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THE ACTION OF LOCAL TEMPERATURE FIELD ON RETARDATION OF CURVILINER CRACK WITH REGARD TO PLASTIC DEFORMATIONS

Abstract

The influence of local temperature field on retardation of the growth of a crack with end plastic zones is studied. We consider the case when the size of the heated area is small compared with the crack's length. The obtained formulas enable to determine the crack opening at the foot of the plastic zone and analyze the in fluence of the directed thermoelstic stress field.

Problem statement. Let's consider a homogeneous isotropic body with the end plastic zone crack. The body's material is accepted as elastico- idealey- plastic subjected to Tresk-Saint Venant's plasticity condition In real materials because of structural and technological factors the crack's surfaces have unevennesses and distortions. Consider an elastico-plastic failure mechanics problem on a crack with end prefracture zones assuming that the crack's contour has roughnesses (small deviations from the linear form). The crack in the plane is assument to be close to the linear form allowing only small deviation of the crack's line from the straight line $y = 0$. The equation of the contour of the end zone crack is accepted in the form $y = f(x)$. It is assumed that the crack satisfies the local symmetry condition. The crack faces are free from external loads. For retardation of crack propagation, on the way of its crack by means of heating of domain S by the thermal source to temperature T_0 the compressible stresses zone is created. It is accepted that the thermoplastic characteristics of the material are temperature independent.

It is assumed that at time $t = 0$ the arbitrary area S on the way of the crack propagation in the place instantly heats up to the constant temperature $T = T_0$.

The remaining part of the plate at initial time has zero temperature. In the case when the typical linear size of the area is small in comparison with the length of the prefracture zone crack or with other typical linear size of the plate in the plan, the effective asymptotic solution of the problem based on representation of thin structure [1] of the crack's end is possible.

Consider the vicinity of the crack's end that is small compared with typical linear size in the plan of the plate but is greater comparatively with typical size of the area S and the typical size of the plastic area. Then the crack on the plane xOy is represented by a semi- infinite through curvilinear cut along $y = f(x), -\infty < x < 0$, free from external loads. Therewith, in the vicinity of the origin of coordinates we'll have a plastic area to be defined.The area S may have any (but finite) sizes and configurations. The stress field typical for thin structure of the crack's end is realized at infinity. This field is assumed to be given and has the form

$$\text{for} \quad z \to \infty \ \Phi(z) = \frac{K_I - iK_{II}}{2\sqrt{2\pi z}}; \quad \Omega(z) = \frac{K_I - iK_{II}}{2\sqrt{2\pi z}}$$

($z = x + iy = re^{i\theta}$; r, θ are polar coordinates; $\Phi(z), \Omega(z)$ are complex potentials [2]).

In the problem under consideration the stress intensity factors K_I, K_{II} representing some functions of the form of plates, boundary conditions are the loading parameters.They are determined from the “global” solution of the problem at no thermal action.

The stated problem consists of determination of opening of the crack faces at the foot of the plastic zone and limiting value of the external load causing the crack growth at the action of the stress field induced by thermal source of .

The boundary conditions of the considered problem are of the form

$$\sigma_n - i\tau_{nt} = 0 \quad \text{for} \quad y = f(x), -\infty < x < 0,$$

$$\sigma_n - i\tau_{nt} = \sigma_s - i\tau_s \quad \text{for} \quad y = f(x), 0 \leq x \leq d,$$

where n, t are natural coordinates, σ_s is the yield point of the material in tension; τ_s is the yield point of the material in shear.

The solution method of the boundary value problem. Represent the stress state in the plane with a crack in the from

$$\sigma_x = \sigma_{x_0} + \sigma_{x_1}, \;\; \sigma_y = \sigma_{y_0} + \sigma_{y_1}, \;\; \tau_{xy} = \tau_{xy_0} + \tau_{xy_1},$$

where $\sigma_{x_0}, \sigma_{y_0}, \tau_{xy_0}$ is the solution of a thermolasticity problem for a crackless plane.

