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On approximate solution of viscous fluid motion equations

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Abstract. *All real fluids are viscous in this or other extent, in other words, they have the property of internal friction. Origin of viscosity forces should be sought in molecular nature of mater's structure. The quantities that we deal with in hydraudynamics, are mean quantities obtained as a result of total account relating to very great quantity of molecules. Taking into account what has been said, the finite differences method for solving viscous fluid motion equations is used.*

Keywords. viscous fluid · boundary condiditions · Reynold number · Laplas equation · solution.

Mathematics Subject Classification (2010): 76D10

1 Introduction

The relevant literature gives references, the solution of some problems of viscous fluid hydromechanics in exact form. Integration of hydromechanics equations in the exact form is succesded very seldom; besides, it should be noted that many exact solutions of viscous fluid hydromechanics equations are of negligible interest, since they may be realized in the presence of boundary conditions hand, a great majority of viscous fluid motions important from point of view of experiments or observations in nature are not amenable to exact hydromechanical analysis. It is quite natural that when it is impossible to solve exactly any problem, for solving this problem approximate methods are used.

All approximate methods of hydromechanics are characterized by one general sign: in these methods, in the main equation or in boundary conditions, a part pf terms or terms or terms are rejected or are not taken account in full measure.

At those cases of viscous fluid low motions, three categories of forces are considered: inertia forces, viscousity forces and pressure forces. The latter forces are internal forces

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and order of their quantity is determined by order of quantity of the first two categories of forces.

As for comporative quantities of inertia forces and viscousity forces, in this direction we are oriented by the Reynold number equal to the $R = lV/\nu$ ratio of the product of characteristic velocity V by the characteristic length l to kinematic viscosity coefficient ν.

According to this, we can speak about two types of approximate solutions of viscous fluid mechanics equations.

The cases of flow wherein the inertia forces are small in comparison with viscosity forces and for which the Reynold number containing kinematic viscosity coefficient in denominator is small belong to the first type. But the Reynolds number will be small in three cases: 1) when the characteristic length l is very small or 2) when characteristic velocity V is very small, or finally, 3) when kinematic viscosity coefficient ν is very large. Thus, for example, the chases of low motions of small particles in comparatively viscous fluids is related to the considered type. Approximate interpretation of motion of hydromechanics of members giving the inertia forces or in simplification of the type of these members.

The another opposite types of flow cover the cases when viscosity forces are small in comparison, with inertia forces and when the Reynold number is very large. For that either characteristic length or characteristic velocity should be very great or fluid's viscosity should be very low. Thus, the cases of rapid motions of great size bodies in low viscous fluids are related to the second type of flows. If in under approximate consideration of the second type flows the viscosity forces are rejected we obviously arrive to ideal fluid motions equations. Therefore, we should consider only that interpretation of second order flows when we partly take into account viscous forces and in the equations we leave only principal members giving viscous forces.

Now we consider a number of specific cases of first type motions, i.e. the motions with Reynold's small numbers.

2 Problem formulation

Let's consider flow of very viscous fluid between two parallel plates and the distance h between them is very small. If we assume that the values of mean velocities of fluid are also small, then the Reynolds number $R = Vh/\nu$ will be very small. We assume that there are no external forces.

Therefore, writing acceleration projection in the perfect form, we get the following viscous fluid flow equations:

$$\left.\begin{array}{l}
\frac{\partial \upsilon_x}{\partial t} + \upsilon_x \frac{\partial \upsilon_x}{\partial x} + \upsilon_y \frac{\partial \upsilon_x}{\partial y} + \upsilon_z \frac{\partial \upsilon_x}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{\nu}{3}\frac{\partial div \upsilon}{\partial x} + \nu \Delta \upsilon_x, \\
\frac{\partial \upsilon_y}{\partial t} + \upsilon_x \frac{\partial \upsilon_y}{\partial x} + \upsilon_y \frac{\partial \upsilon_y}{\partial y} + \upsilon_z \frac{\partial \upsilon_y}{\partial z} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \frac{\nu}{3}\frac{\partial div \upsilon}{\partial y} + \nu \Delta \upsilon_y, \\
\frac{\partial \upsilon_z}{\partial t} + \upsilon_x \frac{\partial \upsilon_z}{\partial x} + \upsilon_y \frac{\partial \upsilon_z}{\partial y} + \upsilon_z \frac{\partial \upsilon_z}{\partial z} = Z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{\nu}{3}\frac{\partial div \upsilon}{\partial z} + \nu \Delta \upsilon_z.
\end{array}\right\} \tag{2.1}$$

