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Analysis of the physical parameters of the Earth's inner core within the mechanics of the deformable body

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Abstract. *It is shown that the distribution of basic physico-mechanical-elastic properties and parameters of the medium (such as elasticity moduli, density, pressure, etc.) in modern models of the Earth is not consistent with the requirements of mechanics. These distributions are only coordinated with integral criteria relative to the total mass of the Earth, its moments of inertia and the data on its natural oscillations in the existing models. It is suggested in the paper that more fundamental local criteria of mechanics of continuum media concerning strength, stability, and elastic wave propagation in deformable media should also be carried out along with these requirements.*
It is found that under the conditions of the accepted distributions of basic physical parameters of the Earth, the existence of the solid core of the Earth as a sphere becomes questionable, i.e. all of the indicated three criteria of mechanics are not carried out. The pressure level exceeds as the tensile strength of the medium, and the critical values of the equilibrium state of buckling at the surface of the Earth's core on geometric forming and "internal" instabilities. Elastic waves with true velocity can't be propagated in the core too.

Keywords. Earth's core · high pressure · instability · elastic waves with actual velocity

Mathematics Subject Classification (2010): 74L15 · 74J30 · 74L10

1 Introduction

Physical parameters of deformable solid media - such as the elasticity moduli, Poisson's ratio, the velocity of propagation of body elastic waves in mechanics are determined under specific conditions (Lyav, 1935; Sedov, 1970). It is required to comply with the conditions of smallness of uniformly distributed homogeneous deformation $\varepsilon << 1$ and the smallness of the ratio $\frac{P}{\mu}$ (where P is a parameter of loading, in particular, pressure; μ

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are moduli of the medium shift; $\varepsilon$is deformation parameter) in classical linear theory of elasticity of isotropic homogeneous media within the framework of which the indicated parameters are interpreted in all theoretical models of the Earth. The condition of uniform distribution of homogeneous deformation should also be controlled in the process of deformation of specific structures (sphere in the considered case). First of all, it is necessary to achieve simultaneous fulfillment of commonly accepted requirements of the mechanics for the media and constructions in solving the problems on distribution of physical and mechanical properties in the Earth's interior, in particular in the solid core. Specific data (Fig.1b) on violating the requirements of the mechanics are suggested for pressure $P = 329$ GPa and shear modulus $\mu = 157$ GPa at a level of the sphere surface shown in publications (Bullen, 1978; Dziewonski and Anderson, 1981; Anderson, 1995; Anderson, 2007; Litasov and Shatskiy, 2016). It can be seen that the value P exceeds the value μ more than 2 times. According to Avsyuk (1973, 2001), Adushkin et al. (2000), Levin (2001) the sphere of solid core takes part in movements within the liquid outer core due to the rotational motion of the Earth and tidal influences. Pressure value can nonuniformly be increased even more due to the resistance to this movement.

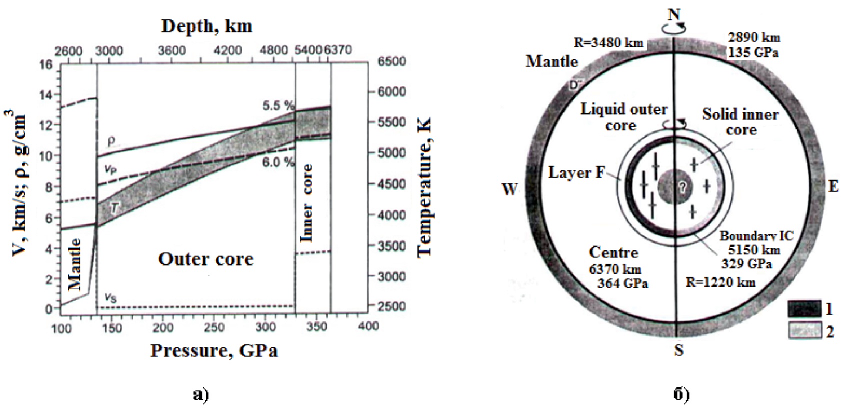

Fig.1.

The conditions of carrying out the requirements concerning uniform distribution of homogeneous deformation are observed in standard laboratory experimental studies of physical and mechanical properties. Methods for conducting experiments, smallness of geometric dimensions of model samples, the actual impossibility of considering the mechanisms of long-term (over geological time) deformation and a number of other reasons don't allow providing possible violations of conditions of the mechanics under conditions of natural occurrence, as well as to exclude from the results of interpretations of influence of uncontrolled perturbations related to the mechanisms of long-term deformability of the structure of the sphere under the conditions of huge value of compression.

