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MECHANICS

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THE STABILITY OF NON-HOMOGENOUS TWO-LAYERED RECTANGULAR PLATES UNDER COMPRESSION IN ANISOTROPY RESISTING CONDITION

Abstract

In this article, the stability of non-homogenous two-layered rectangular plates in anisotropy resisting condition made from different non-homogenous materials is explored. It is assumed that the characteristics of resilient layers are continuous functions of thickness coordinate.

Considering Kirchhoff-Love hypothesis to be valid for the entire thickness of the plate elements, generalized form of the correct expression of moments and general strength and stiffness properties have been determined and voltage stability of the system of equations has been taken by the function. Here resisting elastic anisotropy model was adopted for the environment. A slab-sided state of contraction was examined in detail. Despite the edges of the plate matter of dissolving, the critical force was assigned. To achieve numerical computations, non-homogeneous functions were accepted to be linearly dependent on the thickness coordinate. Calculations were made in the report and appropriate graphics was established.

Introduction

Construction elements like thin single-layered and multi-layered plates and covers made of isotropic homogoneous elastic materials are being widely used in different fields of the engineering. Different types of stability problems regarding these constructions have been investigated largely in the scientific literature. Simple load processes are generally reviewed in this literature and classic theories are used while setting the problems. But in most cases constructions like two-layered plates are made from non-homogeneous elastic isotropic materials and they are located in a complex resisting elastic condition undergoing the influence of different complex loads. However, by using classic mathematical models, it is possible to decrease the difficulty of the problems relative to the other ways, but meanwhile serious errors can occur while defining the critical parameters of the constructions. Therefore more adequate and new methods are required to utilize for the solution of these problems. The reliability and stability problems of these plates have not been studied sufficiently in the literature.

Therefore, in this paper the stability of non-homogenous two-layered rectangular plates in anisotropy resisting condition composed of various non-homogenous materials is investigated.

Let's analyze the stability of non-homogenous two-layered rectangular plates composed of different non-homogenous materials in anisotropy resisting condition .

Coordinate system is shown below: OX and OY axes are in the plane separating the layers of the plate; OZ axis is normal to the plane. (figure 1). *

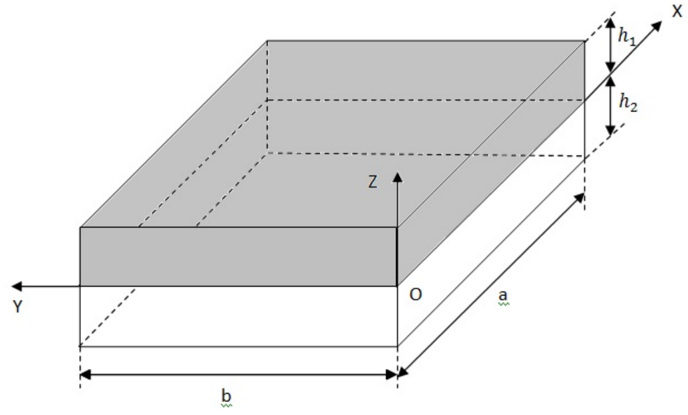

Fig. 1.

It is assumed that the layers of the plate are composed of various non-homogeneous isotropic elastic materials and elastic characteristics of the constituent materials are continuous functions of the thickness coordinate and varying as follow:

$$\lambda_{ij}^{k} = \lambda_{ij}^{k}.\ \ a_{i}^{k}(z) \tag{1}$$

According to Hooke Law, the relation between the strain-stress components is determined as below.

$$\sigma_{11}^{i} = \lambda_{11}^{i}\varepsilon_{11} + \lambda_{12}^{i}\varepsilon_{22},\ \sigma_{22}^{i} = \lambda_{21}^{i}\varepsilon_{11} + \lambda_{22}^{i}\varepsilon_{22},\ \sigma_{12}^{i} = \lambda_{33}^{i}\varepsilon_{12}\ (i = 1, 2), \tag{2}$$

It is accepted that the Kirchhoff-Love hypothesis is valid for the whole thickness of the plates, i.e.

$$\varepsilon_{11} = l_{11} - z\chi_{11}, \quad \varepsilon_{22} = l_{22} - z\chi_{22}, \quad \varepsilon_{12} = l_{12} - z\chi_{12} \tag{3}$$

where $l_{11},\ l_{22},\ l_{12}$ and $\chi_{11},\ \chi_{22},\ \chi_{12}$ are infinite small deformation and bending of the middle plane of the plate.

