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On the dynamics of the oscillating moving load acting on the prestrained bilayer slab resting on a rigid foundation

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Abstract. *Within the scope of the three- dimensional linearized theory of elastic wave propagation in the initially stressed body the dynamics of the oscillating moving load acting on the prestrained bilayered slab resting on a rigid foundation is studied. How the oscillation of this load acts its critical velocity is analyzed. Elasticity relations of the materials of the layers are given through the harmonic potential. The numerical results on the critical velocity and on the influence of the problem parameters on these results are presented and discussed.*

Keywords. Critical velocity · initial stains · bilayered slab · oscillating moving load

Mathematics Subject Classification (2010): 74B05

1 Introduction

Investigations of the dynamics of the oscillating moving load have a great significance because the results of these investigations may be used in many fields of modern industry such as design of roadway coverings and bridges intersected by high-speed trains, aircraft carriers, ballistic systems (rail tools, high-speed precise metal working, memories on magnetic disks) and so on. In [1, 2, 11, 12] and others, with the scope of the classic linear theory of electrodynamics, appropriate problems on dynamics of moving forces acting on the system consisting of a covered layer and half-space have been studied. In these works the motion of layers was described by Kirchhoff and Timoshenko theories, the motion of the half-space on the basis of exacts equations of linear theory of elastic waves. It should be noted that many modern problem on the dynamics of the moving load may not be solved within linear theory of elastic waves. The problems connected with dynamics with prestressed (initially) laminated bodies are among these ones. Within the framework of certain conditions, these problems may be solved by employing of the Three dimensional Lineasized Theory of Propagation of elastic waves in Bodies with initial stresses (TLTPEWBIS). Construction of equations of this theory and their application when studying the problems of dynamics prestressed bodies was considered in the monograph [9]. However, up to now only several papers have been devoted to dynamical response of prestressed laminated half-space to moving load [3, 4, 7, 8]. In the paper [7], dynamic response of the system consisting of a layer and prestressed half-plane was

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considered, in which the motion of the facing layer was described by means of Timoshenkos theory of plates, while the motion of a half-plane by TLTPEWBIS. The solution of the appropriate boundary value problem was found by using the Fourier transformation. The numerical investigations were carried out for the case when mechanical relations for the material of the half-plane were given by harmonic potential. Numerical results on the influence of the problem parameters on the critical velocity was estimated. In [8] the problem analyzed in [7] was studied on the basis of a complex potential. In the paper [3], the results obtained in [7, 8] were extended to the case where the motion of the covering layer is also described by the equations of TLTPEWBIS. The influence of the problem parameters on critical velocity of the moving load is studied. However, these investigations are carried out for the case where the materials of the covering layer and half-space are isotropic. The cases where the mentioned materials are anisotropic (orthotropic) are studied in the paper [4].

Note that in theoretical aspect, the problems considered in the above papers may be assumed as mathematical simulation for theoretical investigation of dynamics of underground high-speed transport systems. Obviously, one can precise and generalize the indicated model from different points of view subject to real facts one of which is vibration of moving forces. Furthermore, in a number of cases, a system consisting of bilayer prestrained slab and rigid foundation is a more adequate model for certain classes of real cases [5] than a system consisting of a covering layer and a half-space. Taking the above mentioned ones into account, in the present paper we attempt to develop investigation [3, 4] for dynamics of oscillating moving load acting on finitely prestrained bilayer slab resting of a rigid foundation. We will consider and analyze numerical results on the influence of the oscillation frequency of the moving load on the values of the critical velocity.

2 Problem statement. Basic equations and relations

Let us consider a bilayer slab resting on a rigid foundation , and determine the position of the slabs points in natural (initial) state by Lagrange coordinates in Cartesian system of coordinates $Ox_1x_2x_3$, $(Oy_1y_2y_3)$. Accept that in natural state, the slab's layers occupy the domain $\Omega^{(2)}$ $\Omega^{(1)}$, where

$$\Omega^{(2)} = \left\{ -\infty < x_1 < +\infty, \ -H^{(2)} < x_2 < 0, \ -\infty < x_3 < +\infty \right\},$$

$$\Omega^{(1)} = \left\{ -\infty < x_1 < +\infty, \ (-H^{(1)} - H^{(2)} < x_2 < -H^{(2)}, \right.$$

$$-\infty < x_3 < +\infty \} \tag{2.1}$$

geometry of indicated domains is shown if Fig.1.