To these equations we can adjoin the continuity equation

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho \upsilon_x)}{\partial x} + \frac{\partial (\rho \upsilon_y)}{\partial y} + \frac{\partial (\rho \upsilon_z)}{\partial z} = 0. \tag{2.2}$$

If we deal with flow of incompressible viscous fluid, then four equations (2.1) and (2.2) are not sufficient for determining five unknown functions $p, \rho, \upsilon_x, \upsilon_y, \upsilon_z$. In this case, it is necessary to take into account thermodynamical properties of the studied processes.

We can impart different forms of equations to viscous incompressible fluid flow equations: in one cases it is suitable to use one form equations, in other cases the another one.

First of all, equations (2.1) and (2.2) for the case of incompressible fluid are simplified as follows:

$$\left.\begin{array}{l} \frac{\partial \upsilon_x}{\partial t} + \upsilon_x \frac{\partial \upsilon_x}{\partial x} + \upsilon_y \frac{\partial \upsilon_x}{\partial y} + \upsilon_z \frac{\partial \upsilon_x}{\partial z} = X - \frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \Delta \upsilon_x, \\ \frac{\partial \upsilon_y}{\partial t} + \upsilon_x \frac{\partial \upsilon_y}{\partial x} + \upsilon_y \frac{\partial \upsilon_y}{\partial y} + \upsilon_z \frac{\partial \upsilon_y}{\partial z} = Y - \frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \Delta \upsilon_y, \\ \frac{\partial \upsilon_z}{\partial t} + \upsilon_x \frac{\partial \upsilon_z}{\partial x} + \upsilon_y \frac{\partial \upsilon_z}{\partial y} + \upsilon_z \frac{\partial \upsilon_z}{\partial z} = Z - \frac{1}{\rho}\frac{\partial p}{\partial z} + \nu \Delta \upsilon_z, \\ \frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial y} + \frac{\partial \upsilon_z}{\partial z} = 0. \end{array}\right\} \tag{2.3}$$

Under these conditions, in the main equations of fluid mechanics (2.3) we may neglect inertia forces in the left hand of these equations, then we get the equations:

$$\left.\begin{array}{l} \frac{\partial p}{\partial x} = \mu\left(\frac{\partial^2 \upsilon_x}{\partial x^2} + \frac{\partial^2 \upsilon_x}{\partial y^2} + \frac{\partial^2 \upsilon_x}{\partial z^2}\right), \\ \frac{\partial p}{\partial y} = \mu\left(\frac{\partial^2 \upsilon_y}{\partial x^2} + \frac{\partial^2 \upsilon_y}{\partial y^2} + \frac{\partial^2 \upsilon_y}{\partial z^2}\right), \\ \frac{\partial p}{\partial z} = \mu\left(\frac{\partial^2 \upsilon_z}{\partial x^2} + \frac{\partial^2 \upsilon_z}{\partial y^2} + \frac{\partial^2 \upsilon_z}{\partial z^2}\right), \\ \frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial y} + \frac{\partial \upsilon_z}{\partial z} = 0. \end{array}\right\} \tag{2.4}$$

Assume that the axis Ox and Oy lie in one of boundary planes, the axis Oz is directed along the perpendicular to these planes so that the boundary plane equations are
$z = 0$ and $z = h$.