The following equations are used to determine the force and moment components:

$$T_{ij} = \int\limits_{-h2}^{0} \sigma_{ij}^{2}dz + \int\limits_{0}^{h1} \sigma_{ij}^{1}dz, \quad M_{ij} = \int\limits_{-h2}^{0} \sigma_{ij}^{2}dz + \int\limits_{0}^{h1} \sigma_{ij}^{1}dz, \tag{4}$$

where h_1 and h_2 -are the thickness of the corresponding layers. After substituting (1) - (3) formulas into the (4), the following general expressions are obtained for the force and moment:

$$T_{11} = \overline{\lambda}_{11}^{2}A_{11}^{0}l_{11} + \overline{\lambda}_{12}^{2}A_{12}^{0}l_{22} - \overline{\lambda}_{11}^{2}\overline{A}_{11}^{1}\chi_{11} - \overline{\lambda}_{12}^{2}\overline{A}_{12}^{1}\chi_{22}, ... \tag{5}$$

$$M_{11}=\overline{\lambda}_{11}^{2}A_{11}^{1}l_{11}+\overline{\lambda}_{12}^{2}A_{12}^{1}l_{22}-\overline{\lambda}_{11}^{2}\overline{A}_{11}^{2}\chi_{11}-\overline{\lambda}_{12}^{2}\overline{A}_{12}^{2}\chi_{22},... \qquad (6)$$

Where the following definitions apply:

$$A_{11}^{k}=\int\limits_{-h_2}^{0}a_1^2(z)z^k dz+\frac{\overline{\lambda}_{11}^{1}}{\overline{\lambda}_{11}^{2}}\int\limits_{0}^{h_1}a_1^1(z)z^k dz, \qquad (7)$$

$$A_{12}^{k}=\int\limits_{-h_2}^{0}a_1^2(z)z^k dz+\frac{\overline{\lambda}_{12}^{1}}{\overline{\lambda}_{12}^{2}}\int\limits_{0}^{h_1}a_1^1(z)z^k dz, \qquad (k=0,1,2)$$

Stability equations

The balancing equation system for the plate is as follow:

$$\frac{\partial T_{11}}{\partial x}+\frac{\partial T_{12}}{\partial y}=0,\ \ \frac{\partial T_{12}}{\partial x}+\frac{\partial T_{22}}{\partial y}=0, \qquad (8)$$

$$\frac{\partial^2 M_{11}}{\partial x^2}+2\frac{\partial^2 M_{12}}{\partial x\partial y}+\frac{\partial^2 M_{22}}{\partial y^2}+T_{11}\frac{\partial^2 W}{\partial x^2}+2T_{12}\frac{\partial^2 W}{\partial x\partial y}+T_{22}\frac{\partial^2 W}{\partial y^2}+K(w)=0, \quad (9)$$

$$\frac{\partial^2 e_{11}}{\partial y^2}+\frac{\partial^2 e_{22}}{\partial x^2}-2\frac{\partial^2 e_{12}}{\partial x\partial y}=0 \qquad (10)$$

Where anisontropic model is defined for the elastic condition as below:

$$K(W)=K_0W-K_1\frac{\partial^2 W}{\partial x^2}-K_2\frac{\partial^2 W}{\partial y^2} \qquad (11)$$

Here $K_0,\ K_1,\ K_2$ are anisontropic coefficients of the elastic condition.

As it can be seen in the equation (8), if the stress function is entered as follow

$$T_{11}=\frac{\partial^2 F}{\partial y^2},\quad T_{22}=\frac{\partial^2 F}{\partial x^2},\quad T_{12}=-\frac{\partial^2 F}{\partial x\partial y}, \qquad (12)$$

the system is becoming equivalent as in the previous case.

To achieve the appropriate form of the equations (9) and (10), they are defined in terms of bending and force components using expression (5), then:

$$\begin{gathered}\ell_{11}=C_{11}T_{11}-C_{12}T_{22}+\chi_{11}(C_{11}\overline{\alpha}_{11}^{2}A_{11}^{1}-C_{12}\overline{\alpha}_{21}^{2}A_{21}^{1})+\\ +\chi_{22}(C_{11}\overline{\alpha}_{12}^{2}A_{12}^{1}-C_{12}\overline{\alpha}_{22}^{2}A_{22}^{1}),\\ \ell_{22}=C_{21}T_{11}-C_{22}T_{22}+\chi_{11}(C_{21}\overline{\alpha}_{11}^{2}A_{11}^{1}-C_{22}\overline{\alpha}_{21}^{2}A_{21}^{1})+\\ +\chi_{22}(C_{21}\overline{\alpha}_{12}^{2}A_{12}^{1}-C_{22}\overline{\alpha}_{22}^{2}A_{22}^{1}),\\ \ell_{12}=C_{33}T_{12}+2C_{33}\overline{\alpha}_{33}^{2}A_{33}^{1}\chi_{22},\end{gathered} \qquad (13)$$

