2014).
14. Soloviev, A.N., Nguyen, Z.CH.: *Opredeleniye uprugikh i dissipativnykh svoystv materialov s pomoshch'yu sochetaniya metoda konechnykh elementov i kompleksnoznachnykh iskusstvennykh neyronnykh setey [Determination of elastic and dissipative properties of materials using a combination of the finite element method and of complex artificial neural networks]*. Vestnik Donskogo gosudarstvennogo tekhnicheskogo universiteta - Bulletin of Don State Technical University. **14** (2), 84-92 (2014).
15. Komlan, Akoussan, Hakim, Boudaoud, El Mostafa, Daya, Yao Koutsawa, Erasmo Carrera: *Sensitivity analysis of the damping properties of viscoelastic composite structures according to the layers thicknesses*. Composite Structures, **149**, 11–25 (2016).
16. Shokrieha, M.M., Rahmatollah, Ghajar, Shajari, A.R.: *The effect of time-dependent slightly weakened interface on the viscoelastic properties of CNT/polymer nanocomposites*. Composite Structures, **146**, 122–131 (2016).
17. Tjahjantoa, D.D., Girlanda, O., Ostlund, S.: *Anisotropic viscoelastic-viscoplastic continuum model for high-density cellulose-based materials*. Journal of the Mechanics and Physics of Solids, **84**, 1–20 (2015).
18. Soloviev, A.N., Shevtsov, M.Y.: *Rekonstruktsiya defektov v uprugikh telakh sochetaniyem geneticheskogo algoritma i metoda konechnykh elementov [Reconstruc-*

tion of defects in elastic bodies combination of genetic algorithm and finite element method]. Vestnik Donskogo gosudarstvennogo tekhnicheskogo universiteta - Bulletin of Don State Technical University. (1), 43–51 (2016).

19. Soloviev, A.N., Shevtsov, M.Y.: *Opredeleniye polnogo nabora uprugikh postoyannykh ortotropnogo kompozita po informatsii o sobstvennykh chastotakh nabora obraztsov [Determination of the full set of elastic constants of orthotropic composite of information about their own set of samples frequencies]*. Ekologicheskiy vestnik nauchnykh tsentrov Chernomorskogo ekonomicheskogo sotrudnichestva Ecological Bulletin scientific centers of the Black Sea Economic Cooperation. (2), 69–77 (2016).
20. Boyarshinov, S.V.: *Osnovy stroitelnoy mekhaniki mashin [Fundamentals of structural mechanics machines]*. Moscow. Mashinostroyeniye, **456** (1973).