Then we accept that velocity of each particle is direct in a parallel way to boundary planes so that

$$\upsilon_z = 0.$$

Finally we note that because of smallness of h change of velocities υ_x and υ_y in direction of the axis Oz will happen more rapidly change of this derivative $\partial \upsilon_x / \partial z$ is greater in comparison with orders of derivatives $\partial \upsilon_x / \partial x$ and $\partial \upsilon_x / \partial y$; just in the same way, order of the derivative $\partial^2 \upsilon_x / \partial z^2$ is greater in comparison with order of the derivatives $\partial^2 \upsilon_x / \partial x^2$ and $\partial^2 \upsilon_x / \partial y^2$. Under these conditions, equations (2.4) take the form:

$$\frac{\partial p}{\partial x} = \mu \frac{\partial^2 \upsilon_x}{\partial z^2}, \quad \frac{\partial p}{\partial y} = \mu \frac{\partial^2 \upsilon_y}{\partial z^2}, \quad \frac{\partial p}{\partial z} = 0, \quad \frac{\partial \upsilon_x}{\partial x} + \frac{\partial \upsilon_y}{\partial y} = 0. \tag{2.5}$$

The third one from the obtained equations show that p depends only on x and y; but then the first equation may be easily integrated:

$$\mu \upsilon_x = \frac{z^2}{2}\frac{\partial p}{\partial x} + zA(x,y) + B(x,y);$$

The functions A and B may be determined from the boundary conditions
$\upsilon_x = 0$ for $z = 0$ and $z = h$.
These conditions give us

$$B(x,y) = 0, \quad A(x,y) = -\frac{h}{2}\frac{\partial p}{\partial x}$$

and therefore,

$$\upsilon_x = -\frac{1}{2\mu}\frac{\partial p}{\partial x} z(h - z). \tag{2.6}$$

Just in the same way easily get

$$\upsilon_y = -\frac{1}{2\mu}\frac{\partial p}{\partial y} z(h - z). \tag{2.7}$$

Finally, the last equation of system (2.5) immediately gives that after substitution of values (2.6) and (2.7), the equation for determining the function $p\left(x,y\right)$:

$$\Delta p=\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}=0. \tag{2.8}$$

Note that form formulas (2.6), (2.7) and (2.8) it immediately follows that

$$\frac{\partial^2 \upsilon_x}{\partial x^2}+\frac{\partial^2 \upsilon_x}{\partial y^2}=0, \quad \frac{\partial^2 \upsilon_y}{\partial x^2}+\frac{\partial^2 \upsilon_y}{\partial y^2}=0.$$

But then it is clear that the formal solution strongly satisfies equations (2.4), because the members that we ignored in these equations vanish identically.

As the first example we consider flow of incompressible fluid between two parallel plane walls. Let the equations of these planes will be

$$z=-h, \quad z=h;$$

respectively. Assume that there are no external forces, the motion is stationary and happens in a parallel way to the axis Ox, such that

$$X=Y=Z=0, \quad \upsilon_y=\upsilon_x=0, \quad \upsilon_x=\upsilon\left(x,y,z\right).$$

Under the made assumptions, the main equations of hydromechanics are strongly simplified:

$$\frac{\partial p}{\partial x}=\mu\left(\frac{\partial^2 \upsilon}{\partial y^2}+\frac{\partial^2 \upsilon}{\partial z^2}\right), \quad \frac{\partial p}{\partial y}=0, \quad \frac{\partial p}{\partial z}=0, \quad \frac{\partial \upsilon}{\partial x}=0. \tag{2.9}$$

The final of these equations shows that υ may depend only on y and z; the middle equations show that p may depend only on x; but then the first equation of (2.9) at whose left hand side there is a function on x, and in the right hand side a function on y and z, may be fulfilled only in the case if the left and right hand sides of this equation are constant variables. So there should be:

$$\frac{\partial p}{\partial x}=const.$$

For determining υ we have the equation:

$$\frac{\partial^2 \upsilon_x}{\partial y^2}+\frac{\partial^2 \upsilon_x}{\partial z^2}=\frac{1}{\mu}\frac{\partial p}{\partial x} \tag{2.10}$$

and the boundary conditions

$$\upsilon=0\,\text{for}\,z=0, \; z=h \tag{2.11}$$

following from the requirement of adhesion of fluid to restricting fixed walls. It is easy to find the solution of equations (2.10) and (2.11) dependent only on z; in fact, in this case we have:

$$\frac{d^2\upsilon}{dz^2}=\frac{1}{\mu}\frac{\partial p}{\partial x},$$

and integration of this equation gives us:

$$\upsilon=\frac{1}{2\mu}\frac{\partial p}{\partial x}z^2+Az+B$$

and therefroe:

$$\upsilon=\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(z^2-h^2\right).$$

