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# On the critical velocity of the moving load acting in the interior of the bi-layered hollow cylinder in the 3D dynamic state

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**Abstract.** *The paper studies critical velocity of the moving load acting on the interior of the bi-layered hollow cylinder in the 3D dynamic state with utilizing the exact equations and relations of the elastodynamics. Under this study it is assumed that in the interior of the cylinder the point located with respect to the cylinder axis moving forces act and the distribution of these forces is non-axisymmetric and is located within a certain central angle. The corresponding mathematical problem is solved with the use of the moving coordinate system and by employing the Fourier transformation with respect to the axial coordinate. Further, the Fourier transforms of the sought values are presented through the Fourier series and each coefficients in these series are determined analytically as a result of the solution to the corresponding equations. Originals of the sought values are determined numerically and according to this determination the critical velocities of the moving load is determined. In the paper, numerical results related to these critical velocities are presented and discussed. In particular, it is established that the values of the critical velocities increase with increasing of thickness of the external layer of the cylinder.*

**Keywords.** critical velocity · bi-layered hollow sylinder · non-axisymmetrical problem

**Mathematics Subject Classification (2010):** 76B55

## 1 Introduction

First of all, we note that the corresponding 3D dynamic problem for the “hollow cylinder + surrounding medium” system has been studied in the paper [1] and the review of the other related investigations are given in the papers [1-4]. Moreover, detailed consideration of the dynamics of the bi-material elastic system has been made in the monograph [5]. It follows from the analysis of the aforementioned works that up to now it was made a few investigations (such as the investigations carried out in the papers [1, 2]) on the 3D dynamics of the moving and oscillating loads acting in the interior of the hollow cylinder surrounded with elastic medium. In the present paper we attempt to develop the investigations started in the papers [1, 2] and to consider the case where the hollow cylinder is surrounded with the other hollow cylinder, i.e. in the present work we consider the problem related to the 3D dynamics of the moving load acting on the interior of the bi-layered hollow cylinder.

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## 2 Formulation of the problem

Consider the aforementioned bi-layered hollow cylinder the sketch of which is illustrated in Fig. 1 and assume that the thicknesses of the walls of the inner and outer cylinders are $h^{(2)}$ and $h^{(1)}$ respectively, and the external radius of the cross section of the inner cylinder is *R*. We associate the cylindrical system of coordinates $Or\theta z$ with the axis of the cylinder and the quantities related to the inner (outer) cylinder we denote by the upper index (2) (by the upper index (1)).

Assume that in the interior of the inner hollow cylinder act a point located with respect to the cylinder axis and that non-uniformly distributed in the circumferential direction moving normal forces act and these forces move with constant velocity $V$ in the $oz$ axis direction (Fig. 1). In the present paper, within this framework we attempt to investigate the non-axisymmetric dynamic response of the bi-layered hollow cylinder to these moving forces and analyze critical velocity of those.

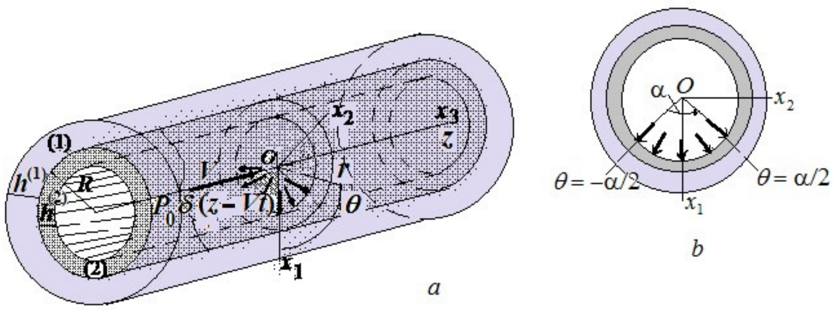


**Fig. 1.** The sketch of the considered system (a) and the sketch of the distribution of the non-axisymmetric normal forces (b)

For this purpose we write the following complete system of field equations of the 3D elastodynamics, as well as the corresponding boundary and contact conditions.

