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# On some particularities of the influence of the fluid viscosity on the frequency response of a viscoelastic plate loaded with this fluid

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**Abstract.** *This paper studies some particularities of the fluid viscosity on the frequency response of the hydro-viscoelastic system consisting of a viscoelastic plate which is in contact with this fluid.The model consisting of the viscoelastic plate and half plane filled with compressible viscous fluid, is employed. The motion of the fluid is described by utilizing linearized Navier-Stokes equations, however the motion of the plate by employing the exact equations of dynamics for viscoelastic bodies. Numerical results are presented and discussed for the case where the viscoelasticity of the plate material is modeled with the use of the fractional exponential operators by Rabotnov. Glycerin is taken as a fluid in these numerical investigations which relate to the normal stress acting on the interface plane between the constituents and on the normal velocity of the fluid on this plane. All the investigations are made for the plane-strain state in the plate and corresponding plane-parallel flow of the fluid.*

**Keywords.** Hydro-viscoelastic system · compressible viscous fluid · compressible inviscid fluid · viscoelastic material · frequency response

**Mathematics Subject Classification (2010):** 74B20 · 74K20

## 1 Introduction

Investigations of the problems related to the vibration of plate + fluid hydro-elastic systems were subject a lot of investigations the review of which is given in the paper by Akbarov and Ismailov (2015). According to this review, in the sense of the accuracy of the employed theories these investigations can be divided into two groups the first of which contains the investigations carried out within the scope of approach plate theories and within the scope of an incompressible inviscid fluid model. However, the second group investigations are made within the scope of the exact equations of motion for the plate and linearized Navier-Stokes equations for the flow of the compressible viscous fluids. At the same time,

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the second group investigations can also be classified as the wave propagation and forced vibration ones. The wave propagation problems under existence of the initial stresses in the plate are made in the papers by Bagno (2015), Bagno et al. (1994) and others, a review of which is given in the survey paper by Bagno and Guz (1997). Detailed consideration of related results was made in the monograph by Guz (2009).

However, the investigations related to the forced vibration of the plate + fluid hydro-elastic systems was started recently and as the subject of the present paper relates to these investigations we consider a brief review of those. Note that the first attempt in this field was made in the work by Akbarov and Ismailov (2014) in which the two-dimensional (plane-strain state) problem on the forced vibration of the pre-strained highly elastic plate + compressible viscous fluid system, was studied. The forced vibration of the system consisting of the elastic plate, compressible viscous fluid and rigid wall was considered in the paper by Akbarov and Ismailov (2015a). Moreover, in the paper by Akbarov and Ismailov (2015b), the dynamics of the moving load acting on the hydro - elastic system considered in the paper by Akbarov and Ismailov (2015b)were investigated. Note that the results obtained in the paper by Akbarov and Ismailov (2014a)were also detailed in the monograph by Akbarov (2015).

Recently, in the paper by Akbarov and Panakhli (2015) the discrete-analytical solution method is proposed for the solution to problems related to the dynamics of the hydro-elastic system consisting of an axially-moving pre-stressed plate, compressible viscous fluid and rigid wall. The concrete numerical results are also presented and discussed.

However, all the investigations reviewed above, were focused on the interaction between an elastic plate and fluid, and therefore, in general, cannot be employed to understand the forced vibration of hydro - viscoelastic systems, such as, the polymer plate loaded with fluids. At the same time, investigations of the vibration of hydro - viscoelastic systems have a great significance because at present polymer composite materials are intensively used in various branches of modern industry related to the building of boats, ships, offshore structures, etc. under the building of which, their interaction behavior with fluids must be takeninto account. In connection with this in the paper by Akbarov and Ismailov (2014b) the first attempt is made in this field and the problem related to the forced vibration of the system consisting of the plate made of a linear viscoelastic material and compressible viscous fluid is considered. However, in the paper by Akbarov and Ismailov (2014b)the related numerical analysis is made for some particular cases and the fluid is modeled as a compressible viscous fluid only. Consequently, the results given and discussed in the paper by Akbarov and Ismailov (2014b)do not allow us to make any conclusion on the influence of the fluid viscosity on the frequency response of the viscoelastic plate + fluid system.

Taking this statement into consideration, in the present paper, the investigations started in the work by Akbarov and Ismailov (2014b)are developed and an attempt is made to explain how the fluid viscosity acts on the frequency response of the hydro-viscoelastic system consisting of a viscoelastic plate which is in contact with this fluid-filled half-plane and how this action depends on the rheological parameters of the plate material. Under this investigation the plate material is described throughRabotnov's fractional exponential operators (Rabotnov, 1980) and plane-strain state in the plate and corresponding plane-parallel flow of the fluid takes place.

## 2 Formulation of the problem

Consider a hydro - viscoelastic system consisting of the plate with $h$ thickness made of viscoelastic material and a half-plane filled by compressible viscous fluid. We introduce the Cartesian system of coordinates $Ox_1x_2x_3$ (Fig. 1) which is associated with the upper free face plane of the plate. Assume that the system is perturbed by the lineal-located time-harmonic normal force with $P_0$ amplitude which acts on the plate's free face plane, according to which, the plane-strain state in the plate and the plane flow of the fluid in the $Ox_1x_2$ plane, occur.