2 Problem statement

This paper presents the results of geomechanical analysis of the data of geophysical studies within the non-classical linearized approach (NLA) (Abasov et al., 2000; Guliyev, 2010). At the same time the numerical data PREM (Dziewonski and Anderson, 1981) are used taking into account the fact that the parameters of the inner core provided in this work taken as a basis in all pre-proposed theoretical models of the medium (Bullen, 1978; Kennett and Engdahl, 1991; Morelli and Dziewonski, 1993; Kennett et al, 1995; Anderson, 2007; Pushcharovsky and Pushcharovsky, 2011). There are only minor differences which have not significant meaning for the conducted geomechanical analysis in various models.

The purpose of geomechanical analysis is to determine the conditions for pressure and strain ensure the correctness of the calculations of physical and mechanical properties of the model of solid core of the Earth on the basis of complex of geophysical data. The pressure and strain values should satisfy certain conditions in determining the physical and mechanical parameters of the medium. Only the results of measurements and calculations obtained in compliance with the conditions of uniform distribution of homogeneous strain is considered reliable. This condition may be violated in different situations.

3 Achievements of theoretical limit of strength

Let's consider the case when the medium is evenly and uniformly deformed prior to the beginning of fracture. In this case, all calculations on the physical and mechanical parameters are correct, if the pressure value does not exceed the theoretical limit of strength. It is shown in NLA (Kuliev, 1988a) that the value of the theoretical limit of the strength of the medium is defined as $P = \mu$ under the conditions of compression (we are interested in this variant of deformation) for a perfect elastic isotropic material. Theoretically, it is the maximum (limit) pressure until the achievement of which the medium is deformed evenly and uniformly without fracture. It is determined from the condition of the loss of the ellipticity of motion equations (4.2). In this case, the conditions $\mu > 0;\ \lambda + \frac{2}{3}\mu > 0$ of classical linear theory of elasticity (here λ, μ are Lame's elasticity moduli) are preserved. It should be noted that the value of the ultimate strength on classical theories of strength is even lower (Rabotnov, 1988), which dramatizes the situation even more. Naturally, proof strength is significantly less than ultimate strength.

4 Instability of equilibrium state

The uniform distribution of strain in the medium may also be violated as a result of the instability (in various forms), without fracture.

NLA allows defining the limits of change of strain within the framework of which the equilibrium of uniformly deformed states is stable. In case of violation of the stability conditions, the change of the equilibrium state of initially homogeneous uniform strain occurs. As a result, the strain in the body is unevenly distributed before reaching the limit strength of the material.

The questions of density distribution of the medium depending on the change of the strain were studied (Guliyev and Askerov, 2007; Guliyev, 2010, 2011, 2013). It is shown that, this dependence is not continuous due to the instability of strain under compression. Therefore, the change of the medium density in the deformable body is not monotone, but spasmodic in certain situations.

Let's consider the problem of stability of solid sphere to concretize the discussion. It is necessary to determine the highest values of surface compressive loads in which the equilibrium state of solid sphere remains stable. Previous theoretical studies (Guz, 1979) shows that this load is determined by solving the problem of axisymmetric form of buckling of isotropic homogeneous sphere. Let's assume that the sphere is filled with homogeneous isotropic medium within the continuum approximation. The external compressive load is given on the surface of the sphere.

The questions of stability of equilibrium state of isotropic sphere under the influence of uniform surface loadings were studied in detail (Guz, 1979, 1986 a). Studies were carried out within the framework of a three-dimensional non-classical linearized theory (NLA), sources of which date back to the incremental theory of mechanics of deformable solid body (Biot, 1965). At the present time, three-dimensional NLA is developed greatly and is used to study various problems of mechanics (Guz, 1979; 1986a,b, 1989; Kuliev, 1988b; Akbarov, 2013, 2015).

The two states of deformable body are considered in NLA. The first state (motion, equilibrium, strain process) is primary or nonperturbed. The second condition is perturbed. All values relating to the second condition are presented as the sum of the corresponding values of the first and second state. Perturbations are considered to be small values compared to the corresponding values of the first (nonperturbed) state. Natural (undeformed) state, which corresponds to the case of lack of pressure and strain in the body is also used to describe the strain in the Lagrangian method.

Deformation is taken in the following form under the uniform initial state

$$u_m^0 = (\lambda_m - 1)\, X_m. \tag{4.1}$$

Here u_m are displacement components along the coordinate axis; λ_m are coefficients of elongation (shortening) along the coordinate axis; $\%_m$ are the Cartesian coordinates.

