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Dynamics of the moving load acting on a metal elastic plate under compressible viscous fluid loading

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Abstract. *The subject of the paper is the study of the dynamics of the moving load acting on the hydro-elastic system consisting of a metal elastic plate and a half-plane filled by a barotropic compressible Newtonian viscous fluid. Under this study the motion of the plate is described by equations of the linear elastodynamics, and the motion of the compressible viscous fluid is described by the linearized Navier-Stokes equations. Numerical results are presented and discussed for the case where the material of the plate is steel, but the fluid material is Glycerin.*

Keywords. moving load, compressible viscous fluid, metal elastic plate, critical velocity.

Mathematics Subject Classification (2010): 74H20

1 Introduction

The review of investigations related to the vibration plate + fluid systems was made in papers [1, 2] and it was noted therein that until recently there was not any study in this field made within the utilizing of the linearized exact equations of motion. In the mentioned sense the first attempts were made namely in the papers [1, 2] in which the frequency response of the system consisting of the elastic [1] and viscoelastic [2] plate and the half-plane occupied with compressible viscous fluid was studied. Under these studies the equations of motion for the plate were written by utilizing the exact linearized equations of elastodynamics and the equations of motion of the fluid were written by utilizing the linearized Navier - Stokes equations.

The other considerable aspect of the investigations regarding the dynamics of the plate-fluid systems is a dynamic response analysis plate-fluid systems induced by a moving load. Results of these investigations are applied for construction of the floating bridges and for determination of their efficiency. As an example for such investigations it can be presented studies carried out in papers [3 - 5] and others listed therein. However in these investigations the fluid reaction to the plate (i.e. to the floating bridge) is taken into consideration without solution of the equations of the fluid motion. It is evident that the approach employed in [3 - 5] is very approximate one and cannot answer the questions how the fluid viscosity, fluid compressibility, plate thickness and the moving velocity of the external force act on the "hydrostatic

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force" acting on the plate and on the fluid flow velocities. To find the answers to these questions it is necessary to solve the corresponding coupled fluid-plate interaction problems within the scope of the exact linearized equations described to the plate and fluid motions. In the mentioned sense, in the present paper the first attempt is made for solution to the problems related to the dynamics of the moving load acting on a system consisting of the metal elastic plate and half-plane filled with compressible viscous fluid.

2 Formulation of the problem and solution method

Consider a system consisting of the plate-layer and half-plane filled with a barotropic compressible Newtonian viscous fluid. We associate the coordinate system $Ox_1x_2x_3$ with the plate and the position of the points of the constituents we determine in this coordinate system. Assume that the plate occupies the region $\{|x_1| < \infty, -h < x_2 < 0\}$, but the fluid occupies the region $\{|x_1| < \infty, -\infty < x_2 < -h\}$. Within this, we consider a motion of the system under consideration in the case where the lineal-located force which moves with the constant velocity V acts on its free face plane of the plate-layer. Assume that the plane-strain state in the plate and the two-dimensional flow of the fluid take place in the Ox_1x_2 plane.

The equations of the plate we take within the scope of the linear theory of elastodynamics, i.e., as follows:

$$\frac{\partial\sigma_{11}}{\partial x_1}+\frac{\partial\sigma_{12}}{\partial x_2}+\sigma_{12}^0\frac{\partial^2 u_1}{\partial x_1^2}=\rho\frac{\partial^2 u_1}{\partial t^2},\ \ \frac{\partial\sigma_{12}}{\partial x_1}+\frac{\partial\sigma_{22}}{\partial x_2}+\sigma_{11}^0\frac{\partial^2 u_2}{\partial x_1^2}=\rho\frac{\partial^2 u_2}{\partial t^2}.$$

$$\sigma_{11}=\lambda(\varepsilon_{11}+\varepsilon_{22})+2\mu\varepsilon_{11},\ \ \sigma_{22}=\lambda(\varepsilon_{11}+\varepsilon_{22})+2\mu\varepsilon_{22},\ \ \sigma_{12}=2\mu\varepsilon_{12},$$

$$\varepsilon_{11}=\frac{\partial u_1}{\partial x_1},\ \ \varepsilon_{22}=\frac{\partial u_2}{\partial x_2},\ \ \varepsilon_{12}=\frac{1}{2}\left(\frac{\partial u_1}{\partial x_2}+\frac{\partial u_2}{\partial x_1}\right). \tag{2.1}$$

Note that in Eq. (2.1) the conventional notation is used.