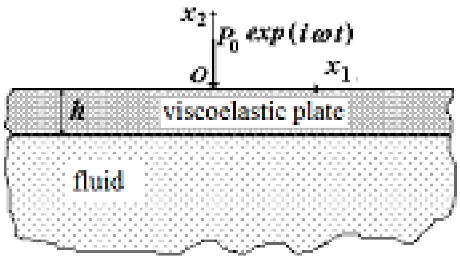



**Fig.1.** The sketch of the hydro-viscoelastic system.

We investigate the forced vibration of the foregoing hydro-viscoelastic system. For this purpose, first we write the governing field equations for the plate motion.

Equation of motion:

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{21}}{\partial x_2} = \rho \frac{\partial^2 u_1}{\partial t^2}, \frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = \rho \frac{\partial^2 u_2}{\partial t^2}, \tag{2.1}$$

Constitutive relations:

$$\sigma_{11} = \lambda^* \varepsilon + 2\mu^* \varepsilon_{11}, \sigma_{22} = \lambda^* \varepsilon + 2\mu^* \varepsilon_{22}, \sigma_{12} = 2\mu^* \varepsilon_{12}, \tag{2.2}$$

where $\lambda^*$ and $\mu^*$ are the following operators:

$$\left\{ \begin{matrix} \lambda^* \\ \mu^* \end{matrix} \right\} \eta(t) = \left\{ \begin{matrix} \lambda_0 \\ \mu_0 \end{matrix} \right\} \eta(t) + \int_0^t \left\{ \begin{matrix} \lambda_1 \\ \mu_1 \end{matrix} \right\} (t-\tau)\eta(\tau)d\tau. \tag{2.3}$$

In Eq. (2.3), $\lambda_0$ and $\mu_0$are the instantaneous values of Lame's constants at $t = 0$, $\lambda_1(t)$ and $\mu_1(t)$ are the corresponding kernel functions for describing the hereditary – viscoelastic properties of the plate material.

Strain-displacement relations:

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}, \varepsilon_{22} = \frac{\partial u_2}{\partial x_1}, \varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_1}{\partial x_2}\right), \varepsilon = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} \tag{2.4}$$

The equations (2.1) – (2.4) are the complete system of equations of the theory of viscoelasticity for isotropic bodies and the notation used in these equations is conventional.

According to Guz (2009), we consider the governing field equations of motion of the Newtonian compressible viscous fluid: the density, viscosity constants and pressure of which are denoted by the upper index (2.1). Thus, we write the equation of motion and other field equations for the fluid.

Linearized Navier-Stokes equations:

$$\rho_0^{(1)} \frac{\partial v_1}{\partial t} - \mu^{(1)} \Delta v_1 + \frac{\partial p^{(1)}}{\partial x_1} - (\lambda^{(1)} + \mu^{(1)}) \frac{\partial e}{\partial x_1} = 0,$$

$$\rho_0^{(1)} \frac{\partial v_2}{\partial t} - \mu^{(1)} \Delta v_2 + \frac{\partial p^{(1)}}{\partial x_2} - (\lambda^{(1)} + \mu^{(1)}) \frac{\partial e}{\partial x_2} = 0, \tag{2.5}$$

Linearized equation of continuity:

$$\frac{\partial \rho^{(1)}}{\partial t} + \rho_0^{(1)} \left( \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2} \right) = 0, \tag{2.6}$$

Constitutive relations:

$$T_{11} = (-p^{(1)} + \lambda^{(1)} e) + 2\mu^{(1)} e_{11}, T_{22} = (-p^{(1)} + \lambda^{(1)} e) + 2\mu^{(1)} e_{22}, T_{12} = 2\mu^{(1)} e_{12}, \tag{2.7}$$

Deformation rate and velocity relations:

$$e_{11} = \frac{\partial v_1}{\partial x_1}, e_{22} = \frac{\partial v_2}{\partial x_2}, e_{12} = \frac{1}{2} \left( \frac{\partial v_1}{\partial x_2} + \frac{\partial v_2}{\partial x_1} \right), e = \frac{\partial v_1}{\partial x_1} + \frac{\partial v_2}{\partial x_2}, \tag{2.8}$$

State equation

$$a_0^2 = \frac{\partial p^{(1)}}{\partial \rho^{(1)}} \tag{2.9}$$

In the equations (2.5) and (2.6),$\rho_0^{(2.1)}$ is the fluid density before perturbation and

$$\Delta = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \tag{2.10}$$

The other notation in equations (2.5) - (2.9) is conventional.

According to Guz (2009), the solution of the system equations (2.5)-(2.10) is reduced to finding the two potentials $\varphi$ and $\psi$ which are determined from the following equations:

$$\left[ \left( 1 + \frac{\lambda^{(1)} + 2\mu^{(1)}}{a_0^2 \rho_0^{(1)}} \right) \Delta - \frac{1}{a_0^2} \frac{\partial^2}{\partial t^2} \right] \varphi = 0, \left( \nu^{(1)} \Delta - \frac{\partial}{\partial t} \right) \psi = 0, \tag{2.11}$$

where $\nu^{(2.1)}$ is the kinematic viscosity, i.e. $\nu^{(2.1)} = \mu^{(2.1)} \Big/ \rho_0^{(2.1)}$.

