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An integrate model for the liquid filtration process and layer-well dynamic relation in the horizontal wells

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Abstract. *A mathematical model has been constructed and the boundary value problem of unsteady horizontal well reservoir filtration has been solved in view of reservoir and permeable pipe string dynamic relation. As a result, it has been determined the formation productivity depending on the depth of the horizontal part of the pipe string and the parameters of the formation-borehole system. A formula depending on the permeability of pipe wall and describing the pressure loss in the horizontal part of the pipeline is derived.*

Keywords. liquid · unsteady filtration · horizontal well · La'place's transfer equation · pressure · rate · dynamic coupling · Bessel's equation

Mathematics Subject Classification (2010): 42A05 · 42B25 · 26D10 · 35A23

1 Introduction

This paper is devoted to the study of nonstationary liquid filtration process in view of the "well-reservoir" and permeable tube interdynamical relation. There are a large number of works on study of filtration properties for horizontal wells [1-6], where it was considered a separate problems on this topics. So, in [1], it was considered just liquid motion in the tube with permeable wall. In[2], it was studied the fluid motion in permeable tube without regarding the fluid filtration process in layer. A motion of non Newtonian liquid is considered in [3] in conical tube with permeable wall, where is not considered the layer radial filtration and liquid motion relation in tube. In [4,5] it was studied the layer filtration proses that does not take into the account the impact of the liquid motion in horizontal and vertical well bore parts. Since all processes of liquid, filtration, motion and flow through permeable wall occur simultaneously, there is a need to study of filtration proses in horizontal well, in view of formation-borehole dynamical relation in permeable tube. Therefore, the study of hydrodynamical processes subject to the formation-borehole dynamical relation in the permeable wells is an essential factor in order to achieve a good exploitation performance of the horizontal wells.

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In this paper, it is constructed an integrate model for non stationary filtration process in layer that takes into the account the permeability of tubes and the "formation-borehole" dynamical relation, which is more adequate to real well conditions.

In the horizontal wells, productivity increases by several times in comparison with the vertical wells [7,18,23,25,16,11]. As a result, oil extraction increases. Increasing the number of horizontally drilled wells requires a study of hydrodynamic processes occurring during their exploitation. Considerable number of papers are devoted to the investigation of filtration processes in the horizontal wells [15,8,6,10]. Most of them are devoted to the study of stationary filtration. But investigation of hydrodynamic processes occurring in horizontal wells, using the dynamic "reservoir-well" relation and the permeability of horizontal part of the pipe string was not studied so much [24,21,3,5].

In this paper, we study the hydrodynamic processes during exploitation of horizontal wells using the dynamic "formation-well" relation and the permeability of pipes' horizontal part that is perforated or equipped the filter system. Well performance largely depends on the length of horizontal part. The problem of finding the effective horizontal part length has of important practical and scientific interest [22,19,13]. We determine the horizontal pipe length starting from the study in combination the reservoir liquid filtration processes, considering boundary permeability through the walls of pipe and fluid flow over the horizontal and vertical sections of well. For other boundary condition in permeable pipe see, [6, 17,9,4], where reservoir-well dynamical relation did not considered.

2 Liquid filtration

Consider a horizontal wellbore located in the center of the circular isotropic formation. We assume that the filtration processes in the formation occurs flatly-radially. Then, ignoring gravity of liquid in the accepted model the filtration equation will be as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \Delta P}{\partial r}\right) = \frac{1}{\chi}\frac{\partial \Delta P}{\partial t}, \quad r_c < r < R_k, \quad t > 0, \tag{2.1}$$

where $\chi = \frac{k}{\mu\beta}$, $\Delta P = P - P_k$, μ is dynamic viscosity, β -permeability index, r -coordinate, t -time, r_c -well radius, R_k -radius of reservoir boundary. Initial and boundary conditions are

$$\Delta P|_{t=0} = P(0) - P_k \quad \text{for} \quad r_c < r < R_k \tag{2.2}$$

$$\Delta P|_{r=r_c} = P_c(t) - P_k \quad \text{for} \quad t > 0 \tag{2.3}$$

$$\Delta P|_{r=R_k} = 0 \quad \text{for} \quad t > 0. \tag{2.4}$$

where $P_c(t)$ is wall pressure at the horizontal part, P_k -pressure on reservoir boundary.

Solution (2.1) with restrictions (2.2)-(2.4) and constant $P_c(t) \equiv P_c(0)$ is following (see, [23]):

$$\Delta P(r,t) = \frac{P_k - P_3(0)}{\ln\frac{R_k}{r_c}} \ln\frac{R_k}{r} + \pi\left(P_k - P_c\right)\sum_{n=1}^{\infty}\frac{J_0\left(x_n\frac{R_k}{r_c}\right)J_0(x_n)}{J_0^2\left(x_n\frac{R_k}{r_c}\right) - J_0^2(x_n)}u\left(x_n\frac{r}{r_c}\right)\exp\left(-\frac{x_n^2\chi t}{r_c^2}\right), \tag{2.5}$$

where

$$u\left(x_n\frac{r}{r_c}\right) = J_0\left(x_n\frac{r}{r_c}\right)Y_0\left(x_n\frac{R_k}{r_c}\right) - J_0\left(x_n\frac{R_k}{r_c}\right)Y_0\left(x_n\frac{r}{r_c}\right),$$

x_n are roots of transcendental equation

$$J_0(x)Y_0\left(x\frac{R_k}{r_c}\right) - J_0\left(x\frac{R_k}{r_c}\right)Y_0(x) = 0;$$

J_0, Y_0 - Bessel functions of first and second order.