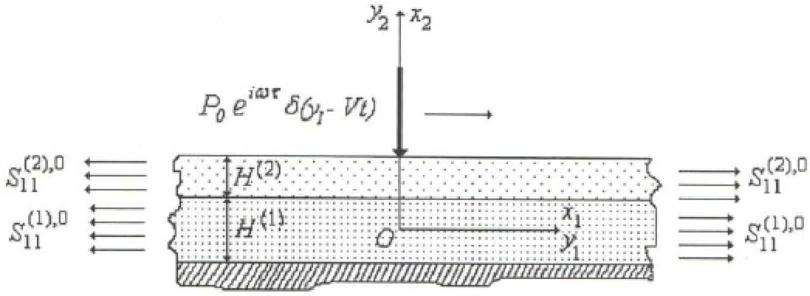

Fig.1.

Along with global system of coordinates $Ox_1x_2x_3$, $(Oy_1y_2y_3)$ to each m-th layer we associate the local system of coordinates $O^{(m)}x_1^{(m)}x_2^{(m)}x_3^{(m)}$, $(Oy_1^{(m)}y_2^{(m)}y_3^{(m)})$, that are obtained from the system of coordinates $Ox_1x_2x_3$, $(Oy_1y_2y_3)$ by parallel transfer along the axis $Ox_2(Oy_2)$, moreover $m = 1, 2$ and $x_1^{(1)} = x_2^{(1)} = x_1$, $x_3^{(1)} = x_3^{(1)} = x_3$, $(y_1^{(1)} = y_1^{(2)} = y_1$, $y_3^{(1)} = y_3^{(2)} = y_3)$

Below, the quantities belonging to the $m - th$ layer, i.e. to the domain $\Omega^{(m)}$ will be denoted by the upper index (m). Furthermore, the quantities belonging to the initial state will be denoted by the upper index "0" . Accept that the materials of the layers are highelastic and these layers before contacting between themselves and with rigid foundation are extended along the axis Ox_1 with uniformly distributed normal forces. Herewith initial deformations in layers are determined by displacements

$$u_i^{(m),0} = (\lambda_i^{(m)} - 1)x_i^{(m)}, \;\; \lambda_i^{(m)} = const_{im}, \; y_i^{(m)} = \lambda_i^{(m)} x_i^{(m)}. \tag{2.2}$$

We will determine the relations of elasticity of materials by means of the harmonic potential

$$\Phi^{(m)} = \frac{1}{2}\lambda^{(m)}(S_1^{(m)})^2 + \mu^{(m)} S_2^{(m)}, \tag{2.3}$$

where

$$S_1^{(m)} = \sqrt{1 + 2\varepsilon_1^{(m)}} + \sqrt{1 + 2\varepsilon_3^{(m)}} - 3,$$

$$S_2^{(m)} = \left(\sqrt{1 + 2\varepsilon_1^{(m)}} - 1\right)^2 + \left(\sqrt{1 + 2\varepsilon_2^{(m)}} - 1\right)^2 + \left(\sqrt{1 + 2\varepsilon_3^{(m)}} - 1\right)^2. \tag{2.4}$$

In expression (3) $\lambda^{(m)}$, $\mu^{(m)}$ denote the material constants $\varepsilon_i^{(m)} (i = 1, 2, 3)$ the principal values of Green's deformation teusov.

Thus, allowing what has been said above, within the bounds of a precewise-homogeneous body with using TLTPEWBIS we study dynamical respouse of the indicated bilyaer slab on vibromotive forces acting on the layer $\Omega^{(2)}$. We consider plane deformation in the plane Oy_1y_2 (i.e. we accept that $\varepsilon_3^{(1)} = \varepsilon_3^{(2)} = \varepsilon_3^{(1),0} = \varepsilon_3^{(2),0} = 0$ and we conduct investigation by using the coordinates connected with initial states. Equations of notion:

$$\frac{\partial Q_{11}^{(m)}}{\partial y_1^{(m)}} + \frac{\partial Q_{12}^{(m)}}{\partial y_2^{(m)}} = \rho^{(m)} \frac{\partial^2 U_1^{(m)}}{\partial t^{(2)}}, \tag{2.5}$$

$$\frac{\partial Q_{21}^{(m)}}{\partial y_1^{(m)}} + \frac{\partial Q_{22}^{(m)}}{\partial y_2^{(m)}} = \rho^{(m)} \frac{\partial^2 U_2^{(m)}}{\partial t^{(2)}}.$$