The velocities $v_1$, $v_2$ and the pressure $p^{(2.1)}$ are expressed by the potentials $\varphi$ and $\psi$ through the following expressions:

$$v_1 = \frac{\partial \varphi}{\partial x_1} + \frac{\partial \psi}{\partial x_2}, v_2 = \frac{\partial \varphi}{\partial x_2} - \frac{\partial \psi}{\partial x_1}, p^{(1)} = \rho_0^{(1)} \left( \frac{\lambda^{(1)} + 2\mu^{(1)}}{\rho_0^{(1)}} \Delta - \frac{\partial}{\partial t} \right) \varphi. \tag{2.12}$$

Assuming that $p^{(2.1)} = -(T_{11} + T_{22} + T_{33})/3$, we obtain:

$$\lambda^{(1)} = -\frac{2}{3} \mu^{(1)}. \tag{2.13}$$

It is also assumed that

$$|v_i| < const. \ |\partial v_i / \partial x_j| < const, \ i; j = 1, 2 \text{ as } x_2 \to -\infty \tag{2.14}$$

and there are no reflected waves from $x_2 = -\infty$.

Moreover, the following boundary and contact conditions are satisfied:

$$\sigma_{21}|_{x_2=0} = 0, \ \sigma_{22}|_{x_2=0} = -P_0 e^{i\omega t}, \ \left. \frac{\partial u_1}{\partial t} \right|_{x_2=-h} = v_1|_{x_2=-h},$$

$$\left. \frac{\partial u_2}{\partial t} \right|_{x_2=-h} = v_2|_{x_2=-h}, \ \sigma_{21}|_{x_2=-h} = T_{21}|_{x_2=-h}, \ \sigma_{22}|_{x_2=-h} = T_{22}|_{x_2=-h}. \tag{2.15}$$

We recall that the main goal of the present investigations is to determine how the viscosity of the fluid affects the vibration of the system under consideration. To achieve this goal we must compare the results obtained within the scope of the foregoing equations and relations for the viscous fluid with the corresponding ones obtained within the scope of the equations and relations for the compressible inviscid fluid. The latter equations and relations are obtained from the equations (2.5) - (2.14) by substituting into these equations $\lambda^{(2.1)} = \mu^{(2.1)} = 0$. Consequently, for the inviscid fluid the potential $\psi$ in (2.11) and (2.12) disappears, i.e. for the inviscid fluid it must be taken that $\psi \equiv 0$, the equations (2.6), (2.8) and (2.9) remain as they are, but the equations (2.5), (2.7), (2.11) and (2.12) are transformed into the equations (2.16), (2.17), (2.18) and (2.19), respectively, as given below.

$$\rho_0^{(1)} \frac{\partial v_1}{\partial t} + \frac{\partial p^{(1)}}{\partial x_1} = 0, \rho_0^{(1)} \frac{\partial v_2}{\partial t} + \frac{\partial p^{(1)}}{\partial x_2} = 0, \tag{2.16}$$

$$T_{11} = T_{22} = -p^{(1)}, T_{12} = 0, \tag{2.17}$$

$$\left[ \varDelta - \frac{1}{a_0^2} \frac{\partial^2}{\partial t^2} \right] \varphi = 0, \tag{2.18}$$

$$v_1 = \frac{\partial \varphi}{\partial x_1}, v_2 = \frac{\partial \varphi}{\partial x_2}, p^{(1)} = -\rho_0^{(1)} \frac{\partial \varphi}{\partial t}. \tag{2.19}$$

For the inviscid fluid, the decay conditions in (2.14) also remain as they are. Moreover, for the inviscid fluid the condition $\partial u_1 / \partial t|_{x_2=-h} = v_1|_{x_2=-h}$ in (2.15) disappears and, according to (2.17), the condition $\sigma_{21}|_{x_2=-h} = T_{21}|_{x_2=-h}$ in (2.15) is replaced with the condition $\sigma_{21}|_{x_2=-h} = 0$.

This completes the formulation of the problem.

## 3 Method of solution

We represent the displacements and the components of the strain tensor related to the plate, and the velocities and components of the strain rate tensor related to the fluid as follows.

$$u_k(x_1, x_2, t) = \bar{u}_k(x_1, x_2) e^{i\omega t}, ..., v_k(x_1, x_2, t) = \bar{v}_k(x_1, x_2) e^{i\omega t}, ...,$$
$$e(x_1, x_2, t) = \bar{e}(x_1, x_2) e^{i\omega t}, k; n = 1, 2. \tag{3.1}$$

Below the over-bar on the amplitudes in (3.1) will be omitted.