To pass from (2.5) to solution (2.1) with variable $P_c(t)$ use the Duhamel integral [25]:

$$\Delta P(r,t) = f_1(0)\Delta P_1(r,t) + \int_0^t \dot{f}_1(\tau)\Delta P_1(r,t-\tau)d\tau, \tag{2.6}$$

where $f_1(t) = P_c(t) - P_k \cdot \Delta P_1(r,t)$ and

$$\Delta P_1 = \frac{\Delta P(r,t)}{P_k - P_c(0)} = -\frac{\ln\frac{R_k}{r}}{\ln\frac{R_k}{r_c}} + \pi\cdot\sum_{n=1}^{\infty}\frac{J_0\left(x_n\frac{R_k}{r_c}\right)J_0(x_n)}{J_0^2\left(x_n\frac{R_k}{r_c}\right) - J_0^2(x_n)}\exp\left(-\frac{x_n^2\chi t}{r_c^2}\right)u\left(x_n\frac{r}{r_c}\right). \tag{2.7}$$

From (2.6) and (2.7) it follows that

$$\Delta P(r,t) = (P_c(0) - P_k)\,\Delta P_1(r,t) - \int_0^t \dot{P}_c(\tau)\Delta P_1(r,t-\tau)d\tau. \tag{2.8}$$

3 Fluid motion in horizontal wellbore part

In the current context, the length of the horizontal wells reaches significant values. Formation fluid passes through walls casing column and starts to flow through. After, the liquid rises up to mouth of well through the vertical portion. Below we compose a mathematical model for permeability process and flow at horizontal section of wellbore.

Consider a horizontal well (Fig. 1). Let's place the origin at the end of the well and direct axis horizontally to the left. The equation of fluid through the permeable wall will as [14]:

$$\frac{\partial Q}{\partial x} = \lambda\left[P_c(x,t) - p(x,t)\right], \tag{3.1}$$

where Q is fluid consumption along the pipe, λ -coefficient characterizing pipe wall permeability, P -liquid pressure at every pipe section, P_c -pressure over the horizontal well part. By [14,15] fluid flow equation through a pipe is

$$\frac{\partial Q}{\partial t} + \frac{Q}{f}\frac{\partial Q}{\partial x} = -\frac{f}{\rho}\frac{\partial P}{\partial x} - \frac{\alpha}{\rho}, \tag{3.2}$$

where t is time, f -pipe passage area, ρ -fluid density, α -coefficient of resistance. By virtue of low velocity, the second term in (3.2) can be neglected (see, [14]). Hence from (3.2) it follows that

$$\frac{\partial Q}{\partial t} = -\frac{f}{\rho}\frac{\partial P}{\partial x} - \frac{\alpha}{\rho}, \tag{3.3}$$

To determine the initial conditions, first consider the steady motion. From (3.1) and (3.3) it follows that

$$\frac{d^2P}{dx^2} - \frac{\alpha\lambda}{f}P = -\frac{\alpha\lambda}{f}P_c(0), \tag{3.4}$$

where $P_c(0)$ is initial value $P_c(0)$. The boundary conditions are

$$Q(0,0) = 0 \tag{3.5}$$

$$Q(0,l) = Q_0, \tag{3.6}$$

where l is length of horizontal portion of tubing column, Q_0 - fluid consumption at entrance into vertical borehole section. From (3.3) it follows that

$$Q(0,x) = -\frac{f}{\alpha}\frac{dP}{dx}. \tag{3.7}$$

Taking into the account (3.7) solution (3.4) satisfying the constrains (3.5) and (3.6) looks as

$$P(0,x) = P_c(0) - \frac{Q_0\,ch\sqrt{\frac{\alpha\lambda}{f}}\,x}{\frac{f}{\alpha}\sqrt{\frac{\alpha\lambda}{f}}\,sh\sqrt{\frac{\alpha\lambda}{f}}\,l}. \tag{3.8}$$