Equations of motion:

$$\frac{\partial\sigma_{rr}^{(m)}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{r\theta}^{(m)}}{\partial\theta}\frac{\partial\sigma_{rz}^{(m)}}{\partial z}+\frac{1}{r}(\sigma_{rr}^{(m)}-\sigma_{r\theta}^{(m)})=\rho^{(m)}\frac{\partial^2u_r^{(m)}}{\partial t^2}$$

$$\frac{\partial\sigma_{r\theta}^{(m)}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{\theta\theta}^{(m)}}{\partial\theta}+\frac{\partial\sigma_{z\theta}^{(m)}}{\partial z}+\frac{2}{r}\sigma_{r\theta}^{(m)}=\rho^{(m)}\frac{\partial^2u_\theta^{(m)}}{\partial t^2}$$

$$\frac{\partial\sigma_{rz}^{(m)}}{\partial r}+\frac{1}{r}\frac{\partial\sigma_{z\theta}^{(m)}}{\partial\theta}+\frac{\partial\sigma_{zz}^{(m)}}{\partial z}+\frac{1}{r}\sigma_{rz}^{(m)}=\rho^{(m)}\frac{\partial^2u_z^{(m)}}{\partial t^2}. \tag{2.1}$$

Elasticity relations:

$$\sigma_{rr}^{(m)}=\left(\lambda^{(m)}+2\mu^{(m)}\right)\frac{\partial u_r^{(m)}}{\partial r}+\lambda^{(m)}\frac{1}{r}\left(\frac{\partial u_\theta^{(m)}}{\partial r}+u_r^{(m)}\right)+\lambda^{(m)}\frac{\partial u_z^{(m)}}{\partial z},$$

$$\sigma_{\theta\theta}^{(m)} = \lambda^{(m)} \frac{\partial u_r^{(m)}}{\partial r} + 2\mu^{(m)} \frac{\partial u_r^{(m)}}{\partial r} + \lambda^{(m)} \frac{1}{r} \left( \frac{\partial u_\theta^{(m)}}{\partial r} + u_r^{(m)} \right) + \lambda^{(m)} \frac{\partial u_z^{(m)}}{\partial z},$$

$$\sigma_{zz}^{(m)} = \lambda^{(m)} \frac{\partial u_r^{(m)}}{\partial r} + \lambda^{(m)} \frac{1}{r} \left( \frac{\partial u_\theta^{(m)}}{\partial r} + u_r^{(m)} \right) + \left( \lambda^{(m)} + 2\mu^{(m)} \right) \frac{\partial u_z^{(m)}}{\partial z},$$

$$\sigma_{r\theta}^{(m)} = \mu^{(m)} \frac{\partial u_\theta^{(m)}}{\partial r} + \mu^{(m)} \left( \frac{1}{r} \frac{\partial u_r^{(m)}}{\partial \theta} - \frac{1}{r} u_\theta^{(m)} \right),$$

$$\sigma_{z\theta}^{(m)} = \mu^{(m)} \frac{\partial u_\theta^{(m)}}{\partial z} + \mu^{(k)} \frac{\partial u_z^{(m)}}{r\partial \theta}, \quad \sigma_{zr}^{(k)} = \mu^{(k)} \frac{\partial u_r^{(k)}}{\partial z} + \mu^{(k)} \frac{\partial u_z^{(k)}}{r\partial \theta}. \tag{2.2}$$

In equations (2.1) and (2.2) the conventional notation of the theory of elasticity is used.

Consider also formulation of the corresponding boundary and contact conditions which can be written as follows.

$$\sigma_{rr}^{(2)} \Big|_{r=R-h^{(2)}} = \begin{cases} -P_\alpha \delta(z - Vt) & \text{for } -\alpha/2 \le \theta \le \alpha/2 \\ 0 \text{ for } \theta \in ([-\pi, +\pi] - (-\alpha/2, \alpha/2)) \end{cases},$$

$$\sigma_{r\theta}^{(2)} \Big|_{r=R-h^{(2)}} = 0, \quad \sigma_{rz}^{(2)} \Big|_{r=R-h^{(2)}} = 0,$$

$$\sigma_{rr}^{(2)} \Big|_{r=R+h^{(1)}} = 0, \quad \sigma_{r\theta}^{(1)} \Big|_{r=R+h^{(1)}} = 0, \quad \sigma_{rz}^{(1)} \Big|_{r=R+h^{(1)}} = 0, \tag{2.3}$$

$$\sigma_{rr}^{(1)} \Big|_{r=R} = \sigma_{rr}^{(2)} \Big|_{r=R}, \quad \sigma_{r\theta}^{(1)} \Big|_{r=R} = \sigma_{r\theta}^{(2)} \Big|_{r=R}, \quad \sigma_{rz}^{(1)} \Big|_{r=R} = \sigma_{rz}^{(2)} \Big|_{r=R},$$