The coefficients from the equations can be expressed with the general stifness charasteristics of (7). Replacing the values of the expression (13) with the (6), the moments can be shown as:

$$M_{11} = r^1_{11}T_{11} + r^1_{12}T_{22} + R^1_{11}\chi_{11} + R^1_{12}\chi_{22},$$
$$M_{22} = r^1_{12}T_{11} + r^1_{22}T_{22} + R^1_{21}\chi_{11} + R^1_{22}\chi_{22}, \quad (14)$$
$$M_{12} = r^1_{33}T_{12} + R^1_{33}\chi_{12}.$$

where the following definitions apply:

$$r^1_{11} = c_{11}\overline{\alpha}^2_{11}A^1_{11} + c_{21}\overline{\alpha}^2_{12}A^1_{12},$$
$$r^1_{12} = -c_{12}\overline{\alpha}^2_{11}A^1_{11} - c_{22}\overline{\alpha}^2_{12}A^1_{12},$$
$$R^1_{11} = \overline{\alpha}^2_{11}A^1_{11}(c_{11}\overline{\alpha}^2_{11}A^1_{11} - c_{12}\overline{\alpha}^2_{21}A^1_{21})+$$
$$+\overline{\alpha}^2_{12}A^1_{12}(c_{21}\overline{\alpha}^2_{11}A^1_{11} - c_{22}\overline{\alpha}^2_{21}A^1_{21}) - \overline{\alpha}^2_{11}A^1_{11},$$
$$R^1_{12} = \overline{\alpha}^2_{11}A^1_{11}(c_{11}\overline{\alpha}^2_{12}A^1_{12} - c_{12}\overline{\alpha}^2_{22}A^1_{22})+$$
$$+\overline{\alpha}^2_{12}A^1_{12}(c_{21}\overline{\alpha}^2_{12}A^1_{12} - c_{22}\overline{\alpha}^2_{22}A^1_{22}) - \overline{\alpha}^2_{12}A^1_{12},$$
$$R^1_{21} = \overline{\alpha}^2_{21}A^1_{21}(c_{11}\overline{\alpha}^2_{11}A^1_{11} - c_{12}\overline{\alpha}^2_{21}A^1_{21})+$$
$$+\overline{\alpha}^2_{22}A^1_{22}(c_{21}\overline{\alpha}^2_{12}A^1_{11} - c_{22}\overline{\alpha}^2_{21}A^1_{21}) - \overline{\alpha}^2_{21}A^1_{21},$$
$$R^1_{12} = \overline{\alpha}^2_{21}A^1_{21}(c_{11}\overline{\alpha}^2_{12}A^1_{12} - c_{12}\overline{\alpha}^2_{22}A^1_{22})+$$
$$+\overline{\alpha}^2_{22}A^1_{22}(c_{21}\overline{\alpha}^2_{12}A^1_{12} - c_{22}\overline{\alpha}^2_{22}A^1_{22}) - \overline{\alpha}^2_{22}A^1_{22},$$
$$r^1_{33} = c_{33}\overline{\alpha}^2_{33}A^1_{33},$$
$$R^1_{33} = 2\overline{\alpha}^2_{33}A^1_{33}c_{33}\overline{\alpha}^2_{33}A^1_{33}, -\alpha^2_{33}A^1_{33}. \quad (15)$$

The equation system below is obtained after substituting the expressions (13) and (14) into (9) and (10):