For finding the stresses $\sigma_{x_0}, \sigma_{y_0}, \tau_{xy_0}$ we solve the thermolasticity problem for an entire plane. As first we determine the temperature distribution in the plane.

For that we solve the boundary value problem of heat- conductivity theory

$$\frac{\partial T}{\partial t} = a\Delta T, \quad T = \begin{cases} T_0 & (x, y \in S) \\ 0 & (x, y \notin S) \end{cases} \text{for } t = 0,$$

where Δ is the Laplace operator; a is the heatconductivity coefficient of the plane's material.

Let for definiteness the area S heated with thermal source be a rectangle with the sides $2x_0$ and , $2y_0$,and the center O_1, of the rectangle S have the coordinates (L, b) .

The stress distribution will have the form

$$T_1(x, y, t) = \frac{T_0}{4}\left[Erf\left(\frac{x - L + x_0}{2\sqrt{at}}\right) + Erf\left(\frac{x_0 + L - x}{2\sqrt{at}}\right)\right] \times$$

$$\times \left[Erf\left(\frac{y - b + y_0}{2\sqrt{at}}\right) + Erf\left(\frac{y_0 + b - y}{2\sqrt{at}}\right)\right];$$

$$Erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp\left(-u^2\right) du$$

For determining the temperature field, for simpli fication of the problem, the perturbed temperature field is not taken account because of the crack .

The stresses $\sigma_{x_0}, \sigma_{y_0}, \tau_{xy_0}$ are expressed by the thermoelastic potential of displacements [3].

Consider some arbitrary realization of the rough (with small deviations from the rectilinear form) surfaces of the crack faces.

As the functions $f(x)$ and $f'(x)$ are small variables, the function $f(x)$ may be represented in the form

$$f(x) = \varepsilon H(x), \ -\infty < x < d,$$

where ε is a small parameter.

We look for the stresses $\sigma_{x_0}, \sigma_{y_0}, \tau_{xy_0}$ and displacements in the form of expansions in small parameter

$$\sigma_{x1}{=}\sigma_x^{(0)} + \varepsilon\sigma_x^{(1)} + ..., \quad \sigma_{y1}{=}\sigma_y^{(0)} + \varepsilon\sigma y + ..., \quad \tau_{xy1}{=}\tau_{xy}^{(0)} + \varepsilon\tau_{xy}^{(1)} + ...$$

$$u = u_0 + \varepsilon u_1 + ..., \qquad \upsilon = \upsilon_0 + \varepsilon\upsilon_1 + ...,$$

Expanding the expressions for the stress in the vicinity of $y = 0$ in series, we find the stresses for $y = f(x)$.

Using the perturbations method, allowing for previous formulas we find boundary conditions for $y = 0, -\infty < x < d$:
in a zero approximation

$$\sigma_y^{(0)} = -\sigma_{y_0}, \ \ \tau_{xy}^{(0)} = -\tau_{xy_0} \quad for\ y = 0, \ \ -\infty < x < 0 \tag{1}$$

$$\sigma_y^{(0)} = \sigma_s - \sigma_{y_0}, \ \ \tau_{xy}^{(0)} = \tau_s - \tau_{xy_0} \quad for\ y = 0, \ \ 0 \le x \le d^0$$

in a first approximation

$$\sigma_y^{(1)} = N, \ \ \tau_{xy}^{(1)} = T \quad for\ y = 0, \ \ -\infty < x < d^1 \tag{2}$$

