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Surface energy dissipation of an AC electric field in a semi-infinite electron plasma with mirror and diffuse boundary conditions

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Abstract. *The response of an electron plasma with an arbitrary degree of degeneracy to an alternating electric field with mirror and diffuse boundary conditions is considered. The value of the surface absorption of the electric field energy is calculated.*

Keywords. Vlasov–Boltzmann equation · energy absorption · electric field

Mathematics Subject Classification (2010): 35Q20, 82D10

1 Introduction

The character of electric field screening near the surface of a conductor is critically important for different problems of surface physics [2], [3], [9], in particular, the problem of propagation of plasma oscillations [1], [6].

Here, we have obtained an analytical solution to the problem on the behavior of a semi-infinite plasma with an arbitrary degree of electron gas degeneracy in an external ac electric field perpendicular to the plasma surface. Such a situation takes place, e.g., when analyzing a solid-state semiconductor plasma. We use the Vlasov–Boltzmann kinetic equation with the Bhatnagar–Gross–Krook (BGK) collision integral for the electron distribution function and Poisson equation for the electric field.

It makes it possible to separate energy absorption into the volume and surface components. Surface absorption is analyzed in detail. A nontrivial character of the dependence of surface absorption on the ratio between the volumetric electron collision frequency and the frequency of the external electric field is demonstrated.

2 Formulation of the Problem and Basic Equations

The general statement of the problem is given in [4], [7], [8]. We will use the τ–model Vlasov–Boltzmann equation

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} = \nu(f_{eq} - f) \tag{2.1}$$

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The behavior of the electric field in plasma is described by Poisson equation

$$\mathrm{div}\mathbf{E} = 4\pi\rho, \quad \rho = e\int (f - f_0) d\Omega_F, \quad d\Omega_F = \frac{(2s+1)d^3p}{(2\pi\hbar)^3}. \qquad (2.2)$$

Here, f is the electron distribution function; f_{eq} is the locally equilibrium Fermi–Dirac distribution function, $f_{eq}(\mathbf{r}, v, t) = \left\{1 + \exp\frac{\mathcal{E}-\mu(\mathbf{r},t)}{kT}\right\}^{-1}$, $f_0 = f_{FD}$ is the unperturbed Fermi–Dirac distribution function, $f_0(v, \mu_0) = f_{FD}(v, \mu_0) = \left\{1 + \exp\frac{\mathcal{E}-\mu_0}{kT}\right\}^{-1}$, $\mathbf{p}=m\mathbf{v}$ is the electron momentum; $\mathcal{E}=mv^2/2$ is the electron kinetic energy; $\mu_0 = const$ and $\mu(\mathbf{r}, t)$ are the unperturbed and perturbed chemical potentials, respectively; e and m are the charge and effective mass of an electron, respectively; ρ is the charge density; $\hbar$ is Planck's constant; ν is the electron scattering frequency; s is the particle spin ($s = 1/2$ for electrons); k is the Boltzmann constant; T is the plasma temperature, which is assumed to be constant; and $\mathbf{E}(\mathbf{r}, t)$ is the electric field in plasma.

Let us consider the condition of mirror reflection of electrons from the boundary of a semi-infinite plasma: $f(x = 0, v_x, v_y, v_z, t) = f_{eq}(x = 0, -v_x, v_y, v_z, t)$, and the condition of diffusive reflection of electrons from the boundary of a semi-infinite plasma: $f(x = 0, \mathbf{v}, t) = f_{eq}(x = 0, \mathbf{v}, t)$ at $v_x > 0$, $e(0) = 1$, $e(+\infty) < +\infty$. The external electric field on the plasma surface is perpendicular to the plasma boundary and varies in time as $\mathbf{E}_{ext}(t) = E_0 e^{-i\omega t}(1, 0, 0)$.

The corresponding self-consistent electric field in plasma has the form $\mathbf{E}(x, t) = E(x)e^{-i\omega t}(1, 0, 0)$.