It is easy to prove that the obtained solution is the solution of equations (2.10) and (2.11) that we need. In fact, assume

$$v = \frac{1}{2\mu}\frac{\partial p}{\partial x}\left(z^2 - h^2\right) + u\left(y, z\right);$$

then it is clear that the function $u\left(y, z\right)$ should satisfy the Laplace equation

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0 \tag{2.12}$$

and the two boundary conditions

$$u = 0 \text{ or } z = \pm h. \tag{2.13}$$

But if we demand that v and consequently u should stay bounded in the considered domain, then $u \equiv 0$ will be the unique solution of equations (2.12) and (2.13).

So, under the made assumptions, the fluid's flow is determined by the following dependence:

$$v = -\frac{1}{2\mu}\frac{\partial p}{\partial x}\left(h^2 - z^2\right). \tag{2.14}$$

3 Method of solution

Now we will construct numerical algorithms for solving the following boundary value problems:

$$\varDelta v \equiv \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = -f\left(x, y\right), \tag{3.1}$$

$$v = \varphi\left(x\right) \tag{3.2}$$

Obviously, the will-posedness of the statement of problem (3.1)-(3.2) doesn't give raise to doubts.

For the given partial equation and the given finite-difference mesh, finite-difference analogue of this equation may be constructed by different methods: 1) expansion of functions in Taylor's series; 2) interpolation of functions by polynomials; 3) integral method; 4) the control volume method. Sometimes all these methods reduce to one and the same finite-difference analogue of the imput equation. There exist different finite-difference schemes by means of which one can solve simplest model equations. The main method for solving equations are physical laws of conservation for example the mass, momentum and energy conservations laws. Partial equations described these conservation laws at a point. Of course, the difference scheme prove close approximation of partial equations in some small domain containing some knots of difference mesh. Convergence of difference scheme to exact solution of partial equations may be considered as convergence in approximation order and convergence in round off errors. Very often, when solving stationary problems, the Gauss-Zeidel iteration methods are used. The error that arises when changing partial equation by its finite-difference analogue is called the approximation error. If equals the difference of exact solutions of the input differential equation and its finite-difference analogue.

In the net domain $\varOmega_h = \bar{\omega}_h \bigcup (OB)_h$, where

$$\bar{D}_h = \dot{\omega}_h \bigcup \underset{h}{\overset{*}{\omega}} \bigcup \varGamma_h,$$

where $\dot{\omega}_h$ and $\underset{h}{\overset{*}{\omega}}$ are regular and irregular meshes, respectively.

We approximate problem (3.1), (3.2) in the following way:

For any $(x, y) \in \dot{\omega}_h$, equation (3.1) is approximated by the following difference equation

$$\bar{L}_h \upsilon_h \equiv \upsilon_{h\bar{x}x} + \upsilon_{h\bar{y}y} = -\bar{f}_h, \tag{3.3}$$

and for any $(x, y) \in \overset{*}{\underset{h}{\omega}}$ we have the following approximation

$$\bar{L}_h \upsilon_h \equiv \upsilon_{h\bar{x}\hat{x}} + \upsilon_{hy\bar{y}} = -\hat{f}_h. \tag{3.4}$$

For $(x, y) \in \Gamma_h$ the take the approximation

$$\upsilon_h \Big|_{\Gamma_h} = \varphi_h. \tag{3.5}$$

Therefore, by (3.3)-(3.5) we get the following difference scheme:

$$R_h \upsilon_h = -f_h, \tag{3.6}$$

$$\upsilon_h \Big|_{\Gamma_h} = \varphi_h, \tag{3.7}$$

where

$$R_h \upsilon_h = \begin{cases} \bar{L}_h \upsilon_h, & if \quad (x, y) \in \dot{\omega}_h, \\ \bar{L}_h \upsilon_h, & if \quad (x, y) \in \overset{*}{\underset{h}{\omega}} \end{cases}$$

$$f_h = \begin{cases} \bar{f}_h, & if \quad (x, y) \in \dot{\omega}_h, \\ \hat{f}_h, & if \quad (x, y) \in \overset{*}{\underset{h}{\omega}}. \end{cases}$$