Here

$$N = 2\tau_{xy}^{(0)}\frac{dH}{dx} - H\frac{\partial\sigma_y^{(0)}}{\partial y}, \ \ T = \left(\sigma_x^{(0)} - \sigma_y^{(0)}\right)\frac{dH}{dx} - H\frac{\partial\tau_{xy}^{(0)}}{\partial y} \tag{3}$$

$$\sigma_{y_0} = -\frac{\mu(1+\nu)\alpha T_0}{4\sqrt{\pi}}\left\{4\sqrt{\pi}A(x,y) + \frac{4}{\sqrt{\pi}}\left[arctg\left(\frac{y-b+y_0}{x-L+x_0}\right) + \right.\right.$$

$$\left. + arctg\left(\frac{y+b-y}{x_0+L-x}\right) + arctg\left(\frac{y_0+b-y}{x-L+x_0}\right) + arctg\left(\frac{y-b+y_0}{x_0+L-x}\right)\right] -$$

$$-\int_0^t \frac{1}{\tau\sqrt{a\tau}}\left[(x-L+x_0)\exp\left(-\frac{(x-L+x_0)^2}{4a\tau}\right) + \right.$$

$$+ (x_0+L-x)\exp\left(-\frac{(x_0+L-x)^2}{4a\tau}\right) \times$$

$$\left.\left. \times \left[Erf\left(\frac{y-b+y_0}{2\sqrt{a\tau}}\right) + Erf\left(\frac{y_0+b-y}{2\sqrt{a\tau}}\right)\right] d\tau\right\};$$

$$\tau_{xy_0} = -\frac{\mu(1+\nu)\alpha T_0}{2\pi}\left\{\ln\frac{(x-x_0-L)^2+(y-b+y_0)^2}{(x-x_0-L)^2+(y-y_0-b)^2} + \right.$$

$$+ \ln\frac{(x-L+x_0)^2+(y-y_0-b)^2}{(x-L+x_0)^2+(y-b+y_0)^2} - \int_0^t \frac{1}{\tau}\left[\exp\left(-\frac{(x-L+x_0)^2}{4a\tau}\right) - \right.$$

$$-\exp\left(-\frac{(x_0+L-x)^2}{4a\tau}\right)\right]\left[\exp\left(-\frac{(y-b+y_0)^2}{4a\tau}\right)-$$

$$-\exp\left(-\frac{(y_0+b-y)^2}{4a\tau}\right)\right]d\tau\Bigg\},$$

where $A(x,y)=\left\{\begin{smallmatrix}1 & (x,y\in S)\\ 0 & (x,y\notin S)\end{smallmatrix}\right.$

μ is the shear modulus of the plate's material; ν is the Poisson ratio; α is the coefficient of linear temperature extension of the plate's material. Here it is taken into account that $d=d+\varepsilon d^1+...$

For solving the boundary value problems at each approximation we use the Kolosov-Muskheleshvili complex potentials. As the stresses in the elastic ideally plastic body are restricted, the solution of boundary value problems (1)-(2) should be sought in the class of everywhere bounded functions.

The solution of boundary value problem (1) is written [2] in the form

$$\Phi_0(z)=\Omega_0(z)=\frac{\sqrt{z-d^0}}{2\pi i}\int\limits_{-\infty}^{d^0}\frac{f_0(x)\,dx}{\sqrt{x-d^0\,(x-z)}}+$$

$$+\frac{1}{2}(\sigma_s-i\tau_s)\left(1-\frac{1}{\pi i}\ln\frac{i\sqrt{d^0}-\sqrt{z-d^0}}{i\sqrt{d^0}+\sqrt{z-d^0}}\right) \tag{4}$$

where $f_0\,(x=-(\sigma_{y_0})(x,0)-i\tau_{xy_0}(x,0))$

Here the functions $\sqrt{z-d^0}$ is analytic exterior to the semi-infinite section for $y=0, x<d^0$ and is positive on the continuation of the section for $x>d^0$.

According to the condition on infinity, we find

$$-\frac{1}{\pi i}\int\limits_{-\infty}^{d^0}\frac{f_0(x)\,dx}{\sqrt{x-d^0}}+\frac{2(\sigma_s-i\tau_s)\sqrt{d^0}}{\pi}=\frac{K_I-K_{II}}{\sqrt{2\pi}} \tag{5}$$

Equation (5) is used to determine the unknown size d^0 of the plastic deformations zone in a zero approximation.