Using the presentation in (3.1) and the dynamic correspondence principle(Fung, 1965)we obtain the following constitutive relations for the amplitudes for the quantities related to the plate.

$$\sigma_{kn} = \left( \varLambda(\omega) \varepsilon(x_1, x_2) \delta_k^n + 2M(\omega) \varepsilon_{kn}(x_1, x_2) \right) e^{i\omega t}, \tag{3.2}$$

where

$$\varLambda(\omega) = \lambda_0 + \lambda_{1c}(\omega) - i\lambda_{1s}(\omega), M(\omega) = \mu_0 + \mu_{1c}(\omega) - i\mu_{1s}(\omega),$$
$$\lambda_{1c}(\omega) = \int_0^\infty \lambda_1(s) \cos(\omega s) ds, \lambda_{1s}(\omega) = \int_0^\infty \lambda_1(s) \sin(\omega s) ds,$$
$$\mu_{1c}(\omega) = \int_0^\infty \mu_1(s) \cos(\omega s) ds, \mu_{1s}(\omega) = \int_0^\infty \mu_1(s) \sin(\omega s) ds. \tag{3.3}$$

We substitute the expressions in (3.1) and (3.2)into the corresponding equations and relations, and replace the derivatives $\partial(\cdot) / \partial t$ and $\partial^2(\cdot) / \partial t^2$with $i\omega(\bar{(\cdot)})$ and $-\omega^2(\bar{(\cdot)})$, respectivelywe obtain the corresponding equations, boundary and contact conditions for theamplitudes. For solution to these equations we employ the Fourier transformation to these equations with respect to the $x_1$ coordinate and taking the problem symmetry with respect to $x_1 = 0$ into account, the amplitudes of the sought values can be represented as follows.

$$\{T_{12}, \sigma_{12}, v_1, u_1, \psi\}(x_1, x_2) = \frac{1}{\pi}\int_0^\infty \{T_{12F}, \sigma_{12F}, v_{1F}, u_{1F}, \psi_F\}(s, x_2)\sin(sx_1)ds,$$

$$\{T_{22}, T_{11}, \sigma_{22}, \sigma_{11}, v_2, u_2, \varphi\}(x_1, x_2) =$$

$$\frac{1}{\pi}\int_0^\infty \{T_{22F}, T_{11F}, \sigma_{22F}, \sigma_{11F}, v_{2F}, u_{2F}, \varphi_F\}(s, x_2)\cos(sx_1)ds \tag{3.4}$$

The equations in terms of the Fourier transformation of the displacements $u_{1F}$ and $u_{2F}$are obtainedas follows

$$Au_{1F} - B\frac{du_{2F}}{dx_2} + C\frac{d^2u_{1F}}{dx_2^2} = 0, Du_{2F} + B\frac{du_{1F}}{dx_2} + G\frac{d^2u_{2F}}{dx_2^2} = 0, \tag{3.5}$$

where

$$A = X^2 - s^2(\Lambda(\omega) + 2M(\omega)), B = s(\Lambda(\omega) + M(\omega)), C = M(\omega),$$

$$D = X^2 - s^2M(\omega), G = \Lambda(\omega) + 2M(\omega), X^2 = \omega^2h^2/c_2^2, c_2 = \sqrt{\mu_0/\rho}. \tag{3.6}$$

Introducing the notation

$$A_0 = \frac{AG + B^2 + CD}{CG}, B_0 = \frac{BD}{CG}, k_1$$

$$= \sqrt{-\frac{A_0}{2} + \sqrt{\frac{A_0^2}{4} - B_0}}, k_2 = \sqrt{-\frac{A_0}{2} - \sqrt{\frac{A_0^2}{4} - B_0}}, \tag{3.7}$$

we can write the solution of the equation (3.5) as follows.

$$u_{2F} = Z_1e^{k_1x_2} + Z_2e^{-k_1x_2} + Z_3e^{k_2x_2} + Z_4e^{-k_2x_2},$$

$$u_{1F} = Z_1a_1e^{k_1x_2} + Z_2a_2e^{-k_1x_2} + Z_3a_3e^{k_2x_2} + Z_4a_4e^{-k_2x_2}, \tag{3.8}$$

where

$$a_1 = \frac{-D - Gk_1^2}{Bk_1^2}, a_2 = -a_1, a_3 = \frac{-D - Gk_2^2}{Bk_2^2}, a_4 = -a_3. \tag{3.9}$$

Using the equations (3.8), (3.4) and (3.2), we also write expressions for the Fourier transformations $\sigma_{12F}$and $\sigma_{22F}$ of the corresponding stresses which enter the boundary and contact conditions in (2.15):

$$\sigma_{12F} = Z_1M(\omega)(k_1a_1 - s)e^{k_1x_2} + Z_2M(\omega)(-k_1a_2 - s)e^{-k_1x_2}$$

$$+Z_2M(\omega)(k_2a_3 - s)e^{k_2x_2} + Z_4M(\omega)(-k_2a_2 - s)e^{-k_4x_2},$$

$$\sigma_{22F} = Z_1(s\Lambda(\omega)a_1 + k_1(\Lambda(\omega) + 2M(\omega)))e^{k_1x_2}$$

$$+Z_2(s\Lambda(\omega)a_2 - k_1(\Lambda(\omega) + 2M(\omega)))e^{-k_1x_2}$$

$$+Z_3(s\Lambda(\omega)a_3 + k_2(\Lambda(\omega) + 2M(\omega)))e^{k_2x_2}$$

$$+Z_4(s\Lambda(\omega)a_4 - k_2(\Lambda(\omega) + 2M(\omega)))e^{-k_2x_2}. \tag{3.10}$$

This completes consideration of the determination of the Fourier transformation of the values related to the plate-layer.