In homogeneous initial state, equation systems of motions take the form within the compressible medium in the Lagrangian coordinates (which coincide with the Cartesian coordinates in the natural state) (Guz, 1986 a,b):

$$\left(\omega_{\mathrm{im}\alpha\beta}\frac{\partial^2}{\partial x_i \partial x_\beta} + \rho\Omega^2\delta_{m\alpha}\right)u_\alpha = 0, i, \beta, \alpha, m = 1,2,3,4; \omega_{im\alpha\beta} = Const. \tag{4.2}$$

Boundary conditions at the surface of the domain S_1 in terms of stress

$$N_i \omega_{ij\alpha\beta}\frac{\partial u_\alpha}{\partial x_\beta} = P_j. \tag{4.3}$$

Here u_α are vector components of disturbance of displacement; P_j is disturbance of surface forces; ρ is the medium density; N_i are components of unit normal vector to the surface of the body in the natural state; $\delta_{m\alpha}$ is the Kronecker symbol; $\omega_{\mathrm{im}\alpha\beta}$ are covariant components of the tensor of the fourth rank characterizing linear, non-linear physical -mechanical properties of the medium and its initial state of stress. In considering the problems of the static, inertial component $\rho\Omega^2\delta_{m\alpha}$ is omitted in the equation (4.2) where Ω is cyclic frequency of harmonic wave.

Various classifications are possible in the formulation of problems of NLT. ”Follower” (non-conservative) and ”dead” (conservative) surface forces are distinguished depending on the nature of the action of surface loads. Surface ”follower” forces are those forces which keep up changes of configurations of body surface in the process of deformation, i.e. they can change the direction of an action and value according to deformation process. An action of liquid and gas is modeled as ”follower” loads in the calculation practice. Surface ”dead” forces retain their original direction and value in the process of deformation. Three various variants of theory are also distinguished in the NLT depending on values of deformation in the initial state (Guz, 1986a) a) theory of large (finite) initial strain; b) the first variant of the theory of small initial strain (shifts and elongation are small in comparison to the unit); c) the second variant of the theory of small initial strain (it is considered that the relationship between the components of the strain tensor and the first derivatives from displacements are linear in addition to the first variant of the theory of small initial strain). Two cases are also distinguished to provide plane harmonic wave. The variation of distance isn’t considered between material particles due to initial strain, and the velocity of wave propagation is called ”natural” in the first variant (Thurston and Brugger, 1964; Guz, 1986b). The variation of distance isn't considered between material particles due to the initial strain, and the velocity of wave propagation is called "true" in the second variant. The formulations of buckling problem are also distinguished for compressible and non-compressible models in the deformable bodies. The problems are considered only for compressible and non-compressible media and case of "true" velocities in the present paper. The generalization of results is of technical nature for other cases.

In the case of the uniform homogeneous deformation of singly connected isotropic media $\lambda_1 = \lambda_2 = \lambda_3$ for all the above-mentioned variants of the theory of the initial strain $\omega_{ij\alpha\beta}$ in a single form (Guz, 1986a)

$$\omega_{ij\alpha\beta} = \lambda_0 \delta_{ij}\delta_{\alpha\beta} + \mu_0 \left(\delta_{i\alpha}\delta_{j\beta} + \delta_{i\beta}\delta_{j\alpha}\right) + S_0 \left(\delta_{ij}\delta_{\alpha\beta} - \delta_{i\alpha}\delta_{j\beta}\right), \tag{4.4}$$

where the designations are respectively introduced for the theory of large initial strain and the first variant of small initial strain theory and the second variant of small initial strain theory

$$\lambda_0 = \lambda_1^2 0_0 - S_0; \mu_0 = \lambda_1^2 b_0 + S_0; \tag{4.5}$$

$$\lambda_0 = \lambda_1^2 0_0 - S_0; \mu_0 = \lambda_1^2 b_0 + S_0; S_0 = \sigma_0; \tag{4.6}$$

$$\lambda_0 = 0_0 - S_0; \mu_0 = b_0 + S_0; S_0 = \sigma_0. \tag{4.7}$$

Values 0_0, b_0, S_0 and $\sigma_0$in terms of $\lambda_1 = \lambda_2 = \lambda_3$ are determined from expressions

$$a_0 = A_{\beta i} - 2\mu_{ij}; b_0 = \mu_{ij}; S_0 = S^0_{\beta\beta}; \sigma_0 = \sigma^0_{\beta\beta}.$$

The summation isn't conducted on indices in these formulae; $\sigma^0_{\beta\beta}$ are normal components of the stress tensor in the initial state.

Explicit algebraic expressions for $A_{\beta i}, \mu_{ij}$ and $S^0_{\beta\beta}$ are obtained in considering the concrete elastic potentials (Guz, 1986a).