$$D_{11}\frac{\partial^4 w}{\partial x^4} + D_{13}\frac{\partial^4 w}{\partial x^2\partial y^2} + D_{12}\frac{\partial^4 w}{\partial y^4} + D_{21}\frac{\partial^4 F}{\partial y^4} + D_{23}\frac{\partial^4 F}{\partial x^2\partial y^2} + D_{22}\frac{\partial^4 F}{\partial y^4}+$$
$$+T_{11}\frac{\partial^2 F}{\partial x^2} + 2T_{12}\frac{\partial^2 w}{\partial x\partial y} + T_{22}\frac{\partial^2 W}{\partial y^2} + K_0 w - K_1\frac{\partial^2 w}{\partial x^2} - K_2\frac{\partial^2 w}{\partial y^2} = 0 \quad (16)$$
$$d_{11}\frac{\partial^4 F}{\partial x^4} + d_{13}\frac{\partial^4 F}{\partial x^2\partial y^2} + d_{12}\frac{\partial^4 F}{\partial y^4} + d_{21}\frac{\partial^4 w}{\partial x^4} + d_{23}\frac{\partial^4 w}{\partial x^2\partial y^2} + d_{22}\frac{\partial^4 w}{\partial y^4} = 0 \quad (17)$$

where the following definitions apply:

$$d_{11} = c_{22}, \qquad d_{12} = c_{11},$$
$$d_{13} = c_{12} + c_{21} - 2c_{33},$$
$$d_{21} = c_{21}\overline{\alpha}^2_{11}A^1_{11} - c_{22}\overline{\alpha}^2_{21}A^1_{21},$$

$$d_{22} = c_{21}\overline{\alpha}_{12}^2 A_{12}^1 - c_{12}\overline{\alpha}_{22}^2 A_{22}^1,$$

$$d_{23} = c_{22}\overline{\alpha}_{11}^2 A_{11}^1 - c_{12}\overline{\alpha}_{21}^2 A_{21}^1 + c_{21}\overline{\alpha}_{12}^2 A_{12}^1 -$$

$$-c_{22}\overline{\alpha}_{22}^2 A_{22}^1 - 4c_{33}\overline{\alpha}_{33}^2 A_{33}^1,$$

$$D_{11} = R_{11}^1, \quad D_{12} = R_{22}^1,$$

$$D_{12} = R_{12}^1 + R_{21}^1 + 2R_{33}^1,$$

$$D_{13} = R_{12}^1 + R_{21}^1 + 2R_{33}^1,$$

$$D_{21} = r_{12}^1, \quad D_{22} = r_{22=1}^1,$$

$$D_{23} = r_{11}^1 + r_{22}^1 - 2r_{33}^1. \tag{18}$$

As it can be seen, the general stability equation of the two layered rectangular plates composed of non-homogeneous isotropic elastic materials in an anistropic resisting condition is indicated in (16) and (17). If the limiting conditions outside the plates are added, the mathematical representation of the problem can be achieved.

Solution of the problem

Now let's analyze the same stability problem of the same plate under compression from up and down ($T_{12} = 0$). In this case stability equations system of (16) and (17) are a little simplified.

$$D_{11}\frac{\partial^4 w}{\partial x^4} + D_{13}\frac{\partial^4 w}{\partial x^2 \partial y^2} + D_{12}\frac{\partial^4 w}{\partial y^4} + D_{21}\frac{\partial^4 F}{\partial y^4} + D_{23}\frac{\partial^4 F}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 F}{\partial y^4} +$$

$$+T_{11}\frac{\partial^2 W}{\partial x^2} + 2T_{12}\frac{\partial^2 W}{\partial x \partial y} + T_{22}\frac{\partial^2 W}{\partial y^2} + K_0 w - K_1\frac{\partial^2 w}{\partial x^2} - K_2\frac{\partial^2 w}{\partial y^2} = 0$$

$$d_{11}\frac{\partial^4 F}{\partial x^4} + d_{13}\frac{\partial^4 F}{\partial x^2 \partial y^2} + d_{12}\frac{\partial^4 F}{\partial y^4} + d_{21}\frac{\partial^4 w}{\partial x^4} + d_{23}\frac{\partial^4 w}{\partial x^2 \partial y^2} + d_{22}\frac{\partial^4 w}{\partial y^4} = 0 \tag{19}$$

The bending i valid as in the equation below if the edges of the plates are pivoted.

$$w = w_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b},$$
$$\Phi = \Phi_{mn}\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}. \tag{20}$$

Here a, b-appropriate length and width of the plate, and m, n is the number of the half-waves in the appropriate directions.