Now we consider the determination of the Fourier transformations of the quantities related to the fluid flow. First, we consider the determination of $\varphi_F$ and $\psi_F$from the Fourier transformation of the equations in (2.11), which takes the relations (2.13) and

$$\varphi_F = \omega h^2 \tilde{\varphi}_F, \psi_F = \omega h^2 \tilde{\psi}_F \tag{3.11}$$

into account and can be written as follows

$$\frac{d^2 \tilde{\varphi}_F}{dx_2^2} + \left( \frac{\Omega_1^2}{1 + i4\Omega_1^2/(3N_w^2)} - s^2 \right) \tilde{\varphi}_F = 0, \frac{d^2 \tilde{\psi}_F}{dx_2^2} - \left(s^2 + iN_w^2\right)_F \tilde{\psi}_F = 0, \tag{3.12}$$

where

$$\Omega_1 = \frac{\omega h}{a_0}, N_w^2 = \frac{\omega h^2}{\nu^{(1)}}. \tag{3.13}$$

The dimensionless number $N_w$ in (3.13) can be taken as a Womersley number and characterizes the influence of the fluid viscosity on the mechanical behavior of the system under consideration. The dimensionless frequency $\Omega_1$ in (3.13) can be taken as the parameter which characterizes the compressibility of the fluid on the mechanical behavior of the system under consideration.

Thus, taking the condition (2.14) into consideration, the solutions to the equations in (4.2) are found as follows

$$\tilde{\varphi}_F = Z_5 e^{\delta_1 x_2}, \tilde{\psi}_F = Z_6 e^{\gamma_1 x_2}. \tag{3.14}$$

where

$$\delta_1 = \sqrt{s^2 - \frac{\Omega_1^2}{1 + i4\Omega_1^2/\left(3N_w^2\right)}}, \gamma_1 = \sqrt{s^2 + iN_w^2}. \tag{3.15}$$

Using (3.14) and (3.15) we obtain the following expressions for the velocities, pressures and stresses of the fluid from the Fourier transformations of the equations (2.5) – (2.12).

$$v_{1F} = \omega h \left[ -Z_5 s e^{\delta_1 x_2} + Z_6 e^{\gamma_1 x_2} \right], v_{2F} = \omega h \left[ Z_5 \delta_1 e^{\delta_1 x_2} - Z_6 s e^{\gamma_1 x_2} \right],$$

$$T_{22F} = \mu^{(1)} \omega \left[ Z_5 \left( \frac{4}{3}\delta_1^2 + \frac{2}{3}s^2 - R_0 \right) e^{\delta_1 x_2} + Z_6 \left( -s\gamma_1 - \frac{2}{3}s\gamma_1 \right) e^{\gamma_1 x_2} \right],$$

$$T_{21F} = \mu^{(1)} \omega \left[ Z_5 s \delta_1 e^{\delta_1 x_2} + Z_6 \left(s^2 + \gamma_1^2\right) e^{\gamma_1 x_2} \right], p_F^{(1)} = \mu^{(1)} \omega R_0 Z_5 e^{\delta_1 x_2}, \tag{3.16}$$

where

$$R_0 = -\frac{4}{3} \frac{\Omega_1^2}{1 + i4\Omega_1^2/(3N_w^2)} - i * N_w^2 \tag{3.17}$$

Substituting expressions (3.8), (3.10) and (3.16) into the boundary and contact conditions in (2.15) we obtain a system of equations with respect to the unknowns $Z_1, Z_2, \ldots, Z_6$through which the sought values are determined.

This completes the consideration of the solution method.

## 4 Numerical results and discussions

Again, using the over-bar notation for the amplitudes of the south values we can represent the stresses, displacements and velocities as follows.

$$\{T_{11}, \sigma_{11}, .., v_2\}(x_1, x_2, t) = Re\left\{e^{i\omega t}\left\{\bar{T}_{11}, \bar{\sigma}_{11}, .., \bar{v}_2\right\}(x_1, x_2)\right\}, \quad (4.1)$$

where $\bar{T}_{11}, \bar{\sigma}_{11}, .., \bar{v}_2$ are calculated through the integrals given in (3.4). We recall that the over-bar is omitted in (3.4).

Under calculation of the improper integrals
$\int_0^\infty f(s)\cos(sx_1)ds$ and $\int_0^\infty f(s)\sin((s)x_1)ds_1$
which enter into foregoing expressions are replaced by the corresponding definite integrals
$\int_0^{S_1^*} f(s)\cos(sx_1)ds$ and $\int_0^{S_1^*} f(s)\sin((s)x_1)ds_1$,
respectively. The values of $S_1^*$ are determined from the convergence requirement of the numerical results. Note that under calculation of the latter integrals, the integration intervals are further divided into a certain number of shorter intervals, which are used in the Gauss integration algorithm. The values of the integrated expressions at the sample points are calculated through the equations (3.10), (3.16) and (2.15). All these procedures are performed automatically with the PC programs constructed by the authors in MATLAB.