Mechanical relations:

$$Q_{ij}^{(m)} = \omega_{ij\alpha\beta}^{(m)} \frac{\partial^2 U_\alpha^{(m)}}{\partial y_1^{(m)}},$$

$$(with\ respect\ to\ \alpha\ and\ \beta\,(=1,2)\,we\ conduct\ summation) \tag{2.6}$$

where

$$\omega_{1111}^{(m)} = \frac{\lambda_1^{(m)}}{\lambda_2^{(m)}}\left(\lambda^{(m)} + 2\mu^{(m)}\right), \;\; \omega_{2222}^{(m)} = \frac{\lambda_2^{(m)}}{\lambda_1^{(m)}}\left(\lambda^{(m)} + 2\mu^{(m)}\right),$$

$$\omega_{1122}^{(m)} = \omega_{2211}^{(m)} = \lambda^{(m)}, \;\; \omega_{1212}^{(m)} = \omega_{2121}^{(m)} = \frac{2\mu^{(m)}\lambda_2^{(m)}}{\lambda_1^{(m)} + \lambda_2^{(m)}},$$

$$\omega_{1221}^{(m)} = \omega_{2112}^{(m)} = \frac{2\mu^{(m)}\lambda_2^{(m)}}{\lambda_2^{(m)}\left(\lambda_1^{(m)} + \lambda_2^{(m)}\right)}, \; m = 1, 2. \tag{2.7}$$

In equations (5)-(7) we accept the following denotation: $Q_{ij}^{(m)}$ - are the perturbation of components of Kirchoff's asymmetric tensor of stresses in the m -th layer $U_j^{(m)}$ are the components of perturbations of displacement vector, $\rho^{(m)}$ is the density of the material of the m-th layer.

Taking into account that at initial state the layers of the slab have only normal stress acting on the areas perpendicular to the axis Oy_1 we derive the following relation

$$\lambda_2^{(m)} = \left\lfloor 2\mu^{(m)} + \lambda^{(m)}\left(2-\lambda_1^{(m)}\right)\right\rfloor \left(\lambda^{(m)} + 2\mu^{(m)}\right)^{-1}. \tag{2.8}$$

Write boundary and non-contact conditions:

$$Q_{21}^{(2)} = 0,\ Q_{21}^{(2)} = -P_0 e^{i\omega t}\delta\left(y_1 - Vt\right)\ as\ y_2^{(2)} = \lambda_2^{(2)} = \lambda_2^{(2)} H^{(2)}/2$$

$$\left\{U_i^{(2)}; Q_{2i}^{(2)}\right\}\Big|_{y_2^{(2)}=-\lambda_2^{(2)}H^{(2)}/2} = \left\{U_i^{(1)}; Q_{2i}^{(1)}\right\}\Big|_{y_2^{(1)}=+\lambda_2^{(1)}H^{(2)}/2}, \tag{2.9}$$

$$U_i^{(1)}\Big|_{y_2^{(1)}=-\lambda_2^{(1)}H^{(1)}/2} = 0. \tag{2.10}$$

Notice that in boundary condition (9) V and ω denote velocity and frequency of harmonic vibration of moving force with quantity P_0 furthermore, $\delta(x)$ denotes Dirace delta function. thus, the problem statement is completed.

3 Solution method

Passing to moving systems of coordinates

$$y_1'^{(m)} = y_1^{(m)} - Vt,\ \ y_2'^{(m)} = y_2^{(m)}. \tag{3.1}$$

And representing all the sought for functions in the form

$$g\left(y_1'^{(m)}, y_2^{(m)}, t\right) = \overline{g}\left(y_1'^{(m)}, y_2^{(m)}\right)e^{i\omega t}. \tag{3.2}$$

From (5),(6) we get following equations with respect to amplitude of the replacement vector components.

$$\omega_{1111}'^{(m)}\frac{\partial^2 U_1^{(m)}}{\partial\left(y_1^{(m)}\right)^2} + \omega_{2112}'^{(m)}\frac{\partial^2 U_1^{(m)}}{\partial\left(y_2^{(m)}\right)^2} + \left(\omega_{1122}'^{(m)} + \omega_{2121}'^{(m)}\right)\frac{\partial^2 U_2^{(m)}}{\partial y_1^{(m)}\partial y_2^{(m)}} =$$

$$\frac{1}{\left(C_2^{(m)}\right)^2}\left(V^2\frac{\partial^2 U_1^{(m)}}{\partial\left(y_2^{(m)}\right)^2} - 2i\omega V\frac{\partial U_1^{(m)}}{\partial y_1^{(m)}} - \omega^2 u_1^{(m)}\right)$$

$$\left(\omega_{1212}'^{(m)} + \omega_{2211}'^{(m)}\right)\frac{\partial^2 U_1^{(m)}}{\partial y_1^{(m)}\partial y_2^{(m)}} + \omega_{1221}'^{(m)}\frac{\partial^2 U_2^{(m)}}{\partial\left(y_1^{(m)}\right)^2} + \omega_{2222}^{(m)}\frac{\partial^2 U_2^{(m)}}{\partial\left(y_2^{(m)}\right)^2} =$$

$$\frac{1}{\left(C_2^{(m)}\right)^2}\left(V^2\frac{\partial^2 U_1^{(m)}}{\partial\left(y_1^{(m)}\right)^2} - 2i\omega V\frac{\partial U_2^{(m)}}{\partial y_1^{(m)}} - \omega^2 u_2^{(m)}\right). \tag{3.3}$$