Substituting (20) into (19):

$$\Phi_{mn} = -w_{mn} D_{mn}^0,$$

$$D_{mn}^0 = \frac{d_{21}\left(\frac{m\pi}{a}\right)^4 + d_{23}\left(\frac{m\pi}{a}\right)^2\left(\frac{m\pi}{8}\right)^2 + d_{22}\left(\frac{m\pi}{8}\right)^4}{d_{11}\left(\frac{m\pi}{a}\right)^4 + d_{13}\left(\frac{m\pi}{a}\right)^2\left(\frac{m\pi}{8}\right)^2 + d_{12}\left(\frac{m\pi}{8}\right)^4} \tag{21}$$

After substituting the (20) expressions into (19) and doing some operations, the characteristic equation as follow is obtained to define the combination of critic loads:

$$T_{11}(1+\eta^2\alpha) = \left(\frac{m\pi}{a}\right)^2 \{D_{11} - D^0_{mn} D_{21} + \eta^2 (D_{13} - \\ -D^0_{mn} + D_{23}) + \eta^4 (D_{12} - D^0_{mn} D_{22}\} + K_0 \left(\frac{a}{\pi m}\right)^2 + K_1 + K_2\eta^2 \quad (22)$$

where the following definitions apply:

$$\eta = \frac{na}{mb}, \qquad \alpha = \frac{T_{22}}{T_{11}}.$$

If the shape of the plate is square, then it is achieved depending on (22) that $(m = n = 1, \quad a = b)$:

$$T_{11}(1+a) = \left(\frac{\pi}{a}\right)^2 \{D_{11}D_{11} + D_{13} + D_{12} - D^0 (D_{21} + D_{23} + D_{22})\} + \\ +K_0 \left(\frac{a}{\pi}\right)^2 + K_1 + K_2 \quad (23)$$

Here $D^0 = \dfrac{d_{21}+d_{23}+d_{22}}{d_{11}+d_{13}+d_{12}}$.

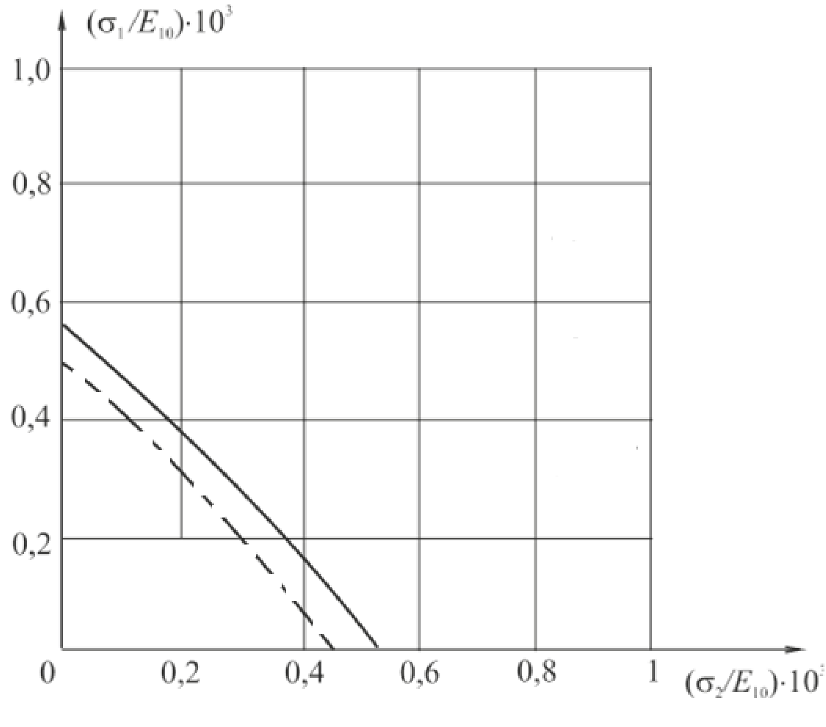

Fig. 2.

$K_0 = 6.48 \cdot 10E4 \ KN/m^3, \quad K_1 = K_2 = 2250.0 \ KN/m$

$- - - - \ \mu_1 = \mu_2 = 0$

____ $\mu_1 = 0.5; \mu_2 = 1$

Numerical computations

To perform numerical computations non-homogeneous functions are shown as the linear function of $a_i^k(z)$ thickness coordinate, i.e.;

$$a_1^1(z) = 1 + \mu_1 \frac{z}{h_1}; \quad a_1^2(z) = 1 + \mu_2 \frac{z}{h_2}. \tag{24}$$

Computations were carried out according to various values of parameters and consequently, it is proven that critical force is dependent on geometric parameters of the plate.

Results of the computations are shown in figure 2. The solution of the existing homogeneous problem is shown with hidden lines. Results indicate that strict errors can occur if the homogeneity is not taken into account, therefore the values of critical parameters can change depending on the values of the parameters from (22). In this case, non-homogeneity decreases the critical value by 8-12.

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