$$u_r^{(1)} \Big|_{r=R} = u_r^{(2)} \Big|_{r=R}, \; u_\theta^{(1)} \Big|_{r=R} = u_\theta^{(2)} \Big|_{r=R}, \; u_z^{(1)} \Big|_{r=R} = u_z^{(2)} \Big|_{r=R}, \;\; 4 \tag{2.4}$$

$$\left| \sigma_{rr}^{(1)} \right|; \left| \sigma_{\theta\theta}^{(1)} \right|; \left| \sigma_{zz}^{(1)} \right|; \left| \sigma_{r\theta}^{(1)} \right|; \left| \sigma_{r\theta}^{(1)} \right|; \left| \sigma_{\theta z}^{(1)} \right|;$$

$$\left| u_r^{(1)} \right|; \left| u_\theta^{(1)} \right|; \left| u_z^{(1)} \right| \to 0, \text{ as } \sqrt{(z - Vt)^2} \to +\infty, \tag{2.5}$$

where in (2.3) $P_\alpha$ is determined from the following relation

$$\int_{-\alpha/2}^{+\alpha/2} P_\alpha (r - h) s \cos\theta d\theta = (r - h) P_0 = const \Rightarrow P_\alpha = P_0 / (2 \sin(\alpha/2)). \tag{2.6}$$

Thus, the investigation of the problem is reduced to the boundary-contact problem (2.1) – (2.5) for solution to which the method developed in the papers [1, 2] is employed. Now we consider some fragments of the application of this method for the problem under consideration.

## 3 Method of solution

As in the papers [1, 2] for solution to the foregoing mathematical problem, according to [6], we use the following representation:

$$u_r^{(m)} = \frac{1}{r}\frac{\partial}{\partial\theta}\Psi^{(m)} - \frac{\partial^2}{\partial r\partial z}X^{(m)}, \;\; u_\theta^{(m)} = -\frac{\partial}{\partial r}\Psi^{(m)} - \frac{1}{r}\frac{\partial^2}{\partial\theta\partial z}X^{(m)} \;,$$

$$u_z^{(m)} = \left(\lambda^{(m)} + \mu^{(m)}\right)^{-1}\left(\left(\lambda^{(m)} + 2\mu^{(m)}\right)\Delta_1 + \mu^{(m)}\frac{\partial^2}{\partial z^2} - \rho^{(m)}\frac{\partial^2}{\partial t^2}\right)X^{(m)},$$

$$\Delta_1 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2}, \quad m = 1, 2. \tag{3.1}$$

In (8) the functions $\Psi^{(m)}$ and $X^{(m)}$ are the solutions of the equations

$$\left(\Delta_1 + \frac{\partial^2}{\partial z^2} - \frac{\rho^{(k)}}{\mu^{(k)}}\frac{\partial^2}{\partial t^2}\right)\Psi^{(m)} = 0,$$

$$\left(\Delta_1 + \frac{\partial^2}{\partial z^2}\right)\left(\Delta_1 + \frac{\partial^2}{\partial z^2}\right) + \rho^{(m)}\frac{\lambda^{(m)} + 3\mu^{(m)}}{\mu^{(m)}\left(\lambda^{(m)} + 2\mu^{(m)}\right)}\times$$

$$\times\left(\Delta_1 + \frac{\partial^2}{\partial z^2}\right)\frac{\partial^2}{\partial t^2} + \left.\frac{\rho^{(m)}}{\mu^{(m)}\left(\lambda^{(m)} + 2\mu^{(m)}\right)}\frac{\partial^4}{\partial t^4}\right]X^{(m)} = 0. \tag{3.2}$$

We introduce a moving cylindrical coordinate system $O'r'\theta'z'$ which is connected with the reference cylindrical coordinate system $Or\theta z$ through the following relations:

$$r' = r, \;\; \theta' = \theta, \;\;\; z' = z - Vt. \tag{3.3}$$

According to relations in (3.3), the operators $\partial^2/\partial^2 t$ and $\partial^4/\partial^4 t$ in the foregoing equations are replaced with the operators $V^2\partial^2/\partial^2 z'^2$ and $V^4\partial^4/\partial^4 z'^4$ , respectively, and in this way, equations rewritten in the moving coordinate system, are obtained. Further, the exponential Fourier transform $f_F\int\limits_{-\infty}^{+\infty} f(z')e^{isz'}dz'$ with respect to the moving coordinate $z'$ (where $s$ is a transformation parameter) is applied to all the equations and relations rewritten with the moving coordinates.