According to [6], we consider the field equations of motion of the Newtonian compressible viscous fluid: the density, viscosity constants and pressure of which are denoted by the upper index (2.1). Thus, the linearized Navier-Stokes and other field equations for the fluid are:

$$\rho_0^{(1)}\frac{\partial\nu_i}{\partial t}-\mu^{(1)}\frac{\partial\nu_i}{\partial x_j\partial x_j}+\frac{\partial p^{(1)}}{\partial x_i}-\left(\lambda^{(1)}+\mu^{(1)}\right)\frac{\partial^2\nu_j}{\partial x_j\partial x_j}=0,$$

$$\frac{\partial p^{(1)}}{\partial t}+\rho_0^{(1)}\frac{\partial\nu_j}{\partial x_j}=0,$$

$$T_{ij}=\left(-p^{(1)}+\lambda^{(1)}\theta\right)\delta_{ij}+2\mu^{(1)}e_{ij},\quad \theta=\frac{\partial\nu_i}{\partial x_1}+\frac{\partial\nu_2}{\partial x_2},$$

$$e_{ij}=\frac{1}{2}\left(\frac{\partial\nu_i}{\partial x_j}+\frac{\partial\nu_j}{\partial x_i}\right).\ \ a_0^2=\frac{\partial p^{(1)}}{\partial\rho^{(1)}}, \tag{2.2}$$

where $\rho_0^{(1)}$ is the fluid density before perturbation. The other notation used in Eq. (2.2) is also conventional.

Assuming that $p^{(1)}$=-$(T_{11}+T_{22}+T_{33})/3$, we obtain that $\lambda^{(1)}=-2\mu^{(1)}/3$. Moreover, we assume that the following boundary and contact conditions are satisfied:

$$\sigma_{21}\left|_{x_2=0}=0,\ \sigma_{21}\right|_{x_2=0}=-P_0\delta\left(x_1-V_t\right),$$

$$\left.\frac{\partial u_1}{\partial t}\right|_{x_2=-h}=\nu_1|_{x_2=-h},\ \ \left.\frac{\partial u_2}{\partial t}\right|_{x_2=-h}=\nu_2|_{x_2=-h},$$

$$\sigma_{21}\left|_{x_2=-h}=T_{21}\right|_{x_2=-h},\ \sigma_{22}\left|_{x_2=-h}=T_{22}\right|_{x_2=-h}, \tag{2.3}$$

where $\delta(\square)$ is the Dirac delta function.

This completes the formulation of the problem. For the solution of this problem, we use the moving coordinate system $x_1'=x_1-Vt$, $x_2'=x_2$, (below we will omit the upper prime on the new moving coordinates) and replacing the derivatives $\partial(\cdot)/\partial t$, and $\partial^2(\cdot)/\partial t^2$ with $-V\partial/\partial x_1$ and $V^2\partial^2/\partial x_1^2$, respectively, we obtain the corresponding equations and boundary and contact conditions for the sought values in the moving coordinate system. For the solution to these equations, we employ the exponential Fourier transformation with respect to the x_1 coordinate

$$f_F(s,x_2)=\int_{-\infty}^{+\infty} f(x_1,x_2)\,e^{-isx_1}dx_1. \tag{2.4}$$

Before the employing the Fourier transformation (4) we introduce the dimensionless coordinates and dimensionless transformation parameter

$$\overline{x_1}=x_1/h,\ \ \overline{x_2}=x_2/h,\ \ \overline{s}=sh. \tag{2.5}$$

Below we will omit the over-bar on the symbols in (5). Moreover, we will also use the notation