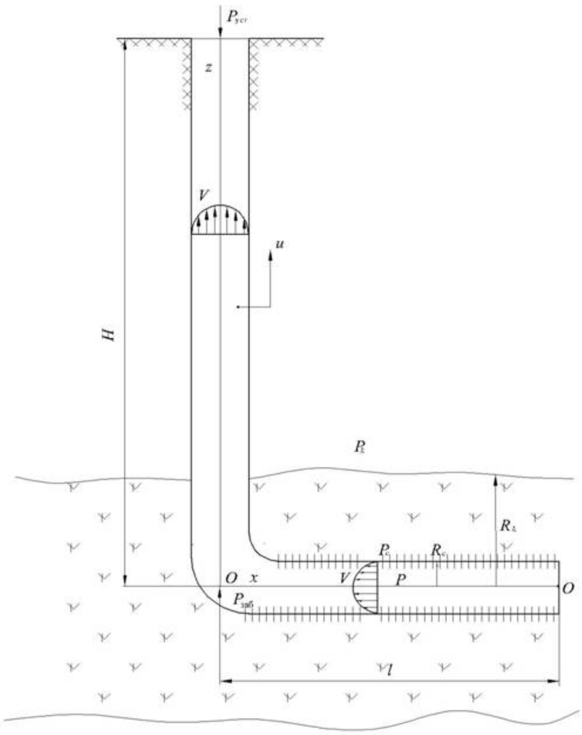

Fig.1. Calculation model.

Put $x = l$ in (3.8):

$$Q_0 = \Big[P_c(0) - P(0,l)\Big] \cdot \frac{f}{\alpha}\sqrt{\frac{\alpha\lambda}{f}}\, th\sqrt{\frac{\alpha\lambda}{f}}\, l, \tag{3.9}$$

Now consider the unsteady flow. The equation of liquid motion has the form (3.2). Initial and boundary conditions are

$$Q(0,x) = \frac{Q_0 ch\sqrt{\frac{\alpha\lambda}{f}}\, x}{sh\sqrt{\frac{\alpha\lambda}{f}}\, l}, \qquad 0 < x < l, \tag{3.10}$$

$$Q(0,t) = 0, \qquad t > 0,$$

$$Q(l,t) = Q_0 + q(t), \qquad t > 0,$$

where $Q_0 + q(t)$ is fluid rate through the pipe section with unknown and time dependent function $q(t)$. Accept the following notation

$$t = \frac{\rho\lambda}{f}\tau, \quad Q = g(x,t)\cdot exp\left(-\frac{\alpha}{\rho}t\right), \quad g(x,t) = G\left(\frac{f}{\rho\lambda}t, x\right).$$

Then

$$\frac{\partial G}{\partial \tau} = \frac{\partial^2 G}{\partial x^2}, \tag{3.11}$$

boundary conditions are

$$G(\tau, 0) = 0, \tag{3.12}$$

$$Q(r,l) = \Big[Q_0 + q\left(\frac{\rho\lambda}{f}\tau\right)\Big] \cdot exp\left(\frac{\rho\lambda}{f}\tau\right) = q_1(\tau), \tag{3.13}$$

Apply the Laplace transformation as in [12,20] by using (3.11) and the boundary conditions (3.12), (3.13), and initial conditions (3.10) it follows that

$$Q(x,t) = \frac{x}{l}\left[Q_0 + q(t)\right] + \sum_{n=1}^{\infty} (-1)^{n+1} \frac{2\pi n}{l^2} \frac{f}{\rho\lambda} sin \frac{\pi n x}{l}$$
$$\times \int_0^t (Q_0 + q(\eta))\, exp\Big[-\left(\frac{\alpha}{\rho} + \frac{\pi^2 n^2}{l^2}\frac{f}{\rho\lambda}\right)(t-\eta)\Big] d\eta$$
$$+ \sum_{n=1}^{\infty} \frac{2.(-1)^{n+1}\pi n Q_0}{\pi^2 n^2 + \frac{l^2\alpha\lambda}{f}} \cdot exp\Big[-\left(\frac{\alpha}{\rho} - \frac{\pi^2 n^2 f}{l^2 \rho\lambda}\right) t\Big]\, sin \frac{\pi n x}{l}.$$

4 Balance of inflow and liquid flow in the horizontal part of well.

The amount of fluid inflow from reservoir per unit time should be equal to the amount passing through cross section of vertical tube. The inflow from reservoir per unit time is

$$Q(r,t) = \frac{2\pi r_c k}{\mu} \int_0^l \frac{\partial P}{\partial r}\, dx. \tag{4.1}$$

Fluid consumption at the end of horizontal pipe is

$$Q(l,t) = Q_0 + q(t). \tag{4.2}$$

Using (4.1) and (4.2) by formula (2.8), we have

$$Q_0 + q(t) = \frac{2\pi l k \left(P_k - P_c(0)\right)}{\mu} \Big[\frac{1}{\ln\frac{R_k}{r_c}} - 2\sum_{n=1}^{\infty} \frac{J_0^2\left(x_n \frac{R_k}{r_c}\right) \exp\left(-\frac{x_n^2 \chi t}{r_c^2}\right)}{J_0^2(x_n) - J_0^2\left(x_n \frac{R_k}{r_c}\right)}\Big]$$
$$- \frac{2\pi l k}{\mu} \int_0^t \dot{P}_c(\tau) \Big[\frac{1}{\ln\frac{R_k}{r_c}} - 2\sum_{n=1}^{\infty} \frac{J_0^2\left(x_n \frac{R_k}{r_c}\right) \exp\left(-\frac{x_n^2 \chi (t-\tau)}{r_c^2}\right)}{J_0^2(x_n) - J_0^2\left(x_n \frac{R_k}{r_c}\right)}\Big] d\tau. \tag{4.3}$$