We assume that the external field is sufficiently weak, so that the linear approximation is applicable.Equations (1) and (2) can be linearized with respect to the absolute Fermi-Dirac distribution function f_0: $f_{eq}(x, P, t) = f_0(P, \alpha) + g(P, \alpha)\delta\alpha(x)e^{-i\omega t}$, where $f_0(P, \alpha) = f_{FD}(P, \alpha) = (1 + e^{P^2-\alpha})^{-1}$, $g(P, \alpha) = e^{P^2-\alpha}/(1 + e^{P^2-\alpha})^2$, $\mathbf{P} = \mathbf{p}/p_T = \mathbf{v}/v_T$. Here v_T is the electron thermal velocity given by $v_T = \sqrt{2kT/m}$ and $\alpha = \mu/kT$ is the reduced chemical potential. The change of the chemical potential is considered to be a small parameter so that representation $\alpha(x, t) = \alpha + \delta\alpha(x)e^{-i\omega t}$ is possible. We linearize the electron distribution function $f(x, P, P_x, t) = f_0(P, \alpha) + g(P, \alpha)h(x, P_x)e^{-i\omega t}$, where $h(x, P_x)$ is a new unknown function and $h(x, P_x) \sim E$.

As a result, we get a system containing new unknown functions and dimensionless variables. The detailed solution is given in [7]. The solution is based on the method of separation of variables, is reduced to obtaining the dispersion function and search eigenfunctions by which we can decompose the resulting analytical solution. Dispersion function determines the range of solutions to the problem

$$\Lambda(z) = 1 - \frac{1}{w_0} - \frac{z^2 - \eta_1^2}{w_0\eta_1^2}\lambda_0(z, \alpha),$$

$$\lambda_0(z, \alpha) = 1 + z\int_{-\infty}^{+\infty} \frac{k(\mu, \alpha)d\mu}{\mu - z}.$$

Constants w_0, η_1^2 and function $k(\eta, \alpha)$ have forms

$$f_0(\eta, \alpha) = (1 + \exp(\eta^2 - \alpha))^{-1}, \quad k(\eta, \alpha) = \frac{f_0(\eta, \alpha)}{2s_0(\alpha)}$$

$$s_0(\alpha) = \int_0^{+\infty} f_0(t, \alpha)dt, \quad s_2(\alpha) = \int_0^{+\infty} t^2 f_0(t, \alpha)dt$$

$$w_0 = 1 - i\frac{\omega}{\nu}, \quad \eta_1^2 = w_0 \frac{\nu^2}{\omega_p^2}\frac{s_2(\alpha)}{s_0(\alpha)}.$$

As a result of the solution, the induced electromagnetic field is represented as the sum of three terms corresponding to the expansion in the spectrum of the dispersion function. In [4], the electric field inside the plasma was obtained as expansions in the eigensolutions of the original system of equations in the case of diffuse boundary conditions. And, in [8], the electric field inside the plasma was obtained as expansions in the eigensolutions of the original system of equations in the case of mirror boundary conditions. In general, the structure of an electric field arising in a plasma can be represented as $e(x) = e_v + e_s(x)$. In the case of mirror boundary conditions, the electric field can be represented in the form

$$e_v = E_\infty,$$

$$e_s(x) = \frac{2\Lambda_1\eta_0}{\Lambda'(\eta_0,\alpha)(\eta_1^2-\eta_0^2)}\left(-\frac{w_0 x}{\eta}\right) +$$

$$+\frac{\Lambda_1}{w_0\eta_1^2 s_0(\alpha)}\int_0^\infty \frac{\eta^2 f_0(\eta,\alpha)}{\Lambda^+(\eta,\alpha)\Lambda^-(\eta,\alpha)}\text{exp}\left(-\frac{w_0 x}{\eta}\right) d\eta.$$

In the case of diffuse boundary conditions, the electric field can be represented in the form

$$e_v = E_\infty,$$

$$e_s(x) = E_d\text{exp}\left(-\frac{w_0 x}{\eta_0}\right) + \int\limits_0^\infty \frac{1}{2\pi i(\eta^2-\eta_1^2)} *$$

$$* \left(C_0 + \frac{C_{-1}}{\eta-\eta_0}\right)\left(\frac{1}{X^+(\eta)} - \frac{1}{X^-(\eta)}\right)\text{exp}\left(-\frac{w_0 x}{\eta}\right) d\eta,$$