Assume that $u \in C^{(4)}(\bar{\omega})$, then by the Taylor formula we get

$$\Delta \upsilon - R_h \upsilon_h = O\left(h^2\right).$$

We represent difference scheme (3.6)-(3.7) in the canonical form:

$$S\upsilon_h \equiv A(t)\, u_h(t) - \sum_{\xi \in ('(t)} B(t, \xi)\, u_h(\xi) = f_h, \tag{3.8}$$

$$\upsilon_h \Big|_{\Gamma_h} = \varphi_h, \tag{3.9}$$

where the arbitrary point $t = (x, y) \in \bar{\omega}_h$,

$$A(t) \equiv \begin{cases} \bar{A}(t) = \frac{4}{h^2}, & 5A;8 \quad t \in \dot{\omega}_h, \\ \hat{A}(t) = \frac{h+h^*}{\hbar h h^*} + \frac{h+h_1}{\hbar_1 h_1 h}, & if \quad t \in \overset{*}{\omega}_h. \end{cases}$$

$$B(t, \xi) = \begin{cases} \bar{B}(t, \xi) = \frac{1}{h^2} \ or \ \frac{1}{h^2} \ or \ \frac{1}{h^2} \quad if \quad 5 \in \dot{\omega}_h, \ \xi \in ('(t), \\ \hat{B}(t, \xi) = \frac{1}{\hbar h} \ or \ \frac{1}{\hbar h^*} \ or \ \frac{1}{\hbar_1 h_1} \ or \ \frac{1}{\hbar_1 h} \ if \ t \in \overset{*}{\omega}_h, \ \xi \in ('(t). \end{cases}$$

Therefore,

$$A(t) > 0, \ B(t, \xi) > 0, \ \forall t \in \omega_h, \quad \forall \xi \in ('(t)$$

$$D(t) \equiv A(t) - \sum_{\xi \in ('(t)} B(t, \xi) \geq 0. \tag{3.10}$$

Obviously, (3.8), (3.9) is a monotone scheme, then the maximum principle holds for this problem and this principle admits to establish solvability of the mesh problem, and in a number of cases to get a priori estimation of its solution, i.e. to prove the scheme's stability.

Let D_h be a mesh domain, S be an operator given on D_h by the relation

$$Sv_h \equiv A(t)\, v_h(t) - \sum_{\xi \in ('(t)} B(t,\xi)\, v_h(\xi).$$

Let D_h be a connected-domain, while $D'_h \leq D_h$ a connected sub-domain, on which the coefficients of the operator S satisfy conditions (3.10). Then if the mesh function given on D_h is not constant on D'_h and $Sv_h(t) \leq 0 \ (Sv_h(t) \geq 0)$ for $t \in D'_h$, then $v_h(t)$ may not accept positive maximum (negative minimum) value on D'_h.

Since the coefficients of difference scheme (3.8) – (3.9) satisfy all requirements of type (3.10), obviously holds the maximum principle for problems (3.8) – (3.9).

Difference problem (3.8), (3.9) is uniquely solvable.

By theorem 1, difference problem (3.8), (3.9) in the case $f_h \equiv 0, \ \varphi_h \equiv 0$ has only a zero solution. Then the approximate inhomogeneous problem has a unique difference from identical zero solution.

Let $D_h = \overset{o}{\underset{h}{D}} \bigcup \overset{*}{\underset{h}{D}}$, where $\overset{o}{\underset{h}{D}}$ is a connected domain, and $D(t) \geq 0$ on $\overset{o}{\underset{h}{D}}, \ D(t) > 0$ on $\overset{*}{\underset{h}{D}}$. Then for the solution of problem (3.8), (3.9) it holds the following estimation:

$$\max_{\bar{D}_h} |v_h(t)| \leq \max_{\Gamma_h} |v_h(t)| + \max_{\bar{D}_h} |U(t)| + \max_{\overset{*}{\underset{h}{D}}} \left| \frac{f_h(t)}{D(t)} \right|, \tag{3.11}$$

where $U(x,y)$ is a majorant function being the solution of the following problem:

$$\begin{aligned} &SU(t) = F_h(t), \ t \in D_h, \ v \geq 0, \ t \in \Gamma_h, \\ &F_h(t) \geq |f_h(t)| \ \ for \ t \in \overset{o}{\underset{h}{D}}, \ F_h(t) \geq 0 \ \ for \ t \in \overset{*}{\underset{h}{D}}. \end{aligned}$$