By Laplace's transform it follows from (4.3) that

$$\overline{P}_c(s) = \frac{P_k}{s} - \frac{\frac{\mu}{2\pi l k}\left(\overline{q}(s) + \frac{Q_0}{s}\right)}{\frac{1}{\ln\frac{R_k}{r_c}} - 2\sum\limits_{n=1}^{\infty} \frac{J_0^2\left(x_n\frac{R_k}{r_c}\right)}{J_0^2(x_n) - J_0^2\left(x_n \frac{R_k}{r_c}\right)} \cdot \frac{s}{s + \frac{\chi x_n^2}{r_c^2}}}, \tag{4.4}$$

where $\overline{P}_c(s)$, $\overline{q}(s)$ are the Laplace transformations of corresponding functions. Applying inverse transform from (4.4) we come to the following

$$P_c(t) = P_k - \frac{\mu}{2\pi l k} \int_0^t (q(\tau) + Q_0)$$
$$\times \left(A^+ \exp\left((\sigma+\omega)(t-\tau)\right) + A^- \exp\left((\sigma-\omega)(t-\tau)\right)\right) d\tau, \tag{4.5}$$

where

$$\sigma = \frac{\alpha_1 + \alpha_2 - (\alpha_1\beta_2 + \alpha_2\beta_1)\ln\frac{R_k}{r_c}}{2\left(1 - (\beta_1+\beta_2)\ln\frac{R_k}{r_c}\right)},$$

$$\omega = \frac{\sqrt{\left(\alpha_1 + \alpha_2 - (\alpha_1\beta_2 + \alpha_2\beta_1)\ln\frac{R_k}{r_c}\right)^2 - 4\alpha_1\alpha_2\left(1 - (\beta_1+\beta_2)\ln\frac{R_k}{r_c}\right)}}{2\left(1 - (\beta_1+\beta_2)\ln\frac{R_k}{r_c}\right)},$$

$s = \sigma \pm i\omega$ are roots of quadratic equation

$$\left(1 - (\beta_1 + \beta_2)\ln\frac{R_k}{r_c}\right)s^2 + \left((\alpha_1 + \alpha_2) - (\alpha_1\beta_2 + \alpha_2\beta_1)\ln\frac{R_k}{r_c}\right)s + \alpha_1\alpha_2 = 0,$$

where

$$\alpha_1 = \frac{\chi x_1^2}{r_c^2}, \quad \alpha_2 = \frac{\chi x_2^2}{r_c^2}, \quad \beta_1 = \frac{2J_0^2\left(x_1\frac{R_k}{r_c}\right)}{J_0^2(x_1) - J_0^2\left(x_1\frac{R_k}{r_c}\right)}, \quad \beta_2 = \frac{2J_0^2\left(x_2\frac{R_k}{r_c}\right)}{J_0^2(x_2) - J_0^2\left(x_2\frac{R_k}{r_c}\right)}$$

and

$$A^{\pm} = \frac{(\sigma \pm \omega + \alpha_1)(\sigma \pm \omega + \alpha_2)\ln\frac{R_k}{r_c}}{2(\sigma \pm \omega)\left(1 - (\beta_1 + \beta_2)\ln\frac{R_k}{r_c}\right) + (\alpha_1 + \alpha_2) - (\alpha_1\beta_2 + \beta_1\alpha_2)\ln\frac{R_k}{r_c}},$$

5 Transient motion of fluid in vertical pipeline section.

Consider the unsteady fluid motion in the vertical pipeline part of the horizontal wellbore. If we place the origin axis z in the lower section, we obtain the following equation of fluid motion [15, 1, 2].

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} - 2hv - g, \tag{5.1}$$

where v is axial fluid velocity, z -coordinate, h -coefficient of resistance, g -acceleration of gravity.

On other hand side,

$$v = \frac{\partial u}{\partial t}, \tag{5.2}$$

where u is movement of any liquid column in cross section.