Below, we will make all mathematical operations in the moving coordinates and will omit the upper primes over these coordinates.

Thus, according to the problem statement the originals of the sought values can be presented through their Fourier transforms by the following relations.

$$\left\{\sigma_{rr}^{(m)}; \sigma_{\theta\theta}^{(m)}; \sigma_{zz}^{(m)}; \sigma_{r\theta}^{(m)}; u_r^{(m)}; u_\theta^{(m)}; \Psi^{(m)}\right\} =$$

$$= \frac{1}{\pi}\int\limits_0^{+\infty}\left\{\sigma_{rrF}^{(m)}; \sigma_{\theta\theta F}^{(m)}; \sigma_{zzF}^{(m)}; \sigma_{r\theta F}^{(m)}; u_{rF}^{(m)}; u_{\theta F}^{(m)}; \Psi_F^{(m)}\right\}\cos(sz)ds,$$

$$\left\{\sigma_{\theta z}^{(m)}; \sigma_{rz}^{(m)}; u_z^{(m)}; X^{(m)}\right\} = \frac{1}{\pi}\int\limits_0^{+\infty}\left\{\sigma_{\theta zF}^{(m)}; \sigma_{rzF}^{(m)}; \sigma_{zF}^{(m)}; u_{zF}^{(m)}; \Psi_F^{(m)}\right\}\sin(sz)ds$$

Substituting the expressions in Eq. (3.4) into the foregoing equations (3.2) and the rewritten relations in the moving coordinate system, we obtain the following equations for the functions $\Psi_F^{(m)}$ and $X^{(m)}$:

$$\left(\Delta_1 - s^2\left(1 - \frac{\rho^{(k)}}{\mu^{(k)}}V^2\right)\right)\Psi_F^{(m)} = 0,$$

$$\left[\left(\Delta_1 - s^2\right)\left(\Delta_1 - s^2\right) - \rho^{(m)}\frac{\lambda^{(m)} + 3\mu^{(m)}}{\mu^{(m)}\left(\lambda^{(m)} + 2\mu^{(m)}\right)} \times\right.$$
$$\left.\times\left(\Delta_1 - s^2\right)\left(s^2V^2\right) + \frac{\left(\rho^{(m)}\right)^2}{\mu^{(m)}\left(\lambda^{(m)} + 2\mu^{(m)}\right)}s^4V^4\right]X_F^{(m)} = 0. \tag{3.4}$$

According to the geometry of the problem under consideration with respect to the circumferential coordinate $\theta$, the Fourier transform of the functions $\Psi_F^{(m)}$ and $X_F^{(m)}$ can be presented in the Fourier series form as follows.

$$\Psi_F^{(m)}(r,s,\theta) = \sum_{n=1}^{\infty}\Psi_{Fn}^{(m)}(r,s)\sin n\theta,$$

$$\Psi_F^{(m)}(r,s,\theta) = \frac{1}{2}X_{F0}^{(m)}(r,s) + \sum_{n=1}^{\infty}X_{Fn}^{(m)}(r,s)\cos n\theta. \tag{3.5}$$

Substituting expressions in (3.6) into equation (3.5), we obtain:

$$\left(\Delta_{1n-}\left(\zeta_1^{(m)}\right)^2\right)\psi_{Fn}^{(m)} = 0,$$

$$\left(\Delta_{1n} - \left(\zeta_2^{(m)}\right)^2\right)\left(\Delta_{1n-}\left(\zeta_3^{(m)}\right)^2\right)X_{Fn}^{(m)} = 0,$$

$$\Delta_{1n} = \frac{d^2}{dr^2} + \frac{d}{rdr} - \frac{n^2}{r^2}, \tag{3.6}$$

where

$$\left(\zeta_1^{(m)}\right)^2 = s^2\left(1 - \frac{\rho^{(m)}V^2}{\mu^{(m)}}\right) \tag{3.7}$$

$\left(\zeta_2^{(m)}\right)^2$ and $\left(\zeta_2^{(m)}\right)^2$ in Eq. (3.7) are determined from the solutions to the following equation.