We can solve problem (3.8)-(3.9) by the Zeilded iterative method. For that we enumerate the modal points contained in $\bar{\omega}_h$ as follows.

We enumerate arbitrarily all nodal points that are on the straight line with maximal ordinate and parallel axis OX, i.e.

$v_{h1,1}, v_{h2,1}, ..., v_{hN_1,1}$ (on the first line).

We enumerate nodal points on the second line in the same way, i.e.

$v_{hN_1+1,2}, v_{hN_1+2,2}, ..., v_{hN_2,2}$, and etc.

Assume that D_h has P parallel lines in all. Then on the P-th line we have the following enumeration of nodal points:

$$v_{hN_{p-1}+1,p}, v_{hN_{p-1},2}, ..., v_{hN_p,p}.$$

Therefore, for regular and irregular nodal points we have the following numerations:

$$v_{h1,1}, v_{h2,1}, ..., v_{hN_p,p}.$$

Then we can write equations (3.8), (3.9) in the form:

$$v_{hn,m} = \sum_{k=1}^{n-1} a_{nk} v_{hk,m} + \sum_{k=n+1}^{N_p} a_{nk} v_{hk,m} + f_{hn,m}.$$

The Zeidel algorithm is of the form:

$$\upsilon_{hn,m}^{(i)} = \sum_{k=1}^{n-1} a_{nk}\upsilon_{hk,m}^{(i)} + \sum_{k=n+1}^{N_p} a_{nk}\upsilon_{hk,m}^{(i-1)} + f_{hn,m} \qquad (n = \overline{1, N_p},\ \overline{1,p})$$

For any $\left\{\upsilon_{hk,m}^{(0)}\right\} \quad (k = \overline{1, N_p},\ \ m = \overline{1,p})$

$V_{hk,m}^{(i)} = \upsilon_{hk,m}^{(i)} - \upsilon_{hk,m} \to 0$ where $i \to \infty$.

Numerical experiment was performed in accordance with above formulas by virtue of the Zeidel scheme.

The obtained results are on the following table:

Nodal points	***Exact solution***	***Numerical solution***	***Absolute error***	***Relative error***
υ(1,3)	0,0001	0,00009	0,00001	10,0000
υ(1,5)	0,0016	0,0015	0,0001	6,2500
υ(1,7)	0,0081	0,0079	0,0002	2,4691
υ(1,9)	0,0257	0,0253	0,0004	1,1713
υ(1,11)	0,0623	0,0621	0,0002	0,6400
υ(3,3)	0,1443	0,1440	0,0003	0,2079
υ(3,5)	0,1111	0,1107	0,0004	0,3600
υ(3,7)	0,1000	0,1009	0,0009	0,9000
υ(3,9)	0,1074	0,1070	0,0004	0,3724
υ(5,5)	0,2099	0,2092	0,0007	0,3334
υ(5,7)	0,1909	0,1904	0,0005	0,2619
υ(5,9)	0,1937	0,1933	0,0004	0,2065
υ(5,11)	0,2229	0,2226	0,0003	0,1345
υ(7,7)	0,2817	0,2813	0,0004	0,1419
υ(7,9)	0,3194	0,3190	0,0004	0,1252
υ(7,11)	0,2861	0,2855	0,0006	0,2097
υ(7,13)	0,3907	0,3902	0,0005	0,1279
υ(9,9)	0,3902	0,3900	0,0002	0,0512
υ(9,11)	0,4329	0,4322	0,0007	0,1617
υ(9,13)	0,5165	0,5160	0,0005	0,0968

The performed experiment shows that a priori properties of exact solution completely affirmed and numerically converges.

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