Considering (4.4) the equation (4.2) and condition (4.3) take the form

$$(\lambda_0 + 2\mu_0)\text{grad div } u - \mu_0 \text{rot rot} u + \rho \Omega^2 u = 0 \tag{4.8}$$

$$[N(\lambda_0 + S_0 divu + (2\mu_0 - S_0)N \cdot \nabla \boldsymbol{u} + (\mu_0 + S_0)N \times rotu] = P. \tag{4.9}$$

In setting "follower" load at the surface the right side of the condition (4.9) takes the form:

$$P = S_0(Ndivu - N\nabla u - N \times rotu) \tag{4.10}$$

Equation (4.8) completely coincides with Lame's equation of classical linear theory of elasticity, if replace Lame's parameters λ and μ to the parameters λ_0 and μ_0 according to (4.5) - (4.7).

It follows from the structure (4.9) and (4.10) that in general such an analogy is absent in the linear theory under the boundary conditions. The analogy holds only in the case of "follower" loads.

Thus, the mathematical problem of stability of an isotropic sphere under uniform compression is formulated in the form of equation (4.8) and the boundary condition (4.9). It is necessary to take $\equiv 0$ in the case of setting the external load on the surface of the sphere in the form of "dead" loads in the right side of the boundary conditions (4.9).

In such formulation, the problem of stability of the equilibrium state of the body of an arbitrary geometrical shape from the compressible media was studied in detail under uniform compression (Guz, 1979, 1986a). It is shown that in case of setting "follower" loads on the whole body surface, state of equilibrium defined by the expression (4.1) is stable under the fulfillment of conditions

$$\lambda_0 + \frac{2}{3}\mu_0 > 0; \mu_0 > 0. \tag{4.11}$$

Conditions (4.11) should always be fulfilled, and therefore, they are considered as the restriction on the structure of the equation of state. It is considered as specific models of the medium a) elastic isotropic body with potential of harmonic type within the theory of large initial strain and stability conditions are obtained in the form:

$$0 < \lambda_1 < 1; \left(\lambda + \frac{2}{3}\mu\right)\left(\lambda + \frac{4}{3}\mu\right)^{-1} < \lambda_1 < 1. \tag{4.12}$$

b) an elastic body with a quadratic potential within the second variant of small initial strain theory and stability condition is obtained in the form:

$$(2 - \lambda_1)\left(\lambda + \frac{2}{3}\mu\right) > 0; \mu + 3(\lambda_1 - 1)\left(\lambda + \frac{2}{3}\mu\right) > 0. \tag{4.13}$$

c) elasto-plastic body (deformation theory) within the second variant of small initial strain theory and stability condition is obtained in the form:

$$P < \mu. \tag{4.14}$$

d) elasto-plastic body (Prandtl- Reuss theory of plasticity) within the second variant of small initial strain theory

$$P < \lambda_1^2\mu. \tag{4.15}$$

It is shown for all the considered models of the medium that equilibrium state is stable in case of setting follower loads on the surface of an isotropic sphere under the fulfillment of conditions (4.11)-(4.15). Herein, the distribution of homogeneous deformation is uniform.

Considering the body in the form of a sphere (medium material is given as: quadratic elastic potential, deformation theory of small elasto-plastic deformation and Prandtl-Reuss theory of plasticity; hereditary-elastic linear body of ageless type; viscous elasto-plastic body), it is shown that in case of "dead" surface loads, there is a critical load P_{kp} (according to the value this load is less than the value μ) in reaching of which the equilibrium state of the sphere defined by the expression (4.1) is unstable. As a result, the distribution becomes uniform in the body of homogeneous deformation. Similar results have also been obtained within the theory of large initial strain using various elastic potentials.

In this case, in general terms it is impossible having taken the inequality for λ_0, μ_0 and S_0, and so that it is ensured the fulfillment of the condition (4.11) regardless of the body shape. Therefore, the following standard equation (Guz, 1979, 1986a) is obtained to define low values of the critical load in the considered problem providing general homogeneous solutions of the equation (4.8) similar to the classical theory of elasticity and requiring the fulfillment of the boundary conditions (4.9) (it is necessary to take $P \equiv 0$ in the right side)

$$2\mu_0(\lambda_0 + \mu_0) + S_0(\lambda_0 + 3\mu_0) = 0. \tag{4.16}$$

Critical forces or strain leading to the buckling of the equilibrium state (4.1) of the sphere are calculated using the formulae (4.5) - (4.7) from equation (4.16).

