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ASYMPTOTIC BEHAVIOR OF THE SOLUTION OF A TORSION PROBLEM OF A TRANSVERSALLY -ISORTOPIC CYLINDRICAL SHELL WITH VARIABLE SHEAR MODULUS

Abstract

In the paper, we study a torsion problem of a transversally –isotropic hollow cylinder when the lateral surfaces are free from stresses, and the elastic characteristics vary according to general power laws on radius. Exact and asymptotic solutions of the torsion problem are constructed. The exact asymptotic expansions of homogeneous solutions are obtained and the stress –strain state of the cylinder is analyzed. It is shown that the solution is composed of two types of solutions: the penetrating solution and the boundary layer type solution. In the case of considerable anisotropy, some boundary layer solutions don't damp and may penetrate rather deep and essentially change the picture of the stress- strain state far from the cylinder endfaces.

In the paper, we study a torsion problem of a transversally –isotropic hollow cylinder when the lateral surfaces are free from stresses, and the elastic characteristics vary according to general power laws on radius. Exact and asymptotic solutions of the torsion problem are constructed. The exact asymptotic expansions of homogeneous solutions are obtained and the stress –strain state of the cylinder is analyzed. It is shown that the solution is composed of two types of solutions: the penetrating solution and the boundary layer type solution. In the case of considerable anisotropy, some boundary layer solutions don't damp and may penetrate rather deep and essentially change the picture of the stress- strain state far from the cylinder endfaces.

1. Consider a torsion problem for a radially- inhomogeneous transversally – isotropic hollow cylinder. The position of the cylinder points in the space is determined by the cylindrical coordinates r, φ, z varying within

$$r_1 \leq r \leq r_2,\ 0 \leq \varphi \leq 2\pi,\ -L \leq z \leq L$$

The equilibrium equations in displacements, at no mass forces have the form [4]:

$$\frac{\partial}{\partial \rho}\left[G(\rho)\left(\frac{\partial u_\varphi}{\partial \rho} - \frac{u_\varphi}{\rho}\right)\right] + \frac{2G(\rho)}{\rho}\left(\frac{\partial u_\varphi}{\partial \rho} - \frac{u_\varphi}{\rho}\right) + G_1(\rho)\frac{\partial^2 u_\varphi}{\partial \xi^2} = 0 \qquad (1.1)$$

Here $\rho = \frac{r}{r_0}$, $\xi = \frac{z}{r_0}$ are new pure variables; $r_0 = \frac{r_1+r_2}{2}$ is the radius of the median surface of the cylinder; $\rho \in [\rho_1; \rho_2]$, $\xi \in [-l; l]$ $\left(\rho = \frac{r_s}{r_0}, l = \frac{L}{r_0}; s = 1; 2\right)$; $u_\varphi = u_\varphi(\rho; \xi)$ is a displacement vector component; $G = G(\rho), G_1 = G_1(\rho)$ are dimensionless elastic characteristics (shear module) considered as positive piecewise – continuous functions of variable ρ.

Assume that the lateral part of the cylinder is free from stresses , i.e.

$$\sigma_{\rho\varphi} = G(\rho)\left(\frac{\partial u_\varphi}{\partial \rho} - \frac{u_\varphi}{\rho}\right)\Big|_{\rho=\rho_s} = 0, (s = 1, 2) \tag{1.2}$$

and on the endfaces the following boundary conditions are given

$$\sigma_{\varphi\xi} = G_1(\rho)\frac{\partial u_\varphi}{\partial \xi}\Big|_{\xi=\pm l} = f^{\pm}(\rho), \tag{1.3}$$

where $f^{\pm}(\rho)$ are sufficiently smooth functions satisfying the equilibrium conditions.

Assume that the shear module of the cylinder are given in the form of the functions

$$G(\rho) = g_0\rho^n, \ G_1(\rho) = g_1\rho^n \tag{1.4}$$

where n is an arbitrary positive number: g_0, g_1 are constants .