$$V'=V/h,\ \nu^{(1)}=\mu^{(1)}/\rho_0^{(1)}. \tag{2.6}$$

For reducing the volume of the paper we do not give here the other details of the solution procedure, which are similar to those given in the papers [1, 2]. Nevertheless, we recall that under the mentioned solution procedure the dimensionless parameters

$$\Omega_1=\frac{V'h}{a_0},\ \ N_w^2=\frac{V'h^2}{\nu^{(1)}},\ M=\frac{\mu^{(1)}V}{\mu h} \tag{2.7}$$

are introduced. Note that the dimensionless number N_w in (7) can be taken as a Womersley number and characterizes the influence of the fluid viscosity on the mechanical behavior of the system under consideration. However, the dimensionless frequency Ω_1 in (7) can be taken as the parameter through which the influence of the compressibility of the fluid on the mechanical behavior of the system under consideration can be characterized. At the same time, the parameter characterizes the ratio of the M characteristic stress caused by fluid viscosity to the shear modulus of the plate material.

Thus, within the scope of the solution procedure discussed in the papers [1, 2], we obtain analytical expression of the sought quantities, after which we determine the originals of those through the expression

$$\{u_1;u_2;\sigma_{11};\sigma_{12};\sigma_{22};\nu_1;\nu_2;T_{11};T_{12};T_{22}\}=\frac{1}{2\pi}Re\left\{\int_{-\infty}^{+\infty}\{u_{1F};u_{2F};\sigma_{11F};\right.$$

$$\left.;\sigma_{12F};\sigma_{22F};\nu_{1F};\nu_{2F};T_{11F};T_{12F};T\ _{22F}\}e^{isx_1}ds\right]. \tag{2.8}$$

The integrals in (8) are calculated numerically for which the infinite interval $[-\infty,+\infty]$ is replaced with the finite one $[-S_1^*,+S_1^*]$. The values of the S_1^* are determined from the convergence criterion of these integrals in (8). Under calculation of the integrals in (8), the interval $[-S_1^*,+S_1^*]$ is divided into a certain number of sorter intervals. Let us denote this number through $2N$. Consequently, the length of the mentioned shorter intervals is S_1^*/N and in each of these shorter intervals the integration is made by the use of the Gauss integration algorithm with the sample points. Consequently, convergence of the mentioned numerical integration can be estimated with respect to the values of S_1^* and N. The various testing of the convergence of the numerical results show that for the quite converge and validate results are obtained in the case where $N=2000$ and $S_1^*=5.0$. We do not here consider examples of the numerical results illustrated this convergence, however note that such examples are given in the paper [1].

This completes the consideration of the solution method.

3 Numerical results and discussions

It follows from the foregoing discussions that the problem under consideration is characterized through the dimensionless parameters Ω_1 N_{w}, and M which are determined by the expressions in (7), λ/μ where λ and μ are the mechanical constants which enter the expression of the elastic relations in Eq. (2.1). Note that the case where $\Omega_1 = 0$ corresponds to the case where the fluid is incompressible, but the case where $1/N_{w} = 0$ corresponds to the case where the fluid is inviscid.

In the numerical investigation we assume that the material of the plate-layer is Steel with mechanical constants: $\mu = 79 \times 10^9 Pa,\ \lambda = 94,4 \times 10^9 Pa$ and density $\rho = 1160 kg/m^3$ [7], but the material of the fluid is Glycerin with viscosity coefficient $\mu^{(1)} = 1,393 kg/(m \cdot s)$ density $\rho_0^{(1)} = 1260 kg/m^3$ and sound speed $a_0 = 1459,5 m/s$ [6]. We also introduce the notation $c_2 = \sqrt{\mu/\rho}$ which is the shear wave propagation velocity in the layer material.