We can neglect the conventional term since that is small [25]. Then it follows from (5.1), (5.2) and $P = E\frac{\partial u}{\partial z}$ that

$$\frac{\partial^2 u}{\partial t^2} = a^2\frac{\partial^2 u}{\partial z^2} - 2h\frac{\partial u}{\partial t} - g. \tag{5.3}$$

Present the motion as sum of two components: as a solid (liquid column) and as a deformation of liquid column part

$$u = u_e + u_r, \tag{5.4}$$

where u_e is movement of liquid column as solid, u_r -reversible deformation of liquid column sections

Equation of liquid column motion as a solid is

$$\frac{\partial^2 u_e}{\partial t^2} = \frac{P_1(0,t) - P_y(t)}{\rho H} - 2h\frac{\partial u_e}{\partial t} - g. \tag{5.5}$$

Initial conditions are

$$u_e\Big|_{t=0} = 0, \qquad t > 0, \tag{5.6}$$

$$\frac{\partial u_e}{\partial t}\Big|_{t=0} = \frac{Q_0}{f}, \qquad t > 0. \tag{5.7}$$

The solution of (5.5) under initial conditions (5.6) and (5.7) is given as

$$u_e(t) = \frac{Q_0}{2hf}\left(1 - \exp(-2ht)\right) + \frac{1}{2h}\int_0^t \Big(1 - \exp(-2h(t-\tau))\Big)\left(-g + \frac{P_1(0,\tau) - P_y(\tau)}{\rho H}\right)d\tau. \tag{5.8}$$

Inserting (5.4) into (5.3) for the equation (5.5) we have expression

$$\frac{\partial^2 u_r}{\partial t^2} = a^2 \frac{\partial^2 u_r}{\partial z^2} - 2h \frac{\partial u_r}{\partial t} - \frac{P_1(0,t) - P_y(t)}{\rho H}. \tag{5.9}$$

Initial and boundary conditions are

$$u_r(0,z) = \frac{\rho g z}{2E}(z - 2H), \quad 0 < z < H, \tag{5.10}$$

$$\left.\frac{\partial u_r}{\partial t}\right|_{t=0} = 0, \qquad 0 < z < H, \tag{5.11}$$

$$u_r(0,t) = 0, \qquad t > 0 \tag{5.12}$$

$$\left.\frac{\partial u_r}{\partial z}\right|_{z=H} = 0, \qquad t > 0. \tag{5.13}$$

Search the solution of equation (5.9) satisfying boundary conditions (5.12) - (5.13) as in the works [1,2]

$$u_r = \sum_{k=1}^{n} \phi_k(t) \sin \frac{(2k-1)\pi z}{2H}, \quad n \in \mathbb{N}. \tag{5.14}$$

Insert this into (5.9) and multiply both sides by $\sin \frac{(2k-1)\pi z}{2H}$. After integrating over $(0, H)$, we get

$$\ddot{\phi}(t) + 2h\dot{\phi}(t) + \frac{(2k-1)^2\pi^2 a^2}{4H^2}\phi(t) = -\frac{4}{(2k-1)\pi} \cdot \frac{P_1(0,t) - P_y(t)}{\rho H}. \tag{5.15}$$

Applying Laplace transformation, it follows from (5.15) that

$$\begin{aligned} \phi_k(t) &= M_k \exp(-ht)\cos\eta_k t + N_k \exp(-ht)\sin\eta_k t \\ &- \frac{1}{\eta_k}\exp(-ht) \cdot \frac{4}{(2k-1)\pi} \int_0^t \exp(h\tau) \frac{P_1(0,\tau) - P_y(\tau)}{\rho H} \sin\eta_k(t-\tau)d\tau, \end{aligned} \tag{5.16}$$

where

$$\eta_k = \sqrt{\frac{(2k-1)^2\pi^2 a^2}{4H^2} - h^2},$$

M_k andN_k are unknown constants. Insert (5.16) into (5.14). Using initial conditions (5.10) and (5.11), we have

$$\begin{aligned} u_r &= \sum_{k=1}^{n} \Big[\left(-\frac{16\rho g H^2}{\pi^3 E (2k-1)^3} \right) \exp(-ht) \left(\cos\eta_k t + \frac{h}{\eta_k t}\sin\eta_k t \right) \\ &- \frac{1}{\eta_k} \frac{4}{(2k-1)\pi} \int_0^t \exp\left(-h(t-\tau)\right) \frac{P_1(0,\tau) - P_y(\tau)}{\rho H} \sin\eta_k(t-\tau)d\tau \Big] \\ &\times \sin\frac{(2k-1)\pi z}{2H}. \end{aligned} \tag{5.17}$$

From (5.4), (5.8) and (5.17) we get

$$\begin{aligned} u &= \frac{Q_0}{2hf}\left(1 - \exp(-2ht)\right) \\ &+ \frac{1}{2h}\int_0^t \left(1 - \exp(-2h(t-\tau))\right)\left(-g + \frac{P_1(0,\tau) - P_y(\tau)}{\rho H} \right) d\tau \\ &+ \sum_{k=1}^{n} \Big[\left(-\frac{16\rho g H^2}{\pi^3 E (2k-1)^3} \right) \exp(-ht) \left(\cos\eta_k t + \frac{h}{\eta_k}\sin\eta_k t \right) \\ &- \frac{1}{\eta_k} \frac{4}{(2k-1)\pi} \int_0^t \exp\left(-h(t-\tau)\right) \frac{P_1(0,\tau) - P_y(\tau)}{\rho H} \sin\eta_k(t-\tau)d\tau \Big] \\ &\times \sin\frac{(2k-1)\pi z}{2H}. \end{aligned} \tag{5.18}$$