We obtain within the second variant of small initial strain theory for elastic isotropic body from (4.16) considering (4.7)

$$P_{kp} = \frac{\mu}{4(1 - 2\nu)}\left(5 - 4\nu - \left(16\nu^2 - 8\nu + 9\right)^{\frac{1}{2}}\right), \tag{4.17}$$

where ν is Poisson's coefficient of the medium.

In case of large initial strain theory and application of harmonic elastic potential using (4.5) from the equation (4.16) we define the critical value of shortening as follows

$$(\lambda_1)_* = 1 + \frac{-5+\nu\left(3+2\nu\right)+\left(\left[5-\nu\left(3+2\nu\right)\right]^2-4\left(1-2\nu\right)\left[4-\nu\left(1+2\nu\right)\right]\right)^{\frac{1}{2}}}{8-2\nu\left(1+2\nu\right)} \tag{4.18}$$

or

$$(\lambda_1)_* = \frac{(3-2\nu)(1+\nu)}{(3-2\nu)(1+\nu)+(1-2\nu)}.$$

Similarly, we can obtain the calculation formulae for the case of quadratic, Murnaghan and other forms of elastic potentials. Formulae (4.17) and (4.18) indicate that the buckling of the equilibrium state is implemented for both small and large deformations and is general in nature.

4.1. Internal instability

Critical values of stress and strain leading to violation of conditions (4.11) cause the phenomenon in the body, which is called the ”internal” instability in theory (Biot, 1965; Guz, 1986a,b). In the case of initially isotropic media, as if the initial pressure plays the role of an internal structure similar to the internal structure of the composite media in the anisotropic approximation within the phenomenological (continuum) approach.

”Internal” instability is studied for an infinite body in the continuum description of materials when a certain load is given on ”infinity”. At the same time the instability is not related to the influence of boundary conditions and geometrical dimensions of the body or structural elements. The critical values of the stress and strain are determined from the study of system types of differential equations (4.2), (4.8) in an infinite domain. The equation system (4.2) loses the property of ellipticity under the conditions of occurrence of the phenomenon of ”internal” instability. In this case, the condition of uniqueness of the solution (4.11) of the linearized problems is violated. The limit value of coefficient of elongation (shortening) is determined from (4.11) λ_1^* by setting the structure of the elastic potential. In the case of the modeling of the deformation process using harmonic elastic potential within the theory of large initial strain (4.5) and (4.11) we get

$$\lambda_1^* = \frac{1+\nu}{2-\nu}, \varepsilon_0^* = \frac{3}{2}\frac{2\nu-1}{(2-\nu)^2}. \tag{4.19}$$

We obtain in case of quadratic elastic potential from (4.11) and (4.12) within large initial strain theory

$$\lambda_1^* = \left(\frac{1+\nu}{2-\nu}\right)^{\frac{1}{2}}, \varepsilon_0^* = \frac{1}{2}\frac{2\nu-1}{2-\nu}. \tag{4.20}$$

We obtain in case of linear elastic isotropic material within the second variant of small initial strain theory

$$P_{kp} = \mu,\ \varepsilon_0^* = \frac{1}{2}\frac{2\nu-1}{1+\nu}. \tag{4.21}$$

ε_0 is a parameter of uniform deformation in the formulae (4.19) - (4.21). It follows from the above mentioned formulae (4.11) - (4.15) and (4.19) - (4.21) that the ”internal” instability occurs within the NLA in uniform deformation (compression) of the isotropic sphere on the level of pressure comparable in value with shear moduli for different elastic potentials obtained within the second variant of small and large initial strain theory.

5 Elastic wave propagation in the deformed medium

The implementation of condition (4.11) also provides validity (not negative values) velocities of propagation of small perturbations (such as the Hadamard conditions (Truesdell, 1975; Guz, 1986b)) in the form of small-amplitude waves in media with initial deformations.

Consequently, equality to zero or invalidity of velocities of propagation of acoustic waves correspond to the phenomenon of "internal" instability of the stressed media.

In case of uniform pre-compression of isotropic medium, the "true" velocities of propagation of elastic waves in it are defined by the expressions (Guz, 1986b)

$$\rho C_l^2 = \lambda + 2\mu - PK_p^R; \; \rho C_S^2 = \mu - PK_S^R. \tag{5.1}$$

Here C_l, C_S are the "true" velocities of quasi-pressure and quasi-shear elastic waves; K_p^R, K_S^R are coefficients of nonlinear action of isotropic medium (Sadovsky and Nikolaev, 1982; Guliyev, 2009). Structures of expressions for K_p^R and K_S^R are concretized by assignment the form of elastic potentials.