$$\mu^{(m)}\left(\zeta^{(m)}\right)^4 - s^2\left(\zeta^{(m)}\right)^2\left[-\rho^{(m)}V^2 - \left(\lambda^{(m)} + 2\mu^{(m)}\right) +\right.$$
$$\left.+\frac{\mu^{(m)}}{\lambda^{(m)} + 2\mu^{(m)}}\left(-\rho^{(m)}V^2 - \mu^{(m)}\right) + \frac{\left(\lambda^{(m)} + \mu^{(m)}\right)^2}{\lambda^{(m)} + 2\mu^{(m)}}\right] +$$
$$s^4\left(\frac{\rho^{(m)}V^2}{\lambda^{(m)} + 2\mu^{(m)}} - 1\right)\left(-\rho^{(m)}V^2 - \mu^{(m)}\right) = 0. \tag{3.8}$$

Thus, the solutions to equations in (3.7) are determined as follows:

$$\psi_{Fn}^{(m)} = A_{1n}^{(m)}I_n\left(\zeta_1^{(m)}r\right) + B_{1n}^{(m)}K_n\left(\zeta_1^{(m)}r\right),$$

$$\chi_{Fn}^{(m)} = A_{2n}^{(m)} I_n\left(\zeta_2^{(m)} r\right) + A_{3n}^{(m)} I_n\left(\zeta_3^{(m)} r\right) +$$
$$+ B_{2n}^{(m)} K_n\left(\zeta_2^{(m)} r\right) + B_{3n}^{(m)} K_n\left(\zeta_3^{(m)} r\right), \quad m = 1, 2. \tag{3.9}$$

Using (2.16), (2.12), (2.7) and (2.2) we determine completely the Fourier transforms of the sought values and using the algorithm developed and applied in the papers [1-4] the originals of these values are determined. Note that one of the main procedures of this algorithm is the determination of the unknown constants $A_{1n}^{(m)}$,$B_{1n}^{(m)}$, $A_{2n}^{(m)}$, $B_{2n}^{(m)}$, $A_{3n}^{(m)}$ and $b_{3n}^{(m)}$ for which it is obtained a complete system of algebraic equations from the boundary and contact conditions in (2.3) and (2.4) respectively.

This completes the consideration of the solution method more detail version of which is given in the papers [1, 2].

## 4 Numerical results

As noted above, the calculation algorithm used under obtaining the numerical results which will be discussed below, are detailed in the works [1 – 5] and therefore here we do not consider this algorithm again. Nevertheless, we note that under obtaining numerical results we take twenty terms in the series in (13) these terms are enough for obtaining convergence numerical results.

Numerical results relate to the critical velocity of the moving load and are obtained for the following two cases:

$$\begin{aligned}&\text{Case 1.} E^{(1)}/E^{(2)} = 0.02, \rho^{(1)}/\rho^{(2)} = 0.01,\ \nu^{(1)}/\nu^{(2)} = 0.25\\&\text{Case 2.} E^{(1)}/E^{(2)} = 0.5, \rho^{(1)}/\rho^{(2)} = 0.5,\ \nu^{(1)}/\nu^{(2)} = 0.3\end{aligned} \tag{4.1}$$

**Table 1.** The values of the dimensionless critical velocity obtained for the case where $E^{(1)}/E^{(2)} = 0.02$, $\rho^{(1)}/\rho^{(2)} = 0.01$ and $\nu^{(1)}/\nu^{(2)} = 0.25$ under various values of the ratios $h^{(2)}/R$ and $h^{(1)}/h^{(2)}$

| $h^{(2)}/R$ | $h^{(1)}/h^{(2)}$ | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 5 | 7 | ∞ [1] |
| 0.1 | 0.8710 / 0.3970 | 0.8700 / 0.4200 | 0.8700 / 0.4365 | 0.8700 / 0.4474 | 0.8700 / 0.4494 | 0.8700 / 0.4494 |
| 0.2 | 0.8900 / 0.5467 | 0.8900 / 0.5600 | 0.8900 / 0.5648 | 0.8900 / 0.5662 | 0.8900 / 0.5663 | 0.8900 / 0.5663 |
| 0.3 | 0.9000 / 0.6532 | 0.9000 / 0.6600 | 0.9000 / 0.6613 | 0.9000 / 0.6615 | 0.9000 / 0.6615 | 0.9000 / 0.6615 |
| 0.5 | 0.8038 / 0.4174 | 0.8052 / 0.4384 | 0.8052 / 0.4758 | 0.8052 / 0.5637 | 0.8052 / 0.6122 | 0.8052 / 0.6250 |