where

$$E_\infty = C_0 = \frac{\Lambda_1}{\Lambda_\infty}, \quad E_d = \frac{C_0(\eta_1/(\eta_0^2-\eta_1^2)+\alpha^-)}{X(\eta_0)(\eta_1\alpha^+-\eta_0\alpha^-)},$$

$$C_{-1} = -\frac{C_0[\eta_1 + \alpha^-(\eta_0^2-\eta_1^2)]}{(\eta_1\alpha^+-\eta_0\alpha^-)}, \quad \alpha^\pm = \frac{X(\eta_1) \pm X(-\eta_1)}{2},$$

$$X(z) = \frac{1}{z}\text{exp}V(z), \quad V(z) = \frac{1}{\pi}\int\limits_0^\infty \frac{\zeta(\tau)d\tau}{\tau - z}$$

$$\zeta(\tau) = \frac{1}{2i}\ln G(\tau) - \pi.$$

The detailed description of the function $G(\tau)$ is given in [4], [7].

3 Energy Absorption

Let us consider the electron response in a metal layer to an external ac electric field. We will calculate the electric field energy absorbed in a cylindrical area with the base area S and thickness a. The external ac electric field $E_0 \exp(-i\omega t)$ is applied perpendicular to the layer surface.

Absorption in a cylindrical volume with the base area S and thickness a is given by a well-known expression [8]

$$Q = \frac{S}{2}\mathrm{Re}\int\limits_0^a j(x)E^*(x)dx.$$

Here, $j(x)$ is the current density and the asterisk denotes a complex conjugate.

Since we are considering a one-dimensional problem, the equation for the electric field has the form $\frac{dE}{dx} = 4\pi q$ where q is the charge density. All quantities are assumed to depend on time as $\exp(-i\omega t)$, i.e., $E = E(x)\exp(-i\omega t)$. The continuity equation for the one-dimensional charge–current system is given by $\frac{dj(x)}{dx} - i\omega q(x) = 0$.

In [7], the value of the absorption of the electric field energy in a plasma with an arbitrary degree of degeneracy with diffuse boundary conditions was calculated. Energy absorption can be separated into volumetric and surface components. Volume absorptions in the case of diffuse and mirror boundary conditions are equal. It is of interest to compare the surface absorption of electric field energy in plasma in the case of diffuse and mirror boundary conditions.

With the help of some calculations, we obtain [7] that in the case of mirror boundary conditions the quantity Q_s is

$$Q_s = \frac{v_T S E_0^2}{8\pi} q_s,$$

where

$$q_s = \sqrt{\frac{s_2(\alpha)}{s_0(\alpha)}}\mathrm{Im}\left\{\frac{\Omega\sqrt{\varepsilon}}{\sqrt{\varepsilon^2+\Omega^2}}\sqrt{\varepsilon + i\Omega}\right\}.$$

In the case of diffuse boundary conditions, the quantity Q_s is

$$Q_s = \frac{v_T S E_0^2}{8\pi} q_s,$$

where

$$q_s = \frac{\Omega}{\varepsilon}\mathrm{Im}\left\{\frac{1}{w_0}\cdot\left[C_0V_1 + \alpha^+(\eta_0^2 - \eta_1^2) - \frac{C_{-1}}{C_0}(C_0 + \eta_0\alpha^+ - \eta_1\alpha^-)\right]\right\}.$$

The quantity Q_s corresponds to surface absorption. For a sufficiently broad plasma layer (with a width exceeding the electron mean free path), Q_s independent of the layer thickness.

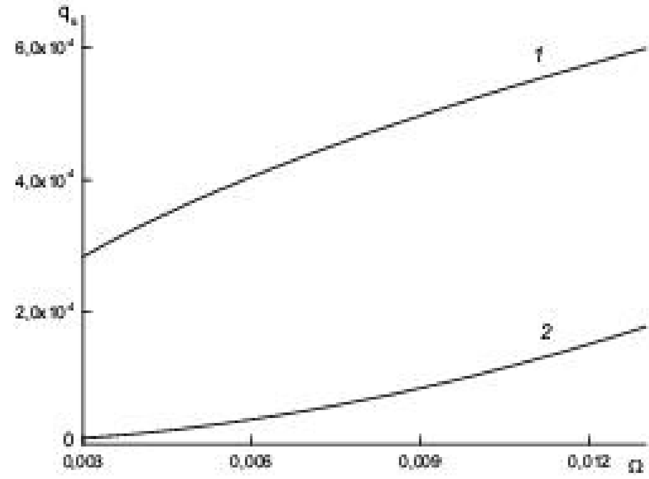

Fig. 1 Surface absorption at $\varepsilon = 0.0001$ and $\alpha = -1$ in the case of mirror boundary conditions (1) and in the case of diffuse boundary conditions (2)