Note that while writing equations(13), the prime over the coordinates y_1, y_2 and the dash over the sought for functions $U_1^{(m)}U_2^{(m)}$ was onite; and the notation $\omega_{nk\alpha\beta}'^{(m)} = \omega_{nk\alpha\beta}^{(m)}/\mu^{(m)}, c_2^{(m)} = \sqrt{\mu^{(m)}/\rho^{(m)}}$ was introduced. this omission will take place and later on, and in the first boundary condition in (9) ; contact conditions in (10) will remain in the above written form.

However the second boundary condition in (9) after passages (11) and (12) are transformed into the following form:

$$Q_{22}^{(2)} = -P_0\delta\left(y_1^{(2)}\right)\ \text{ for }\ y_2^{(2)} = +\lambda_2^{(2)}H^{(2)}/2. \tag{3.4}$$

Thus, investigation of the considered problem is reduced to the solution of equations (13) within the boundes of boundary conditions (9), (first condition in (9),(14) and contact conditions(10). for solving the indicated boundary value problem we use Fourier's exponential transformation with respect to y_1 coordinates

$$f_F\left(s, y_2^{(m)}\right) = \int_{-\infty}^{+\infty} f\left(y_1, y_2^{(m)}\right) e^{-isy_1} dy_1 \tag{3.5}$$

After making some transformations we obtain the following equation for Fourier transformation of the sought-for quantities:

$$\left(-\psi^{(m)} - S^{-2}\omega_{1111}^{\prime(m)}\right) U_{1F}^{(m)} + \omega_{2112}^{\prime(m)} \frac{d^2 U_{1F}^{(m)}}{d\left(\widetilde{y}_2^{(m)}\right)^2} +$$

$$+\left(\omega_{1212}^{\prime(m)} + \omega_{2211}^{\prime(m)}\right) i\overline{s} \frac{d^2 U_{2F}^{(m)}}{d\overline{y}_2^{(m)}} = 0,$$

$$\left(-\psi^{(m)} - s^{-2}\omega_{1221}^{\prime(m)}\right) U_{2F}^{(m)} + \omega_{2222}^{\prime(m)} \frac{d^2 U_{2F}^{(m)}}{d\left(\overline{y}_2^{(m)}\right)^2} +$$

$$+i\overline{s}\left(\omega_{1212}^{\prime(m)} + \omega_{2211}^{\prime(m)}\right) \frac{dU_{1F}^{(m)}}{d\overline{y}_2^{(m)}} = 0, \tag{3.6}$$

where

$$\psi^{(m)} = \frac{\left(C_2^{(1)}\right)^2}{\left(C_2^{(m)}\right)^2}\left(-\overline{s}^2 C^2 + 2\Omega C\overline{s} - \Omega^2\right),$$

$$C = \frac{V}{C_2^{(1)}},\ \Omega = \frac{\omega H^{(2)}}{C_2^{(1)}},\ \overline{y}_2^{(m)} = \frac{y_2^{(m)}}{H^{(2)}},\ \overline{s} = sH^{(2)}. \tag{3.7}$$

From equations (16) we get:

$$\frac{d^2 U_{2F}^{(m)}}{d\left(\overline{y}_2^{(m)}\right)^4} + a_1^{(m)} \frac{d^2 U_{2F}^{(m)}}{d\left(\overline{y}_2^{(m)}\right)^2} + b_1^{(m)} U_{2F}^{(m)} = 0, \tag{3.8}$$

$$\frac{dU_{1F}^{(m)}}{d\overline{y}_2^{(m)}} = i\left(a^{(m)} + U_{2F}^{(m)} + b^{(m)} \frac{d^2 U_{2F}^{(m)}}{d\left(\overline{y}_2^{(m)}\right)^2}\right), \tag{3.9}$$