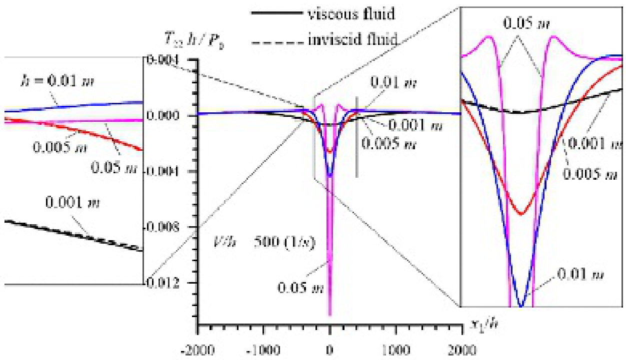

Fig.1. Distribution of the $T_{22}h/P_0$ with respect to the x_1/h.

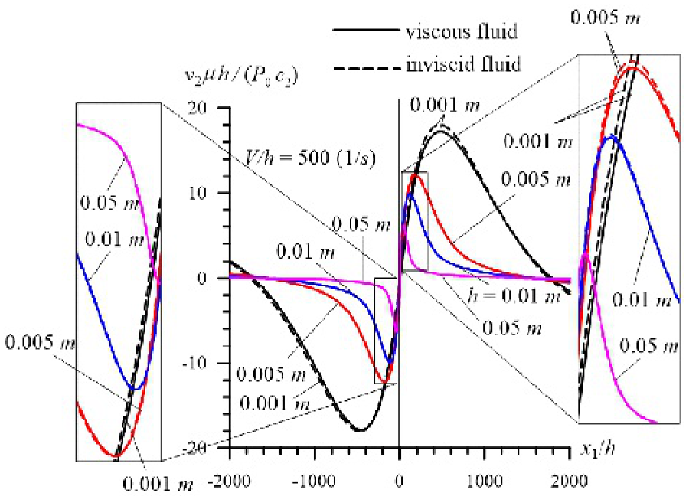

Fig. 2. The distribution of the $\nu_2\mu h/(P_0 c_2)$ with respect of the x_1/h.

Thus, after selection of these materials, the foregoing dimensionless parameters can be determined through the two quantities: h (the thickness of the plate-layer) and V (the velocity of the external moving load). Numerical results which will be discussed below relate to the normal stress acting on the interface plane between the fluid and plate-layer and to the velocities of the fluid (or of the plate-layer) on the mentioned interface plane in the directions of the Ox_1 and Ox_2 axes.

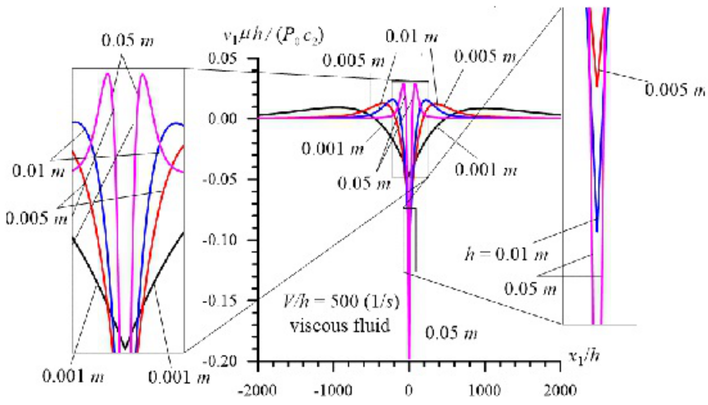

Fig. 3. The distribution of the $\nu_1\mu h/(P_0c_2)$ with respect of the x_1/h in the viscous fluid case.