Pressure at every liquid column cross-section is

$$P_1 = E\frac{\partial u}{\partial z}. \tag{5.19}$$

Then from (5.19), (5.18) we get

$$\begin{aligned} P_1(t,z) = \sum_{k=1}^{n} \Big[& \frac{8\rho g H}{\pi^2(2k-1)^2}\exp(-ht)\left(\cos\eta_k t + \frac{h}{\eta_k}\sin\eta_k t\right) \\ & + \frac{1}{\eta_k}\frac{2E}{\rho H^2}\int\limits_0^t \exp\left(-h(t-\tau)\right)\left(P_1(0,\tau) - P_y(\tau)\right)\sin\eta_k(t-\tau)d\tau\Big] \\ & \times \cos\frac{(2k-1)\pi z}{2H}. \end{aligned} \tag{5.20}$$

Find the pressure at end of vertical section of the well from (5.20) and at $z = 0$

$$\begin{aligned} P_1(0,t) = \sum_{k=1}^{n} \Big[& \frac{8\rho g H}{\pi^2(2k-1)^2}\exp(-ht)\left(\cos\eta_k t + \frac{h}{\eta_k}\sin\eta_k t\right) \\ & + \frac{1}{\eta_k}\frac{2E}{\rho H^2}\int\limits_0^t \exp\left(-h(t-\tau)\right)\left(P_1(0,\tau) - P_y(\tau)\right)\sin\eta_k(t-\tau)d\tau\Big]. \end{aligned} \tag{5.21}$$

From obtained integral equation (5.2) it needs to find $P_1(0,t)$. Applying Laplace transform, it follows from (5.21) that

$$\begin{aligned} P_1(0,t) = & -2A_1\int\limits_0^t P_y(\tau)\exp(-h(t-\tau))\sin\omega_1(t-\tau)d\tau \\ & -\frac{80\rho g H}{\pi^2}\cdot\frac{-\omega_1^2 + 0.1\eta_1^2 + 0.9\eta_2^2}{2\omega_1^2 + \frac{4E}{\rho H^2} - (\eta_1^2+\eta_2^2)}\exp(-ht)\left(\cos\omega_1 t + \frac{h}{\omega_1}\sin\omega_1 t\right) \\ & -2A_2\int\limits_0^t P_y(\tau)\exp(-h(t-\tau))\sin\omega_2(t-\tau)d\tau \\ & -\frac{80\rho g H}{\pi^2}\cdot\frac{-\omega_2^2 + 0.1\eta_1^2 + 0.9\eta_2^2}{2\omega_2^2 + \frac{4E}{\rho H^2} - (\eta_1^2+\eta_2^2)}\exp(-ht)\left(\cos\omega_2 t + \frac{h}{\omega_1}\sin\omega_2 t\right), \end{aligned} \tag{5.22}$$

where

$$A_1 = \frac{E\left(-4\omega_1^2 + 2\eta_1^2 + \eta_2^2\right)}{\omega_1\left[2H^2\rho\omega_1^2 - H^2\rho\left(\eta_1^2+\eta_2^2\right) + 4E\right]},\ A_2 = \frac{E\left(-4\omega_2^2 + 2\eta_1^2 + \eta_2^2\right)}{\omega_2\left[2H^2\rho\omega_2^2 - H^2\rho\left(\eta_1^2+\eta_2^2\right) + 4E\right]},$$

$$\omega_{1,2} = \sqrt{\frac{1}{2}\left(\eta_1^2 + \eta_2^2 - \frac{4E}{H^2\rho}\right) \pm \sqrt{\frac{1}{4}\left[(\eta_1^2 - \eta_2^2)^2 + \frac{16E^2}{H^4\rho^2}\right]}}.$$

From (5.22) we find bottom-hole pressure depending on wellhead pressure vibration.

Wellhead pressure always pulsates. Let pulsating constituent be as

$$P_y(t) = P_0 + \Delta P_0\cos\nu t, \tag{5.23}$$

where ν is vibration frequency of wellhead pressure, P_0 is constant component, ΔP_0 -maximum value of pulsating of wellhead pressure.