where

$$a^{(m)} = \frac{-\left(\psi^{(m)} + \overline{s}^2\omega_{1212}^{\prime(m)}\right)}{\overline{s}\left(\omega_{1212}^{\prime(m)} + \omega_{2211}^{\prime(m)}\right)},\ b^{(m)} = \frac{\omega_{2222}^{\prime(m)}}{\overline{s}\left(\omega_{1212}^{\prime(m)} + \omega_{2211}^{\prime(m)}\right)},$$

$$a_1^{(m)} = \left[b^{(m)}(-\psi^{(m)} - \overline{s}^2\omega_{1111}^{\prime(m)}) + a^{(m)}\omega_{2112}^{\prime(m)} +\right.$$

$$\left.+\overline{s}\left(\omega_{1122}^{\prime(m)} + \omega_{2121}^{\prime(m)}\right)\right](\omega_{2112}^{\prime(m)} + b^{(m)})^{-1}$$

$$b_1^{(m)} = a^{(m)}(-\psi^{(m)} - \overline{s}^2\omega_{1111}^{\prime(m)})(\omega_{2112}^{\prime(m)} b^{(m)})^{-1}. \tag{3.10}$$

We find the solution of equation (18) in the form:

$$U_{2F}^{(m)} = A_1^{(m)} e^{K_1^{(m)}\overline{y}_2^{(m)}} + A_2^{(m)} e^{-K_1^{(m)}\overline{y}_2^{(m)}} +$$

$$+A_3^{(m)} e^{K_2^{(m)}\overline{y}_2^{(m)}} + A_1^{(m)} e^{-K_2^{(m)}\overline{y}_2^{(m)}}, \tag{3.11}$$

where

$$K_1^{(m)} = \sqrt{-\frac{a_1^{(m)}}{2} + d_1^{(m)}},\ K_2^{(m)} = \sqrt{-\frac{a_1^{(m)}}{2} - d_1^{(m)}},$$

$$d_1 = \sqrt{-\frac{a_1^{(m)}}{4} - b_1^{(m)}}. \tag{3.12}$$

Thus, from equations (21), (19) and (6) we find Fourier transformations of all sought-for quantities. We calculate the unknown $A_1^{(1)}(S), ...,\ A_4^{(1)}(S), A_1^{(2)}(S), ..., A_4^{(2)}(S)$ contained in these transformations from boundary conditions $(9)_1$, (14) and contact conditions from (10), that form a closed system of algebraic equations.

$$f(y_1, y_2) = \frac{1}{2\pi} \int\limits_{-\infty}^{+\infty} f_F\,(s, y_2)\, e^{isy_1} ds. \tag{3.13}$$

From algebraic equations we find the mentioned unknowns and then use the inverse transformation(23), for defining the originals of the sought-for functions. Now consider the calculation of integral (23). Notice that in the case when $\Omega = 0$ (as in papers [8,9] or in the case when $C = 0$ (as in paper [11,12] calculation of the integrals(23) is reduced to calculation of the integrals for $\frac{1}{\pi}\int\limits_0^{+\infty} f_F\,(s, y_2)\cos(s, y_1)ds$ for $U_2^{(m)}$, $\sigma_{22}^{(m)}, \sigma_{11}^{(m)}, \varepsilon_{22}^{(m)}, \varepsilon_{11}^{(m)}$ and $U_1^{(m)}$ $\sigma_{12}^{(m)}$, $\varepsilon_{12}^{(m)}$. However in the case when simultaneously $C > 0$ and $\Omega > 0$ such conclusions violate with the member $2\Omega c\overline{S}$, that is contained in the expression for $\psi^{(m)}$ (17). therefore, in this case integral (23) should be calculated without above simplification, i.e. using the following formula:

$$\frac{1}{2\pi}\int\limits_0^{+\infty} (\cdot) e^{isx_1} ds \approx \frac{1}{2\pi}\int\limits_{-S^*\int}^{+S^*} (\cdot)\cos(sx_2)ds + \frac{i}{2\pi}\int\limits_{-S^*}^{+S^*} (\cdot)\sin(sx_1)ds. \tag{3.14}$$

And the values of S^* are calculated from requirements of numerical convergence of the appropriate algorithm. Introduce the following denotation:

$$\sigma_{ijc}^{(m)} = \frac{1}{2\pi}\int\limits_{-S^*}^{+S^*} \sigma_{ijc}^{(m)}\cos(sy_1)ds,\ \sigma_{ijc}^{(m)} = \frac{1}{2\pi}\int\limits_{-S^*}^{+S^*} \sigma_{ijF}^{(m)}\sin(sy_1)ds,$$

$$U_{ic}^{(m)} = \frac{1}{2\pi}\int\limits_{-S^*}^{+S^*} U_{iF}^{(m)}\cos(sy_1)ds,\ U_{ic}^{(m)} = \frac{1}{2\pi}\int\limits_{-S^*}^{+S^*} U_{iF}^{(m)}\sin(sy_1)ds.$$