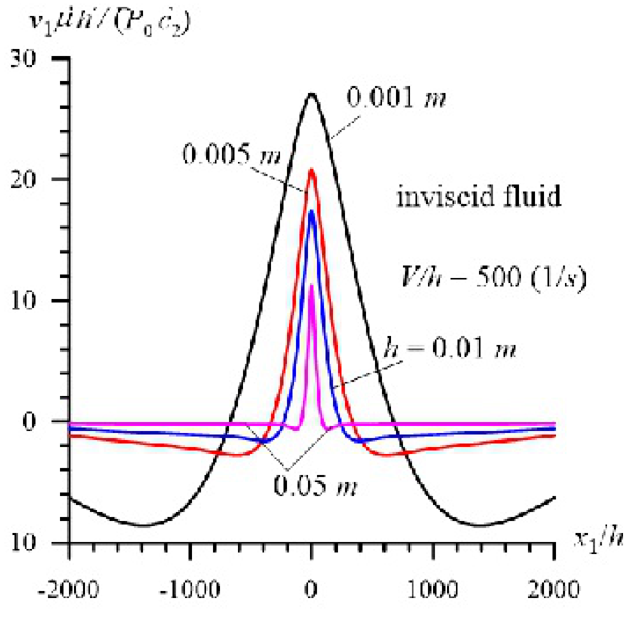

Fig. 4. The distribution of the $\nu_1\mu h/(P_0c_2)$ with respect of the x_1/h in the inviscid fluid case.

Thus, first we investigate the distribution of the studied quantities $T_{22}h/P_0$, $\nu_2\mu h/(P_0c_2)$ and $\nu_1\mu h/(P_0c_2)$ on the interface plane with respect to the dimensionless coordinate x_1/h . We recall that here the coordinate x_1 is determined with respect to the moving coordinate system and, according to the coordinate transformation $x_1' = x_1 - Vt$, $x_2' = x_2$ which was introduced in the beginning of the previous section (the upper prime over the moving coordinates was omitted), the change in the values of the x_1/h (i.e. of the x_1'/h) can also be considered as a change in the values of the dimensionless time Vt/h. Consequently, the distribution of the foregoing quantities with respect to the moving dimensionless coordinate x_1/h can also be considered as the change of those at some fixed point in the frame of the fixed coordinate system with respect to the dimensionless time Vt/h . Graphs of these distributions are given in Fig. 1 (for the $T_{22}h/P_0$), Fig. 2 (for the $\nu_2\mu h/(P_0c_2)$), Fig. 3 (for the $\nu_1\mu h/(P_0c_2)$ in the viscous fluid case) and Fig. 4 (also for the in the inviscid fluid case).

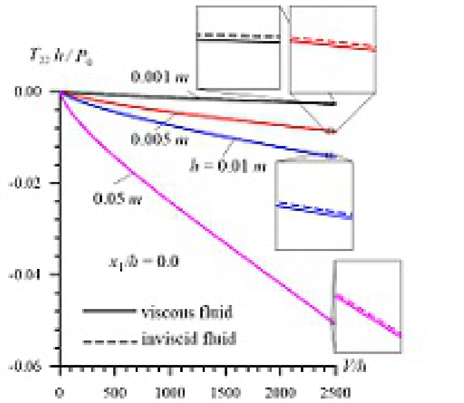

Fig. 5. The graphs of the dependence between $T_{22}h/P_0$ and V/h

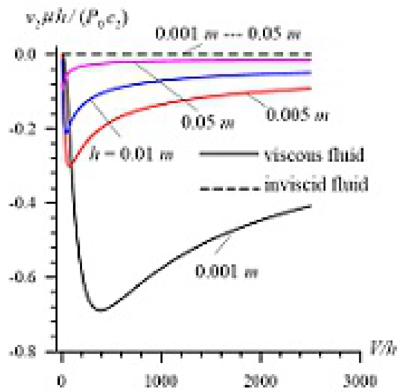

Fig. 6. The graphs of the dependence between $\nu_2\mu h/(P_0c_2)$ and V/h

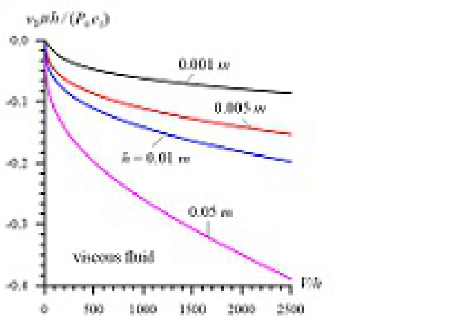

Fig. 7. The graphs of the dependence between $\nu_1\mu h/(P_0c_2)$ and V/h in the viscous fluid case