We derive conditions under the implementation of which the velocities of propagation of elastic waves are true in pre uniformly strained isotropic medium using the formula (5.1) and work results (Guliyev, 2009). Accordingly, pressure elastic wave couldn't be propagated with true velocity in cases of the second variant of small and large initial strain theory in the quadratic elastic potential in terms of implementation

$$\frac{P}{\mu} \geq \frac{2\left(1-\nu^2\right)}{(1-2\nu)(3-\nu)}; \; \frac{P}{\mu} \geq \frac{2\left(1-\nu^2\right)}{(1-2\nu)(5-3\nu)} \tag{5.2}$$

in the stressed isotropic medium. This condition for shear elastic waves takes the form

$$\frac{P}{\mu} \geq \frac{1+\nu}{2-\nu}; \; \frac{P}{\mu} \geq \frac{1+\nu}{3(1-\nu)}. \tag{5.3}$$

It is also necessary to have numerical information on the elasticity moduli of the 3rd order along with the data of Lame coefficients λ and μ to obtain the numerical estimation in case of using Murnaghan potential.

6 Conclusions

Based on the results obtained in the previous sections (formulae (11) - (4.21) for the theoretical limit of strength and instability of the equilibrium state and formulae (5.1) - (5.3) for propagation of elastic waves in the deformable media), the appropriate calculations are performed. Numerical values of critical forces and elongations are shown in Table 1 corresponding to the buckling of the equilibrium state (4.1) in setting of "dead" forces and "internal" instability on the surface of the sphere. The results for $\frac{P_*}{\mu}$ are calculated on formula (4.17), for $(\lambda_1)_*$ on (4.18), and λ_1^* on (4.19). They show that the equilibrium state of the sphere is unstable both within the theory of small and large initial strain in the considered type of loading. The critical values of forces and coefficient of elongation (shortening) in obtaining of which "internal" instability is respectively implemented under small and large initial strain in the sphere are shown (lines of 2 and 4 of Table 1). It follows from the comparison of results of the second and fourth lines to the results of the third and fifth lines of Table 1 that the buckling of the equilibrium state of elastic homogeneous isotropic sphere on a geometric forming in case of influence of "dead" loads on its surface precedes the "internal" instability. The equilibrium state of the sphere is stable on geometric forming in case of influence of ”follower” loads on the surface. Therefore, the ”internal” instability occurs without preliminary forming in this case. It should be emphasized that it is clear from the formulae of critical forces and elongation that they don’t depend on the geometric parameters of the sphere and buckling mode. An exhaustive explanation is given to this case (Guz, 1986a). The boundary surface is one of the coordinate surfaces of the spherical system of coordinates in the considered problems. Eigen-values should not depend on the geometric parameters of the problem due to the nature of Lame’s equations (4.8) (which includes derivatives of the same order) and the indicated case. The critical loads will depend on the geometric parameters (for example, thin-walled parameters) in case of considering the problems of stability of bodies bounded by several coordinate surfaces. The lack of effects of plastic and viscous properties of the material (formulae (14) and (4.15)) on the value of the critical parameters is related with the fact that inelastic deformation is incompressible due to the adopted laws of state, and inelastic deformation does not occur due to uniform compression in the initial state.

Calculation results implemented on formulae (5.2)-(5.3) are shown in Table 2. The data relating to the second variant of small initial strain theory is given in the numerator but in denominations - large initial strain theory.

The numbers given in lines 4 and 5 of the table show that if these values are exceeded, the conditions (5.2) and (5.3) aren’t fulfilled within the considered variants of NLT, i.e. elastic pressure and shear waves can’t be propagated in the medium with true velocity accordingly. The subscript in $P\ l$ – indicates that these values relate to pressure and S to shear waves. Contrary to that it follows from the data of Fig. 1a, b that velocities of pressure and shear elastic waves in the sphere are true in PREM in conditions $P \geq 2\mu$. It shows once again that the data on the physical and mechanical, acoustic and density characteristics in the theoretical models should be distributed in accordance with relevant requirements of the mechanics of deformable media with initial stress considering nonlinear laws of state. The obtained results relate to the data of the inner core. At the same time, they predict that it is necessary to process and interpret the relevant geological and geophysical data on the basis of non-linear (at least within NLT) theories considering preliminary deformation of the medium in solving the problem on the distribution of mantle and lithosphere parameters.

Data on the composition of the inner core material indicate its anisotropy (Fig. 1b) (Litasov and Shatskiy, 2016). Naturally, phenomenon of ”internal” instability will occur at much lower levels of loads and strain than in the isotropic approximations in the anisotropic medium because of the smallness of the shear stiffness.