$$\left\{\widetilde{\sigma}_{22}^{(m)};\ \widetilde{\sigma}_{11}^{(m)};\ \widetilde{U}_{22}^{(m)}\right\} = \left\{\left|\widetilde{\sigma}_{22}^{(m)}\right| e^{i\alpha_{22}^{(m)}}, \left|\widetilde{\sigma}_{11}^{(m)}\right| e^{i\alpha_{11}^{(m)}}, \left|\widetilde{U}_2^{(m)}\right| e^{i\alpha_2^{(m)}}\right\},$$

$$\left\{\widetilde{\sigma}_{12}^{(m)};\ \ \widetilde{U}_1^{(m)}\right\} = \left\{i\left|\widetilde{\sigma}_{12}^{(m)}\right| e^{i\alpha_{21}^{(m)}}, i\left|\widetilde{U}_1^{(m)}\right| e^{i\alpha_1^{(m)}}\right\}, \tag{3.15}$$

where

$$\left|\widetilde{\sigma}_{ij}^{(m)}\right| = \sqrt{\left(\sigma_{ijc}^{(m)}\right)^2 + \left(\sigma_{ijc}^{(m)}\right)^2},\ \left|\widetilde{U}_i^{(m)}\right| = \sqrt{\left(U_{ic}^{(m)}\right)^2 + \left(U_{is}^{(m)}\right)^2}$$

$$\tan\alpha_{ii}^{(m)} = \frac{\sigma_{iis}^{(m)}}{\sigma_{iic}^{(m)}};\ \tan\alpha_2^{(m)} = \frac{U_{2s}^{(m)}}{\sigma_{2c}^{(m)}};$$

$$\tan\alpha_{12}^{(m)} = \frac{\sigma_{12s}^{(m)}}{\sigma_{12c}^{(m)}};\ \tan\alpha_1^{(m)} = \frac{U_{1c}^{(m)}}{\sigma_{1s}^{(m)}}. \tag{3.16}$$

After above mentioned ones the values of stress and displacements are determined from the expression

$$\left\{\sigma_{ij}^{(m)};\ U_i^{(m)}\right\} = \mathrm{Re}\left(\left\{\widetilde{\sigma}_{ij}^{(m)};\ \widetilde{U}_i^{(m)}\right\} e^{i\omega t}\right) \tag{3.17}$$

according to which we get

$$\sigma_{ij}^{(m)} = \left|\widetilde{\sigma}_{ij}^{(m)}\right| \cos\left(\alpha_{ij}^{(m)} + \omega t\right),\ U_i^{(m)} = \left|\widetilde{U}_i^{(m)}\right| \cos\left(\alpha_i^{(m)} + \omega t\right). \tag{3.18}$$

One of the basic issues of dynamics of moving loads is investigation of stress and displacement distribution. along with this, before this investigation it is necessary to study the issues of critical velocity i.e. to determine the value of critical velocity and to determine influence of changes of problem parameters on these values. In the paper, we will stop namely on the issues of critical velocity.

State an algorithm for defining critical velocity. In this connection notice that numerical investigations show that for each chosen value of dimensionless velocity $U_{iF}^{(m)}$, $\sigma_{ijF}^{(m)}$ the subintegrand quantities $SH^{(2)}$ a have singular points with respect to. And these are simultaneously the solutions of the equation

$$\det\left\|\alpha_{nm}\left(C\left(SH^{(2)}\right)\right)\right\| = 0,\ \ n, m = 1, 2, ..., 8, \tag{3.19}$$

where α_{nm} are the coefficients of unknowns in algebraic system of equations obtained from boundary condition (9) and contact conditions (10). consequently, the singularity order (denote as r) of integrated quantities consider with the order of the roods of equation (29). It is known that if $0 \leq r < 1$, integral (24) may be calculated by using the well known algorithm. For $r = 1$ calculation of the algorithm is performed in the sense of Cancly's principal value. In the case $r > 1$ the integral doesn't accept definite value , and the velocity corresponding to this case is determined as " critical velocity" At critical velocity, the resonance-type phenomenon occurs.Obviously , the critical velocity corresponds to local minimum (or maximum) of the function $C = C(SH^{(2)})$, satisfying equation (29). Now briefly we consider the special case of the problem under consideration and state main problem of mechanics related to these cases.