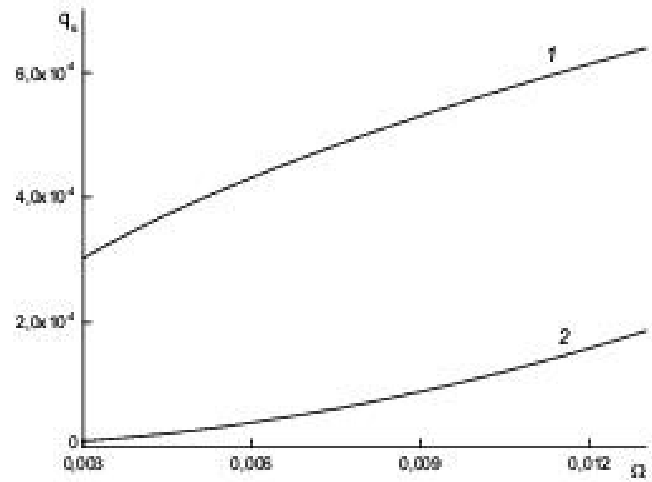

Fig. 2 Surface absorption at $\varepsilon = 0.0001$ and $\alpha = 0$ in the case of mirror boundary conditions (1) and in the case of diffuse boundary conditions (2)

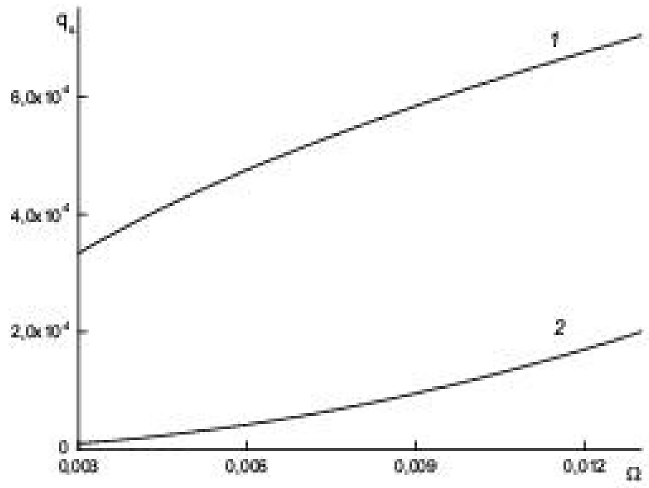

Fig. 3 Surface absorption at $\varepsilon = 0.0001$ and $\alpha = 1$ in the case of mirror boundary conditions (1) and in the case of diffuse boundary conditions (2)

Fig. 1 shows the plots of the surface absorption q_s in the case of $\varepsilon = 0.0001$ and $\alpha = -1$, graphs 1, 2 correspond to the values, respectively, in the case of mirror boundary conditions and in the case of diffuse boundary conditions.

Fig. 2 shows the plots of the surface absorption q_s in the case of $\varepsilon = 0.0001$ and $\alpha = 0$, graphs 1, 2 correspond to the values, respectively, in the case of mirror boundary conditions and in the case of diffuse boundary conditions.

Fig. 3 shows the plots of the surface absorption q_s in the case of $\varepsilon = 0.0001$ and $\alpha = 1$, graphs 1, 2 correspond to the values, respectively, in the case of mirror boundary conditions and in the case of diffuse boundary conditions.

Fig. 1 - 3 shows that for $\varepsilon = 0.0001$ and $\alpha = -1, 0, 1$ the values of surface absorption in the case of mirror boundary conditions are greater than the values of surface absorption in the case of diffuse boundary conditions. The graphs show that as the chemical potential of the growth surface absorption increases.

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