It should be noted that the results presented in this article are obtained without considering the influence of temperature, the distribution of which is shown in Fig. 1a. The consideration of temperature influence on critical values of instability worsens the situation. Buckling process is implemented at significantly lower pressure level under the influence of temperature fields. Therefore, the consideration of temperature will not provide a qualitative impact on the conclusion on the insufficiency of interpretation of geophysical data within the classical theory. The consideration of temperature is necessary to solve specific problems of the local distribution of the considered parameters.

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Table 1.

ν	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.41**	**0.45**
$\frac{P^*}{\mu}$	1	1	1	1	1	1	1
$\frac{P_*}{\mu}$	0.5	0.53	0.57	0.60	0.64	0.64	0.65
λ_1^*	0.5	0.58	0.67	0.76	0.88	0.89	0.94
$(\lambda_1)_*$	0.75	0.79	0.84	0.89	0.94	0.94	0.97

Table 2.

ν	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.41**	**0.45**
K_P^R	$\frac{-1.5}{-2.5}$	$\frac{-1.3182}{-2.1364}$	$\frac{-1.1668}{-1.8333}$	$\frac{-1.0385}{-1.5769}$	$\frac{-0.9286}{-1.3571}$	$\frac{-0.9184}{-1.3369}$	$\frac{-0.8793}{-1.2586}$
K_S^R	$\frac{-1}{-1.5}$	$\frac{-0.8636}{-1.2273}$	$\frac{-0.75}{-1}$	$\frac{-0.6538}{-0.8077}$	$\frac{-0.5714}{-0.6429}$	$\frac{-0.5638}{-0.6277}$	$\frac{-0.5345}{-0.569}$
$\frac{P_l}{\mu}$	$\frac{0.6667}{0.4}$	$\frac{0.8534}{0.5266}$	$\frac{1.1429}{0.7273}$	$\frac{1.6852}{1.1098}$	$\frac{3.2308}{2.2105}$	$\frac{3.5689}{2.4518}$	$\frac{6.2549}{4.3699}$
$\frac{P_S}{\mu}$	$\frac{0.5}{0.3333}$	$\frac{0.5789}{0.4074}$	$\frac{0.6667}{0.5}$	$\frac{0.7647}{0.6190}$	$\frac{0.875}{0.7778}$	$\frac{0.8868}{0.7966}$	$\frac{0.9355}{0.8788}$

Fig. 1. *a* - profiles of the density distribution, velocities of sonic waves in the Earth's core on model PREM [Dziewonski, Anderson, 1981] and also the temperature [Nimmo, 2015].The numbers show the change in density and V_P at the boundary of the inner core in $\%.b$ - scheme of structure of the Earth's core reflecting the main results of the seismological researches.

F - layer with the reduced velocities V_P, isotropic structure of the upper layer of the inner core with the differences in the hemispheres, the presence of an additional inner core (in question) are shown. The amplitude of the anisotropy of seismic waves is indicated by icons in the polar and equatorial directions according to the data of papers [Deuss, 2014; Souriau, Calvet, 2015] with changes. 1 - low velocity, light damping; 2 - high velocity, heavy damping. [Fig. 1a, b taken from Litasov, Shatskiy, 2016].

References

1. Abasov, M.T., Kuliev, G.G., Dzhevanshir, R.D.: *Development model of the Lithosphere*. Doclady Russian Academy of Sciences, **70** (2), 129–135 (2000).
2. Adushkin, V.V., An, V.A., Kvazik, P.B.: et al. The rotation of the inner core from seismic records of nuclear explosions. *Thesis of reports "The inner core of the Earth. Geophysical data on process in the core"*, IPE RAS, Moscow, Russia (2000).
3. Akbarov, S.D.: Stability Loss and Buckling Delamination: Three-Dimensional Linearized Approach for Elastic and Viscoelastic Composites, *Springer, Berlin, Germany* (2013).
4. Akbarov, S.D.: Dynamics of Pre-Strained Bi-Material Elastic Systems: Linearized Three-Dimensional Approach, *Springer, Switzerland* (2015).
5. Anderson, D.: New Theory of the Earth. *Cambridge University Press, New York, USA* (2007).
6. Anderson, O.L.: Equations of State of Solids for Geophysics and Ceramic Science, *Oxford University Press, New York, USA* (1995).
7. Avsyuk, Yu.N.: *Motion of the inner core*. Proceedings of the USSR Academy of Sciences, **212** (5), 1103–1105 (1973).