Insert (5.23) into (5.22), after integration we have

$$
\begin{aligned}
P_1(0,t) = -2A_1\Big\{ & \frac{P_0}{h^2+\omega_1^2}\left[\omega_1\left(1-\exp(-ht)\cos\omega_1 t\right) - h\exp(-ht)\sin\omega_1 t\right] \\
& + \frac{\Delta P_0}{2[h^2+(\omega_1+\nu)^2]}\left[h\left(\sin\nu t - \exp(-ht)\sin\omega_1 t\right)\right] \\
& + \frac{\Delta P_0}{2[h^2+(\omega_1+\nu)^2]}\left[(\omega_1+\nu)\left(\cos\nu t + \exp(-ht)\cos\omega_1 t\right)\right] \\
& - \frac{\Delta P_0}{2[h^2+(\omega_1-\nu)^2]}\left[h\sin\nu t + \exp(-ht)\sin\omega_1 t\right] \\
& - \frac{\Delta P_0}{2[h^2+(\omega_1-\nu)^2]}\left[(\omega_1-\nu)\cos\nu t - \exp(-ht)\cos\omega_1 t\right]\Big\} \\
& + \frac{80\rho g H}{\pi^2}\,\frac{-\omega_1^2+0.1\eta_1^2+0.9\eta_2^2}{2\omega_1^2+\frac{\phi E}{\rho H^2}-(\eta_1^2+\eta_2^2)}\exp(-ht)\left(\cos\omega_1 t + \frac{h}{\omega_1}\sin\omega_1 t\right) \\
& - 2A_2\Big\{\frac{P_0}{h^2+\omega_2^2}\left[\omega_2\left(1-\exp(-ht)\cos\omega_2 t\right) - h\exp(-ht)\sin\omega_2 t\right] \\
& + \frac{\Delta P_0}{2[h^2+(\omega_2+\nu)^2]}\left[h\left(\sin\nu t - \exp(-ht)\sin\omega_2 t\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
& + \frac{\Delta P_0}{2[h^2+(\omega_2+\nu)^2]}\left[(\omega_2+\nu)\left(\cos\nu t + \exp(-ht)\cos\omega_2 t\right)\right] \\
& - \frac{\Delta P_0}{2[h^2+(\omega_2-\nu)^2]}\left[h\sin\nu t + \exp(-ht)\sin\omega_2 t\right] \\
& - \frac{\Delta P_0}{2[h^2+(\omega_2-\nu)^2]}\left[(\omega_2-\nu)\cos\nu t - \exp(-ht)\cos\omega_2 t\right]\Big\} \\
& + \frac{80\rho g H}{\pi^2}\,\frac{-\omega_2^2+0.1\eta_1^2+0.9\eta_2^2}{2\omega_2^2+\frac{\phi E}{\rho H^2}-(\eta_1^2+\eta_2^2)}\exp(-ht)\left(\cos\omega_2 t + \frac{h}{\omega_2}\sin\omega_2 t\right). \qquad (5.24)
\end{aligned}
$$

It follows from (5.24) that natural vibration of liquid column decay after a while, only forced oscillation remains. Therefore, from (5.24) we get

$$
\begin{aligned}
& P_1(0,t) = \\
& \quad - 2A_1\Big\{\frac{P_0}{h^2+\omega_1^2}\omega_1 + \frac{\Delta P_0}{2[h^2+(\omega_1+\nu)^2]}\left[h\sin\nu t + (\omega_1+\nu)\cos\nu t\right] \\
& \quad - \frac{\Delta P_0}{2[h^2+(\omega_1-\nu)^2]}\left[h\sin\nu t + (\omega_1-\nu)\cos\nu t\right]\Big\} \\
& \quad - 2A_2\Big\{\frac{P_0}{h^2+\omega_2^2}\omega_2 + \frac{\Delta P_0}{2[h^2+(\omega_2+\nu)^2]}\left[h\sin\nu t + (\omega_2+\nu)\cos\nu t\right] \\
& \quad - \frac{\Delta P_0}{2[h^2+(\omega_2-\nu)^2]}\left[h\sin\nu t + (\omega_2-\nu)\cos\nu t\right]\Big\}.
\end{aligned} \qquad (5.25)
$$

Expenditure Q_2 is found by the formula

$$
Q_0 = \frac{2\pi k}{\mu}\cdot\frac{\int\limits_0^t \left(P_k - P_c(0,x)\right)dx}{\ln\frac{R_k}{r_c}}.
$$

As a first step of approximation assume $P_c(0,x) = P_c(0,0)$. Then

$$
Q_0 = \frac{2\pi k l}{\mu}\cdot\frac{P_k - P_c(0,0)}{\ln\frac{R_k}{r_c}}. \qquad (5.26)
$$

Integrating (5.3) for the case of steady fluid motion, we get

$$P_1(0,l) - P_y(0) = \left(\frac{2hQ_0}{f} + g\right)\rho H. \tag{5.27}$$

Then it follows from (3.9), (5.26) and (5.27) that

$$Q_0 = \frac{\frac{2\pi kl}{\mu} \cdot (P_k - P_y(0) - H\rho g)}{\ln \frac{R_k}{r_c} + \frac{2h}{f}\frac{2\pi kl}{\mu}H\rho + \frac{2\pi kl\alpha}{\mu f}\sqrt{\frac{\alpha\lambda}{f}}\,\tanh\,\sqrt{\frac{\alpha\lambda}{f}}\,l}. \tag{5.28}$$

Fluid expenditure in the lower tubing of vertical well part is

$$Q = f\frac{\partial u}{\partial t}\Big|_{t=0}. \tag{5.29}$$

Then from (5.29) and (5.18), (5.25) we can find the fluid expenditure in lower tubing section. Hence it follows from (4.2) and (5.29) that

$$q(t) = f\frac{\partial u}{\partial t}\Big|_{t=0} - Q_0. \tag{5.30}$$

From (5.30) and (5.18), (5.25) we find

$$\begin{aligned} q(t) &= Q_0\left(\exp(-2ht) - 1\right) \\ &+ f\int_0^t \exp\left(-2h(t-\tau)\right)\Big(-g + \frac{P_1(0,\tau) - P_y(\tau)}{\rho H}\Big)d\tau. \end{aligned} \tag{5.31}$$

Now, using (4.5) and (5.28), (5.31) we can easily find $P_c(t)$.