Assuming that the volumetric expansion-compressionof the plate material is purely elastic, i.e.$(\lambda * + \frac{2}{3}\mu *)\varphi(t) = (\lambda_0 + \frac{2}{3}\mu_0)\varphi(t)$,according to which the relation, the viscoelasticity of this material can be expressed only with the

$$\mu * \varphi(t) = \mu_0\left[\varphi(t) - \frac{3\beta_0}{2(1+\nu_0)}\Pi_\alpha^*\left(-\frac{3\beta_0}{2(1+\nu_0)} - \beta_\infty\right)\varphi(t)\right], \quad (4.2)$$

operator, where(Rabotnov 1980)

$$\Pi_\alpha^*(x)\varphi(t) = \int_0^\infty \Pi_\alpha(x, t-\tau)\varphi(\tau)d\tau, \Pi_\alpha(x,t)$$
$$= t^{-\alpha}\sum_{p=0}^{\infty}\frac{(x)^p t^{p(1-\alpha))}}{\Gamma((1+\mathrm{p})(1-\alpha))}, 0 \le \alpha < 1. \quad (4.3)$$

In (4.3) $\Gamma(x)$is the gamma function. Moreover, the constants$\alpha$, $\beta_0$and $\beta_\infty$ in (4.2) are the rheologicalparameters of the plate material.

According to Rabotnov (1980), it is obtained that

$$\mu_c = \mu_0\left[1 - \frac{3}{2(1+\nu_0)}(d+\beta_{01})^{-1}\Pi_{\alpha c}(-\beta_{01} - \beta_\infty, \omega)\right],$$
$$\mu_c = -\mu_0\frac{3}{2(1+\nu_0)}(d+\beta_{01})^{-1}\Pi_{\alpha s}(-\beta_{01} - \beta_\infty, \omega), \quad (4.4)$$

where

$$\Pi_{\alpha c}(-\beta_{01} - \beta_\infty, \omega) = \frac{\xi^2 + \xi\sin\frac{\pi\alpha}{2}}{\xi^2 + 2\xi\sin\frac{\pi\alpha}{2} + 1},$$
$$\Pi_{\alpha s}(-\beta_{01} - \beta_\infty, \omega) = \frac{\xi\cos\frac{\pi\alpha}{2}}{\xi^2 + 2\xi\sin\frac{\pi\alpha}{2} + 1}, \beta_{01} = \frac{3}{2(1+\nu_0)}. \quad (4.5)$$

In (4.5) the following notation is used

$$\xi = (Q\Omega)^{\alpha-1}, Q = \frac{c_{20}}{h(\beta_{01} + \beta_\infty)^{\frac{1}{1-\alpha}}}, \Omega = k_1 R\frac{c}{c_2}, c_2 = \sqrt{\mu_0/\rho_0}. \quad (4.6)$$

Moreover, in the works by Akbarov and Ismailov (2014b) and Akbarov (2014) it is established that the long-term value of the shear modulus for the selected operator are determined as follows:

$$\mu_\infty = \lim_{t\to\infty} \mu* = \mu_0 \left(1 - \frac{3}{2(1+\nu_0)} \frac{1}{(3/(2(1-\nu_0)) + d)}\right), \tag{4.7}$$

where the notation

$$d = \frac{\beta_\infty}{\beta_0}. \tag{4.8}$$

is used. The expressions (4.7) and (4.8) show that the constant $d$ characterizes the long-term values of the elastic constants.

Considerable discussions of the mechanical meaning of the parameters $Q$(4.6) and $d$(4.8) are given in the papers by Akbarov (2014) and Akbarov and Ismailov (2014b)where it was established that $Q$ and $d$ can be taken as the characteristic creep time and the parameter characterizing the long term values of the viscoelastic material, respectively.

It follows from the foregoing discussions that the problem under consideration is characterized not only with the dimensionless rheological parameters $Q$ and $d$, but also with the dimensionless parameters $\Omega_{\text{-}1}$, $N_w$ in (3.13). Moreover, it is also introduced the parameter $M_\mu = \mu^{(2.1)}\omega/\mu_0$. Note that the case where $\Omega_{\text{-}1} = 0$corresponds to the case where the fluid is incompressible, but the case where $1/N_w = 0$corresponds to the case where the fluid is inviscid.

In the numerical investigations we assume that the instantaneous values of the Lame constants and the density of the plate material are taken as those related to Lucite, i.e. according to Guz and Makhort (2000) we assume that $\mu_0 = 1.86 \times 10^9 Pa$, $\lambda_0 = 3.96 \times 10^9 Pa$ and the density $\rho_0 = 1160 kg/m^3$, but the material of the fluid is Glycerin with viscosity coefficient $\mu^{(2.1)} = 1,393 kg/(m \cdot s)$, density $\rho = 1260 kg/m^3$and sound speed $a_0 = 1927 m/s$ (Guz, 2009).