Case 1. $\Omega = 0, C \neq 0$. We get a problem on fores moving with constant velocity and influencing on bilayer slab resting on rigid foundation.In this case, as in [8, 9], the basic issuse of the investigation is determination of the value of critical velocity (denote then by C_{or} and determine distribution of contact stresses for $C < C_{or}$

Case 2. $\Omega \neq 0, C = 0$ We get a stadionary dynamic problem for the system under consideration subjected to the action of forces harmonically changing in time. According to [11, 12] the basic issuce of this case is to determine the resonance value of frequency Ω at which displacement and stress get their own absolutely maximal value.

Case 3. $\Omega \neq 0, C \neq 0$ First of all note that all the results belofing to this case are not obtained by simple superposition of previous two cases. the stated one is agfirmed with the expression $\psi^{(m)}$ cited in (17). and the main goal of the investigation is to determine the influeme of the frequenag Ω on the value of C_{or}, and also to determine the influence of velocity on "resonance" value of the frequency Ω. In all above cases, specific studies are based on calculation of integral (24) and the algorithm developed in the papers [11,12] is used.

4 Numerical results and their discussions

Accept that $\lambda^{(1)}/\mu^{(1)} = \lambda^{(2)}/\mu^{(2)} = 1.5$ and we carry out all numerical investigations for this case. For illustrating competence of the used algorithms of PC programmes, at first we consider case 1 that was also considered in the papers [6,8,9] and within the bounds of asscemtious $\mu^{(2)}/\mu^{(1)} = 2$, $\lambda_1^{(2)}/\lambda_1^{(1)} = 1.0$ $\rho^{(1)}/\rho^{(2)} = 0.5$ we study influence of change of $H^{(1)}/H^{(2)}$ on the value of the critical velocity C_{or}. According to the known state, at subsonic mode motion, with growth of $H^{(1)}/H^{(2)}$ the value of (C_{or}) should, approach to the appropriate value of C_{or}, obtained for the system composed of a facing layer and a half-space that were considered in [6,8,9]. The appropriate results are given in Table 1, and they affirm the stated states.

Now let's consider numerical example related to the case 3. Notice that in case 1 the value of critical velocity is determined according, to the graphs of dependence between C and $SH^{(2)}$ structured only for the values of $SH^{(2)} > 0$ as the function $DSH^{(2)} = \det \left\| \alpha_{nk} \left(SH^{(2)} \right) \right\|$ is an even function of argument of $SH^{(2)}$. Hoverer in case 3, as it was stated above, indicating evenness of the function $D\left(SH^{(2)}\right)$ and thereby the evenness of the function $C = C\left(SH^{(2)}\right)$ doesint hold. Therefore in case 3, when determining the values of C_{or} it is necessary to use the graph of the function $C = C\left(SH^{(2)}\right)$ that is constructed not only for the value of $SH^{(2)} < 0$. For illustrating the above stated one, Left;s consider the graphs of the function $C = C\left(SH^{(2)}\right)$ cited in Fig. 2 and that were constructed within the bounds of suppositions $\mu^{(2)}/\mu^{(1)} = 0,5, \;\; H^{(1)}/H^{(2)} = 0,5 \;\; \rho^{(1)}/\rho^{(2)} = 1,6, \;\; \lambda_1^{(2)}/\lambda_1^{(1)} = 1.0$

As seen from the graph, in the case $\Omega = 0.0$ the branches of the function $C = C\left(SH^{(2)}\right)$ constructed for $SH^{(2)} < 0$ and for $SH^{(2)} > 0$ are symmetric with respect to the strightline, determined with the equation $SH^{(2)} = 0.0$ However in the cases $\Omega > 0.0$ the indicated symmetry violates, and the value of dimensionless velocity corresponding to the case $dc/d\left(SH^{(2)}\right) = 0$ for the branch constructed for $SH^{(2)} < 0$ becomes less than for the branch constructed for $SH^{(2)} < 0$. Consequently, the value of least of oritical velocity is determined according to the branch of the function $C = C\left(SH^{(2)}\right)$ constructed for $SH^{(2)} < 0$. Therewith the value of C_{or} decreases with increasing Ω. However, the value of critical velocity determined by the branch of the function $C = C\left(SH^{(2)}\right)$ constructed for $SH^{(2)} > 0$ increases clue to increase of Ω.

From practical point of view the least critical velocities are more important than the subsequent (in quantity) critical velocities. Therefore, we consider influmence of change of problem parameters on its value, i.e. the least value of critical velocity. Therewith we accept $\mu^{(2)}/\mu^{(1)} = 0,5$, $\rho^{(1)}/\rho^{(2)} = 0,2$ and study influence of initial strains of of slab's layers i.e. the values of parameters $\lambda_1^{(2)}$ $\lambda_1^{(1)}$ and influence of $H^{(1)}/H^{(2)}$ on the value of C_{or} for different values of Ω. Note that numerical result for theis investigation are given in Tables 2,3 and 4 for the values of $\Omega > 0.0; 0,1$ and 0,2 respectively. the conclusions from the analysis of these results are cited in the following section.