8. Avsyuk, Yu.N.: Extraterrestrial factors affecting tectogenesis. *In: Fundamental Problems of Global Tectonics (Edited by Pushcharovsky, Yu.M.), Scientific World, Moscow, Russia* (2001).
9. Biot, M.A.: Mechanics of Incremental Deformation, *Willey, New York, USA* (1965).
10. Bullen, K.E.: The Density of the Earth. *Mir, Moscow, Russia* (1978).
11. Dziewonski, A.M., Anderson, D.L.: *Preliminary reference Earth model*. Phys. Earth Planet. Inter., **25** (4), 297–356 (1981).
12. Guz, A.N.: Stability of Elastic Bodies Under Uniform Compression, *Naukova Dumka, Kyiv, Ukraine* (1979).
13. Guz, A.N.: Fundamentals of Three-Dimensional Theory of Stability of Deformable Bodies, *Vishcha shkola, Head Publishing House, kyiv, Ukraine* (1986a).
14. Guz, A.N.: *Elastic Waves in Bodies with Initial Stresses*. *Propagation Patterns, (2^{nd} Volume), Naukova Dumka, Kiyev, Ukraine* (1986b).
15. Guliyev, H.H.: *Nonlinear actions of elastic medium and their effect on the propagation velocity of elastic waves*. Proceedings of the NAS of Azerbaijan, *Earth Sciences* (2), 31–39 (2009).
16. Guliyev, H.H.: *A new theoretical conception concerning the tectonic processes of the Earth*. New Concepts in Global Tectonics Newsletter, (56), 50–74 (2010).
17. Guliyev, H.H.: *Fundamental role of deformations in internal dynamics of the Earth*. New Concepts in Global Tectonics Newsletter, (61), 33–50 (2011).
18. Guliyev, H.H.: *Deformations, corresponding to processes of consolidation, deconsolidation and phase transitions in internal structures of the Earth*. Geophysical Journal, **35** (3) 166–176 (2013).
19. Guliyev, H.H., Askerov, A.D.: *The solution of nonlinear problem on increase of environment density of the Earth depths and its instability*. Proceedings of the NAS of Azerbaijan, Series of Earth Sciences, (1), 38–50 (2007).
20. Kennett, B.L.N., Engdahl, E.R.: *Traveltimes for global earthquake location and phase identification*. Geophysical Journal International, **105** (2), 429–465 (1991).
21. Kennett, B.L.N., Engdahl, E.R., Buland, R.: *Constraints on seismic velocities in the Earth from traveltimes*. Geophysical Journal International, **122** (1), 108–124 (1995).
22. Kuliev, G.G.: *A new approach to calculation of the theoretical ultimate strength of materials*. Strength of Materials, **20** (5), 623–629 (1988a).
23. Kuliev, G.G.: Fundamentals of the Mathematical Theory of the Stability of Wells. *Elm, Baku, Azerbaijan* (1988b).
24. Levin, B.V.: The role of the Earth's inner core movements in the tectonic processes, *In: Fundamental Problems of Global Tectonics (Edited by Pushcharovsky, Yu.M.) Scientific World, Moscow, Russia* (2001).
25. Litasov, K.D., Shatskiy, A.F.: *Composition of the Earth's core: A review*. *Russian Geology and Geophysics*, **57** (1), 31–62 (2016).
26. Lyav, A.I. : The Mathematical Theory of Elasticity, *ONTI, Moscow, Russia* (1935).
27. Morelli, A., Dziewonski, A.M.: *Body-wave traveltimes and a spherically symmetric P- and S-wave velocity model*. Geophysical Journal International, **112** (2), 178–194 (1993).
28. Nimmo, F.: Energetics of the Core, (8^{th} Volume), In: *Treatise on Geophysics*, (2^{nd} Edition), (Edited by Schubert G.), Elsevier, Oxford (2015).
29. Pushcharovsky, Yu.M., Pushcharovsky, D.Y.: *When, how and why were the Earth's geospheres formed*. Priroda (5), 25–31 (2011).
30. Rabotnov, Y.N.: Mechanics of Deformable Solids, *Nauka, Moscow, Russia* (1988).
31. Sadovsky, M.A., Nikolaev, A.V.: *New methods of seismic exploration. Prospects of development*. Bulletin of the Academy of Sciences of USSR, **52** (1), 57–64 (1982).
32. Sedov, L.I.: Mechanics of the Continuum Medium. *(1^{st} Volume), Nauka, Moscow, Russia* (1970).

33. Thurston, R., Brugger, K.: Third-order elastic constants and velocity of small amplitude elastic waves in homogeneously stressed media, *Physical Review,* **133**(6A), 1604-1610 (1964).
34. Truesdell, K.: Initial Course of Rational Mechanics of Continuum Media. *Nauka, Moscow, Russia* (1975).