Using the obtained solution in a zero approximation of the elastico-plastic problem, calculate the displacements ϑ_0-iu_0 of the plastic area faces for $y=0, 0\le x\le d^0$

$$\vartheta_0-iu_0=\pm\frac{4}{\pi E}\left\{(\sigma_s-i\tau_s)\left[2\sqrt{d^0\,(d^0-x)}+x\ln\frac{\sqrt{d^0}-\sqrt{d^0-x}}{\sqrt{d^0}-+\sqrt{d^0-x}}\right]+\right.$$

$$\left.+\sqrt{d^0-x}\int\limits_{-\infty}^{d^0}\frac{f_0(x)\,dx}{\sqrt{d^0-x}}\right\} \tag{6}$$

At that the displacement ϑ_0-iu_0 at the crack end (for $y=0, x=0$) becomes equal

$$\vartheta_0-iu_0=\pm\frac{2}{\pi E}\left[\frac{\pi(K_I-K_{II})}{4(\sigma_s-i\tau_s)}+\sqrt{d^0}\int\limits_{-\infty}^{d^0}\frac{f_0(x)\,dx}{\sqrt{d^0-x}}\right] \tag{7}$$

After finding the solution in a zero approximation, we look for the solution in a first approximation. We find the functions N and T from formula (3).

The solution of boundary value problem (2) is written in the form

$$\Phi_1(z) = \Omega_1(z) = \frac{\sqrt{z-d^1}}{2\pi i} \int_{-\infty}^{d^0} \frac{(N - iT)\,dx}{\sqrt{x-d^1}\,(x-z)} \tag{8}$$

Therewith the following solvability condition of the boundary value problem should be fulfilled

$$\frac{1}{\pi i} \int_{-\infty}^{d^0} \frac{(N - iT)\,dx}{\sqrt{x-d^1}} = 0 \tag{10}$$

Condition (10) is used to determine the unknown parameter d^1 using the obtained solution in a first approximation of the stated elasticplastic problem, we find the displacements $\vartheta_1 - iu_1$ of the plastic area faces for $y = 0, 0 \leq x \leq d^1$:

$$\vartheta_1 - iu_1 = \pm\frac{4}{\pi E}\sqrt{x-d^1} \int_{-\infty}^{d^0} \frac{(N - iT)\,dx}{\sqrt{d^1-x}} \tag{11}$$

Therewith the displacement $\vartheta_1 - iu_1$ at the crack's end at the foot of the plastic zone (for $y = 0, x = 0$) becomes equal

$$\vartheta_1^0 - iu_1^0 = \frac{4\sqrt{d^1}}{\pi E} \int_{-\infty}^{d^0} \frac{(N - iT)\,dx}{\sqrt{d^1-x}} \tag{12}$$

For the displacements at the crack end for $y = 0, x = 0$ we have

$$\vartheta^0 - iu^0 = \vartheta_0^0 - iu_0^0 + \varepsilon\left(\vartheta_1^0 - iu_1^0\right) = \left(\vartheta_0^0 - \varepsilon\vartheta_1^0\right) - i\left(\vartheta_1^0 - \varepsilon u_1^0\right) \tag{13}$$

As the crack propagation criterions we use the criterion of critical opening of the crack faces

$$V(x_0) = \sqrt{\left(\vartheta^0\right) + \left(u^0\right)} = \delta_c \tag{14}$$

where is the fracture toughness of the plane determined experimentally.

The obtained relations (14),(7),(12),(13) permit to calculate the influence of local change of temperature near the crack end on the crack growth in an elastico-plastic sheet structural element in any form of the area S. Recall that K_I, K_{II} describe the stress filed from the distances of the crack end that greater in comparison with the sizes of the area S and the plastic zone.

References

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