6 Results and Discussions

Fig. 2 and Fig. 3 shows the results of numerical calculations by formulas (5.24) and (5.28) under the assumptions of parameters:

$$\begin{aligned} &R_k = 75m, \quad r_c = 0.075m, \quad \mu = 10^{-3}Pa \cdot san, \quad \lambda = 1\frac{m^2}{Pa \cdot san}, \\ &k = 5 \cdot 10^{-12}, \quad 10^{-12}, 10^{-13};\; P_k = 25 \cdot 10^6 Pa;\; P_0 = 10^6 Pa; \\ &\Delta P_0 = 10^5 Pa, \quad a = 10^3\frac{m}{san}, \quad H = 1500, 2000m, \quad E = 10^9\frac{N}{m^2}, \\ &f = 3 \cdot 10^{-3}m^2; \quad h = 10^{-3}san^{-1}; \quad \alpha = 10^{-3}\frac{kg}{m^3}, \quad g = 9.8\frac{m}{san^2}. \end{aligned}$$

As is evident from Fig. 2, the bottom well pressure is decaying and oscillatory. Over time, the natural oscillations of the system remain.

Fig. 3 presents dependence of producing capacity versus of horizontal pipe length to reservoir boundary radius $\xi = \frac{l}{R_k}$. Fig. 3 shows that increase of horizontal pipe length brings to well production increase. In a certain value of length that depends on the permeability the increase slow, and that stabilizes.

This is of practical importance, since the formula of expenditure considering the filtration properties of the reservoir allows us to determine the required length of the horizontal part of the pipe string.

Fig. 3 also shows that decreasing in-place permeability the producing capacity increases almost linearly depending on the length of the horizontal part of the wellbore. This means at low permeability the length of the horizontal wellbore part should be taken as long as possible.

6.1 Conclusion.

A mathematical model has been constructed for the unsteady filtration of liquid in horizontal wells. Here is dynamic presented formation-well connection on the permeability of horizontal well-wall part of pipe string. Formulas are obtained for determining the performance of the reservoir and the bottom hole pressure on the filtration properties of the formation and formation -borehole system parameters.

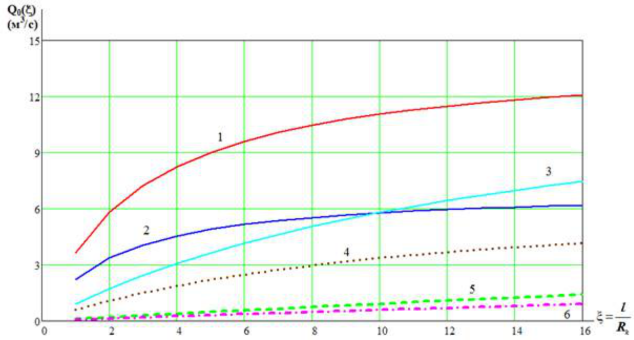

Fig.2. Dependence of formation productivity from power of layer.

1. $k = 5.10^{-12}m^2$, $H = 1500m$; 2. $k = 5.10^{-12}m^2$, $H = 2000m$;
3. $k = 10^{-12}m^2$, $H = 1500m$; 4. $k = 10^{-12}m^2$, $H = 2000m$;
5. $k = 10^{-13}m^2$, $H = 1500m$; 6. $k = 10^{-13}m^2$, $H = 2000m$

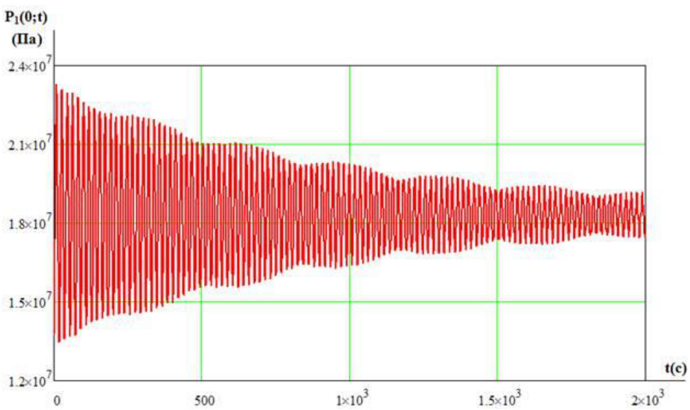

Fig.3. Bottomhole pressure dynamics.

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