**Table 2.** The values of the dimensionless critical velocity obtained for the case where $E^{(1)}/E^{(2)} = 0.5$, $\rho^{(1)}/\rho^{(2)} = 0.5$ and $v^{(1)}/v^{(2)} = 0.3$ under various values of the ratios $h^{(2)}/R$ and $h^{(1)}/h^{(2)}$

| $\frac{h^{(2)}}{R}$ | $h^{(1)} / h^{(2)}$ | | | | | |
|---|---|---|---|---|---|---|
| | 1 | 2 | 3 | 5 | 7 | ∞ [1] |
| 0.1 | $\frac{0.9280}{0.5111}$ | $\frac{0.9301}{0.6043}$ | $\frac{0.9270}{0.6681}$ | $\frac{0.9255}{0.8526}$ | $\frac{0.9246}{0.8547}$ | $\frac{0.8565}{0.8547}$ |
| 0.2 | $\frac{0.9300}{0.6680}$ | $\frac{0.9272}{0.7642}$ | $\frac{0.9273}{0.8151}$ | $\frac{0.9280}{0.8739}$ | $\frac{0.9246}{0.8742}$ | $\frac{0.8785}{0.8743}$ |
| 0.3 | $\frac{0.9320}{0.7823}$ | $\frac{0.9310}{0.8483}$ | $\frac{0.9310}{0.8809}$ | $\frac{0.9310}{0.8962}$ | $\frac{0.9310}{0.8963}$ | $\frac{0.9042}{0.8918}$ |
| 0.5 | $\frac{0.9421}{0.9022}$ | $\frac{0.9420}{0.9205}$ | $\frac{0.9398}{0.9232}$ | $\frac{0.9420}{0.9396}$ | $\frac{0.9420}{0.9396}$ | $\frac{0.9432}{0.9396}$ |

These results are obtained for various values of the ratious $h^{(2)}/R$ and $h^{(1)}/h^{(2)}$ and are given in Tables 1 and 2 for Case 1 and Case 2 respectively. Note that in these tables the lower numbers indicates the first critical velocity, however, the upper number indicate the second critical velocity. Thse two types subsonic critical velocities is characteristic ones for the 3D problems for the cylindrical systems. Moreover, in these tables it is also indicated the corresponding critical velocities obtained for the system consisting of the hollow cylinder and surrounding elastic medium in the corresponding cases and these results are obtained in the paper [1]. According to the well-known physicomechanical consideration, in the subsonic cases, with increasing the ratio $h^{(1)}/h^{(2)}$ the results obtained in the case under consideration must approach the corresponding ones obtained for the "hollow cylinder +surrounding infinite medium" system. Thus, the results are given in Tables 1 and 2 confirm this prediction and this confirmation can also be taken as illustrating the trustiness of the used algorithm and PC programs used in the present investigation.

Analyze of the numerical results also shows that a decrease in the thickness of the outer cylinder, i.e. a decrease in the values of the ratio $h^{(1)}/h^{(2)}$ causes to decrease the values of the critical velocities. It should be noted that the magnitude of this decrease is more significant with respect to the first critical velocities. Moreover, it follows from the results that the magnitude of the mentioned decreasing becomes more considerable for relatively great values of the ratio $h^{(2)}/R$ .

With this, we restrict ourselves to consideration of the numerical results obtained for problem under consideration and note that this consideration will be continued in the further works by the author.

## 5 Conclusions

Thus, in the present paper, the 3D dynamic problem of the moving load acting in the interior of the bi-layered hollow cylinder is studied with employing 3D exact equations of elastodynamics. It is assumed that the forces acting in the interior of the inner layer of the cylinder is point located with respect to the axial coordinate and is distributed along a certain arc within the corresponding central angle. The corresponding mathematical problem is solved with employing the moving coordinate system and with the application of the Fourier transform with respect to the axial coordinate of the moving coordinate system. Further, the Fourier transforms of the sought values are presented in the Fourier series form with respect to the circumferential coordinate of the cylindrical system of coordinates. The coefficients of these series are unknown functions with respect to the radial coordinates the analytical expressions for which are determined through the solution of the corresponding equations. The originals of the Fourier transforms are found numerically. Numerical results on the critical velocity of the moving load are presented and discussed. It is established that a decrease of the thickness of the external layer of the cylinder causes to decrease of the values of the critical velocity. Moreover, it is established that the magnitude of the mentioned decreasing become more considerable with an increase of the external radius of the inner layer cross section.

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