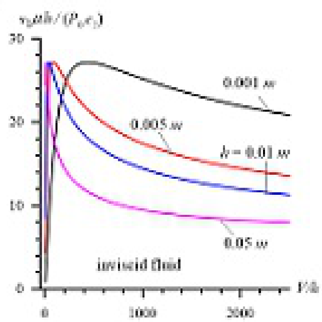

Fig. 8. The graphs of the dependence between $\nu_1\mu h/(P_0c_2)$ and V/h in the inviscid fluid case

Note that these graphs are constructed in the case where $V/h = 500\,(1/s)$ for various values of the h. In Fig. 1 and Fig. 2 the results related to the viscous and corresponding inviscid fluid cases are given simultaneously. Here and below under "inviscid fluid case" ("viscous fluid case") we will understand the case where the selected fluid (i.e. Glycerin) is modeled as inviscid (viscous) one. However, the results obtained for the $\nu_1\mu h/(P_0c_2)$ in the viscous fluid case incompatible with those obtained in the inviscid fluid case. Therefore the results obtained for the $\nu_1\mu h/(P_0c_2)$ in the viscous and inviscid fluid cases are given separately in Fig. 3 and Fig. 4 respectively. The mentioned incompatibility can be explained with disappear of the contact condition $\left.\frac{\partial u_1}{\partial t}\right|_{x_2=-h} = \left.\nu_1\right|_{x_2=-h}$ in (3) for the inviscid fluid case. Consequently, according to the results given in Fig. 3 and Fig. 4, we can conclude that the distribution of the velocity $\nu_1 \mu h/(P_0 c_2)$ cannot be described within the scope of the inviscid fluid model not only in the quantitative sense, but also in the qualitative sense.

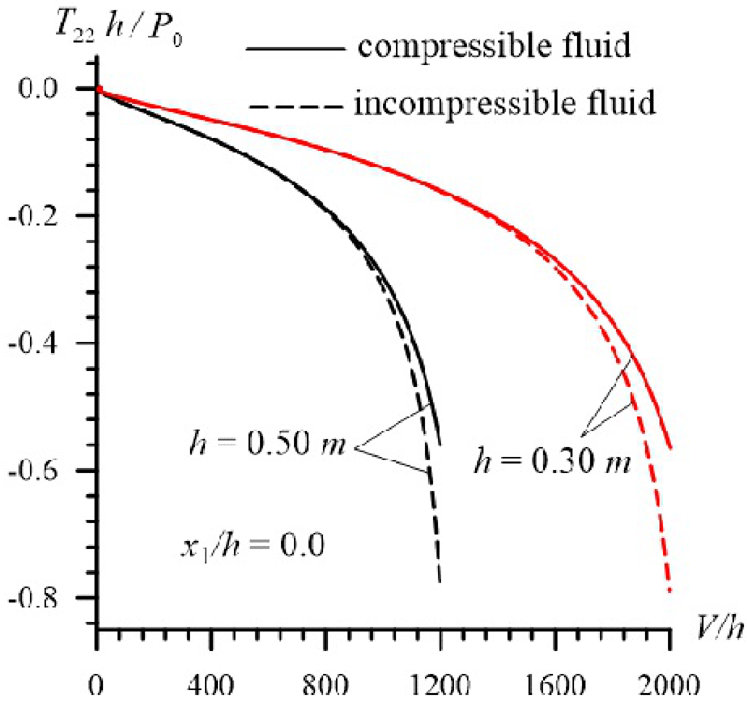

Fig. 9. The influence of the fluid compressibility on the values of the stress $T_{22}h/P_0$