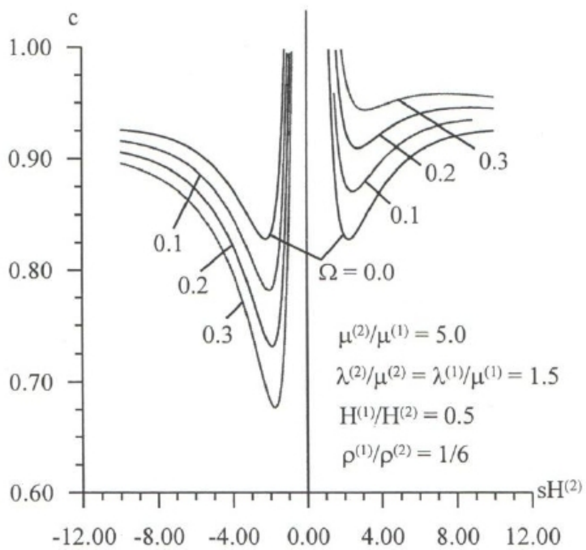

Fig.2.

Table 1.

$H^{(1)}/H^{(2)}$						
0.5	1.0	1.5	2.0	4.0	6.0	∞
$2{*}0.9084$	$2{*}0.8812$	$2{*}0.8651$	$2{*}0.8556$	$2{*}0.8431$	$2{*}0.8415$	0.8415[8.9]
						0.8370[6]

Table 2.

$2*\lambda_1^{(2)}/\lambda_1^{(1)}$	$H^{(1)}/H^{(2)}$		
	0.5	2.0	5.0
1.0/1.0	0.8266	0.7142	0.6871
1.02/1.0	0.8657	0.7646	0.7393
1.05/1.0	0.9306	0.8326	0.8088
1.07/1.0	0.9551	0.8740	0.8505
1.0/1.02	0.8263	0.7152	0.6898
1.0/1.05	0.8259	0.7037	0.6771
1.0/1.07	0.8393	0.7068	0.6846
1.0/1.10	0.8399	0.7113	0.6937
1.0/1.15	0.8411	0.7184	0.7042
1.0/1.20	0.8427	0.7253	0.7113

Table 3.

$2{*}\lambda_1^{(2)}/\lambda_1^{(1)}$	$H^{(1)}/H^{(2)}$		
	0.5	2.0	5.0
1.0/1.0	0.7800	0.6270	0.5603
1.02/1.0	0.8204	0.6793	0.6121
1.05/1.0	0.8772	0.7501	0.6805
1.07/1.0	0.9130	0.7932	0.7213
1.0/1.02	0.7794	0.6278	0.5639
1.0/1.05	0.7786	0.6128	0.5190
1.0/1.07	0.7950	0.6158	0.5285
1.0/1.10	0.7953	0.6203	0.5420
1.0/1.15	0.7963	0.6276	0.5625
1.0/1.20	0.7976	0.6348	0.5804

Table 4.

$2{*}\lambda_1^{(2)}/\lambda_1^{(1)}$	$H^{(1)}/H^{(2)}$		
	0.5	2.0	5.0
1.0/1.0	0.7293	0.5268	0.3729
1.02/1.0	0.7709	0.5800	0.4188
1.05/1.0	0.8294	0.6521	0.4796
1.07/1.0	0.8663	0.6960	0.5161
1.0/1.02	0.7285	0.5272	0.3763
1.0/1.05	0.7273	0.5095	0.3812
1.0/1.07	0.7353	0.5121	0.3165
1.0/1.10	0.7779	0.5161	0.3287
1.0/1.15	0.8444	0.5228	0.3479
1.0/1.20	0.9022	0.5295	0.3658

5 Conclusion

Thus, from the analytics of the obtained results we can make the following conclusions:

-the value of C_{or} monotonically increases with growth of the thickness of the lover layer whose rigidity is less than the rigidity of the facing layer;

-the initial extension of the upper layer of the slab increases the value of C_{or} , however the influence of initial extension of the lower layer on the value of C_{or} is nonmonotone:

The increase of vibrations of the moving load, i.e. the increase of the values of Ω reduces to decrease of critical velocity.

Investigation and analysis of distributed stresses acting on the surfaces of layer's contacts will be conducted in other work of the author.

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