Thus, after selection of these materials, the dimensionless parameters $\Omega_{\text{-}1}$, $N_w$ and $M_\mu$ can be determined through the following two quantities: $h$ (the thickness of the plate-layer), and $\omega$ (the frequency of the time-harmonic external forces). The numerical results, which are discussed below, relate to the normal stress $T_{22}$ acting on the interface plane between the fluid and plate and to the velocity $v_2$ of the fluid on the interface plane in the direction of the $Ox_2$ axis.

Making investigations on the convergence of the numerical results it is established that the case where $S_1^* = 5$ and $N* = 100$ (the number of shorter intervals into which is divided the integrated interval $[0, S_1^*]$ ) is quite sufficient for obtaining verified numerical results. Therefore, in the present numerical investigation we assume that $S_1^* = 5$ and $N* = 100$.

Thus, we consider the graphs of the frequency response of the dimensionless stress $T_{22}h/P_0$ given in Fig. 2 which are obtained in the cases where $h = 0.005\,m$, respectively, and graphs of the frequency response of the dimensionless velocity $v_2\mu h/(P_0 c_2)$ given in Fig. 3which are also obtained in the case where $h = 0.005\,m$.

In these figures the graphs grouped by the letter a (b) show the influence of the rheological parameter $Q(d)$ under a fixed value of the rheological parameter $d(= 1)$ $(Q(= 10))$. Under construction of the foregoing graphs it is assumed that $\omega t = 0$ and $x_1/h = 0$. Note that in these figures and in others which will be considered below, the results related to the viscose (solid lines) and inviscid (dashed lines) cases are given simultaneously in order to compare these results and, according to this comparison, to make corresponding conclusions on the influence of the fluid viscosity on the studied frequency responses. Note that here and below under "viscous fluid case" ("inviscid fluid case") it is assumed that the fluid, i.e. Glycerin, which is selected for the present investigations, is modeled through the constitutive relations in (2.7) (through the constitutive relations in (17)).

Thus, it follows from Fig.2 that for all the cases under consideration the absolute values of the stress $T_{22}h/P_0$ increase monotonically with $\omega$. Moreover, it follows from these results that in the inviscid fluid case the absolute values of the stress $T_{22}h/P_0$ decrease with increasing of the rheological parameters $Q$ and $d$. This conclusion occurs for the viscous fluid case with respect to the influence of the parameter $d$ on the values of $T_{22}h/P_0$. However, in the viscous fluid case the character of the influence of the parameter $Q$ on the values of the stress $T_{22}h/P_0$ has complicated (i.e., non-monotonic) character. With all this, the magnitude of the influence of the rheological parameters $Q$ and $d$ on the values of the stress $T_{22}h/P_0$ in the inviscid fluid case is greater significantly than that in the viscous fluid case.

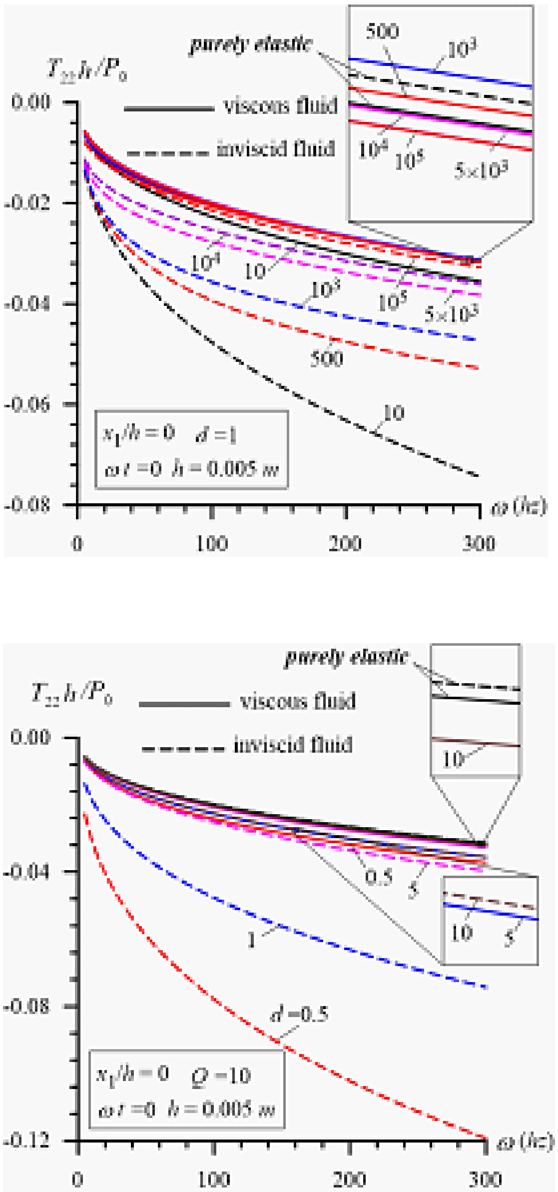
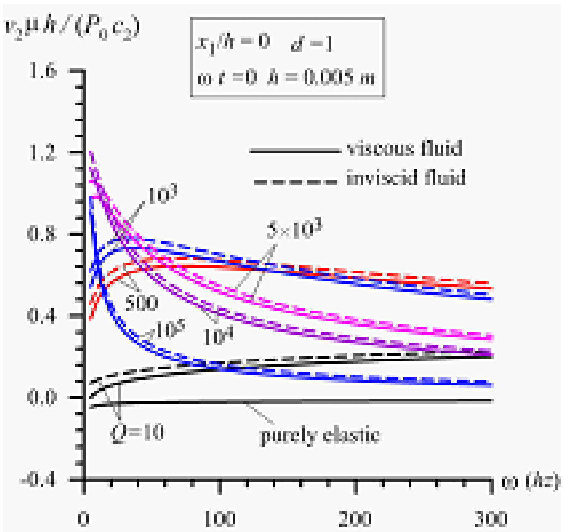