The analysis of the graphs in these figures shows that the attenuation of the investigated quantities with $|x_1/h|$ takes place more rapidly and the width of the action area of the moving load decrease with increasing of the plate thickness h under fixed value of the velocity of the moving load. We again note that the foregoing results can also be estimated as the change of the studied quantities with respect to time at a certain fixed point of the interface plane. For instance, we consider a point which is in a distance L from the origin of the fixed coordinate system. According to the relation $x_1 = L - Vt = 0$, we determine the time $t^* = L/V$ at which the moving load achieves this point. Consequently, the left (right) branch of the graphs given in Fig. 1 - Fig. 4 which illustrate the change of the studied quantities with respect to the x_1/h under $x_1/h \leq 0$ (under $x_1/h \geq 0$) can also be taken as the change of those with respect to time t under $t \geq t^*$(under $t \leq t^*$) at the point which is in a distance L from the origin of the fixed coordinate system.

Now we consider the graphs of the dependence between the studied quantities and the velocity V/h . These graphs for the stress $T_{22}h/P_0$ and for velocities $\nu_2 \mu h/(P_0 c_2)$ and $\nu_1 \mu h/(P_0 c_2)$ are given in Fig. 5, Fig. 6, Fig. 7 and Fig. 8 which are constructed for various values of the h . Under construction of these graphs the values of the studied quantities are calculated at $x_1/h = 0$.

It follows from these graphs that in the case under consideration the influence of the fluid viscosity on the values of the stress $T_{22}h/P_0$ is insignificant, but on the values of the fluid flow velocity is very significant.

Now we consider the results which illustrate the influence of the fluid compressibility on the values of the studied quantities. We recall that the influence of the fluid compressibility is characterized through the parameter Ω_1 (7). Numerical results show that the influence of the fluid compressibility on the studied quantities becomes considerable in the cases where $\Omega_1 \geq 0.25$. However, in the cases where the influence of the fluid viscosity on the distribution of the stress $T_{22}h/P_0$ and velocity $\nu_2\mu h/(P_0c_2)$ disappears almost completely. Under obtaining results related to the incompressible fluid model we assume that $\Omega_1 = 0.0$. Basing this reason, we investigate the influence of the fluid compressibility on the values of the studied quantities within the scope of the inviscid fluid case. Thus, according to the foregoing discussions, an increase in the values of the velocity must increase the difference between the results obtained within the scope of the compressible and incompressible fluid models. However, the investigations shows that there exists such value of the velocity of the moving load under which the absolute values of the studied quantities become infinite and the resonance type event takes place. Note that the existence of the critical velocity is characteristic one for dynamics of the moving load acting on the layered medium. The review of the investigations related to critical velocity of the moving load acting on bi-material elastic systems was made in a paper [8]. However, up to now, we have not found any investigation on the critical velocity of the moving load action on the hydro-elastic systems. Consequently, the results related to the critical velocity, which will be discussed here, are the first attempts on the investigations of the critical velocity of the moving load acting on the hydro-elastic systems. We introduce a notation V_{cr}/a_0 for illustration of the values of the dimensionless critical velocity. Numerical investigations show that the values of the V_{cr}/a_0 are the same for each studied quantities and for each point, i.e. for each value of the x_1/h at which the values of these quantities are calculated. Numerical investigations also show that the values of V_{cr}/a_0 do not depend on the plate thickness h but depend on the compressibility or incompressibility of the fluid. Moreover, it is established that the values of the V_{cr}/a_0 depend also on the mechanical properties of the fluid and of the plate materials. For the selected fluid and plate-layer material we obtain that$V_{cr}/a_0 = 0.3262$ for the incompressible fluid model case and $V_{cr}/a_0 = 03476$ for the compressible fluid model case. Consequently, the compressibility of the fluid causes to increase of the values of the critical velocity.

Now we consider the graphs of the dependence among $T_{22}h/P_0$ and the velocity V/h constructed for the compressible and incompressible fluid models in the case where $V/h < V_{cr}/h$ These graphs are given in Fig. 9 from which follows that the fluid compressibility causes to decrease of the absolute values of the pressure acting on the interface plane between the plate and fluid.

With this we restrict ourselves to analysis of the numerical results and note that the study of the problems which are similar to that considered here will be continued in the further works by the author of the present paper.

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