**Fig. 2.** Frequency response of $T_{22}h/P_0$ for various values of the rheological parameters $Q$ (a) and $d$ (b) in the case where $\omega t = 0$ and $h = 0.005\,m$.

Consequently, according to the foregoing results, we can conclude that as a result of the fluid viscosity the absolute values of the stress $T_{22}h/P_0$ in the viscoelastic plate case decrease significantly. The results obtained for the inviscid and viscous fluid cases approach to each other with increasing of the rheological parameters and in the case where the plate material is purely elastic one the absolute values of the $T_{22}h/P_0$ obtained for the inviscid fluid case become less than the corresponding ones obtained for the viscous fluid case.



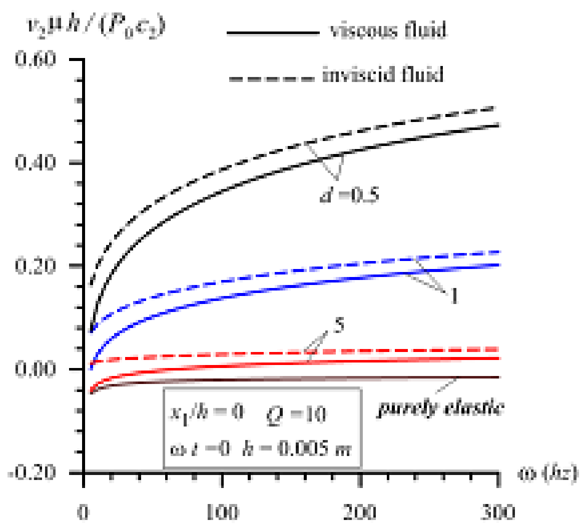




**Fig. 3.** Frequency response of $v_2\mu h/(P_0c_2)$ for various values of the rheological parameters $Q$ (a) and $d$ (b) in the case where $\omega t = 0$ and $h = 0.005\,m$.

Now we consider the graphs given in Fig. 3 which illustrate the influence of the rheological parameters $Q$ and $d$ on the frequency response of the dimensionless velocity $v_2\mu h/(P_0c_2)$. According to Akbarov and Ismailov (2015a), we recall that under inviscid fluid loading of the purely elastic plate and under $\omega t = 0$ it is obtained that $v_2\mu h/(P_0c_2) = 0$ for each $\omega$,

$h$ and $x_1/h$. But under viscous fluid loading of the purely elastic plate, the noted values of $v_2\mu h/(P_0c_2)$ are different from zero. Consequently, Fig. 3 shows that as a result of the viscoelasticity of the plate material, the values of $v_2\mu h/(P_0c_2)$ in the inviscid fluid case become different from zero. Moreover, Fig. 3a shows that the dependence of these values of $v_2\mu h/(P_0c_2)$ on the rheological parameter $Q$ under fixed $d(=1)$, is non-monotonic, i.e. at first, to a certain value of $Q$, an increase in its value causes an increase in the values of $v_2\mu h/(P_0c_2)$. However, after the "certain value" of $Q$, further increases of $Q$ cause the absolute values of $v_2\mu h/(P_0c_2)$ to decrease and these values approach the corresponding values of $v_2\mu h/(P_0c_2)$ obtained for the purely elastic plate case.

It follows from the results illustrated in Fig. 3b that the dependence between the rheological parameter $d$ and $v_2\mu h/(P_0c_2)$ under fixed $Q(=10)$ is monotonic, i.e. a decrease in the values of $d$ causes an increase in the absolute values of $v_2\mu h/(P_0c_2)$.

With this we restrict ourselves to consideration of the numerical results.

## 5 Conclusions

Analysis of the presented numerical results allows us to draw the following concrete conclusions:

1 The influence of the fluid viscosity on the frequency response of the stress and velocity depends significantly of the values of the rheological parameters $Q$ (4.6) and $d$ (4.8);
2 The magnitude of the mentioned influence increases with decreasing of the rheological parameters $Q$ and $d$;
3 The magnitude of the fluid viscosity on the absolute values of the studied quantities in the relatively small values of the rheological parameters $Q$ and $d$ is very significant and it is necessary to this take into consideration under corresponding applications.

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