We'll seek the solution of equation (1.1) in the form [1,2]:

$$u_\varphi(\rho,\xi) = \upsilon(\rho)\, m(\xi) \tag{1.5}$$

where the function $m(\xi)$ is subjected to the condition

$$m''(\xi) - \mu^2 m(\xi) = 0, \tag{1.6}$$

and the parameter μ is determined after fulfilment of boundary conditions on the lateral surface.

Substituting (1.5) in (1.1),(1.2), allowing for (1.4),(1.6),we have:

$$\upsilon''(\rho) + \frac{(n+1)}{\rho}\upsilon'(\rho) + \left[\frac{g_1}{g_0}\mu^2 - \frac{(n+1)}{\rho^2}\right]\upsilon(\rho) = 0, \tag{1.7}$$

$$g_0\rho^n\left(\upsilon'(\rho) - \frac{\upsilon(\rho)}{\rho}\right)\Big|_{\rho=\rho_s} = 0, \tag{1.8}$$

$$(s = 1; 2).$$

The general solution of (1.7) has the form:

$$\upsilon(\rho) = \rho^{-\frac{n}{2}}\left[C_1 J_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right) + C_2 Y_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right)\right], \tag{1.9}$$

where $J_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right), Y_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right)$ are the Bessel functions of first and second kinds respectively; C_1, C_2 are arbitrary constants.

Using the Hooke's law, we can represent the stresses $\sigma_{\rho\varphi}, \sigma_{\varphi\xi}$ in the form

$$\sigma_{\rho\varphi} = g_0\rho^{\frac{n}{2}-1}\left[C_1\left(\mu\rho\sqrt{\frac{g_1}{g_0}}J'_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right) - \left(1+\frac{n}{2}\right)J_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right)\right) + \right.$$

$$\left. + C_2\left(\mu\rho\sqrt{\frac{g_1}{g_0}}Y'_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right) - \left(1+\frac{n}{2}\right)Y_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right)\right)\right] m(\xi), \tag{1.10}$$

$$\sigma_{\varphi\xi} = g_1\rho^{\frac{n}{2}}\left[C_1 J_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right) + C_2 Y_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right)\right] m'(\xi).$$

Satisfying the homogeneous boundary conditions (1.8) with respect to C_1, C_2 ,we get the linear system of algebraic equations:

$$\left\{\begin{array}{l} \left[\mu\rho_1\sqrt{\frac{g_1}{g_0}}J'_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_1\right)-\left(1+\frac{n}{2}\right)J_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_1\right)\right]C_1+ \\ +\left[\mu\rho_1\sqrt{\frac{g_1}{g_0}}Y'_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_1\right)-\left(1+\frac{n}{2}\right)Y_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_1\right)\right]C_2=0, \\ \left[\mu\rho_2\sqrt{\frac{g_1}{g_0}}J'_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_2\right)-\left(1+\frac{n}{2}\right)J_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_2\right)\right]C_1+ \\ +\left[\mu\rho_2\sqrt{\frac{g_1}{g_0}}Y'_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_2\right)-\left(1+\frac{n}{2}\right)Y_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_2\right)\right]C_2=0. \end{array}\right. \tag{1.11}$$

From the condition of existence of nontrivial solutions of the system (1.11), we get the following characteristic equation:

$$\triangle\left(\mu,\rho_1,\rho_2\right)=\mu^2\rho_1\rho_2\frac{g_1}{g_0}L^{(1;1)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\right)-\mu\left(1+\frac{n}{2}\right)\sqrt{\frac{g_1}{g_0}}\left[\rho_1L^{(1;0)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\right)+\right.$$

$$\left.+\rho_2L^{(0;1)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\right)\right]+\left(1+\frac{n}{2}\right)^2L^{(0;0)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\right)=0, \tag{1.12}$$

where

$$L^{(i;j)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\right)=J^{(i)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_1\right)\cdot Y^{(j)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_2\right)-$$

$$-J^{(j)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_2\right)\cdot Y^{(i)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_1\right);\quad i,j=0;1$$

The transcendental equation (1.12) determines the denumerable set μ_k and the constants C_{1k}, C_{2k} corresponding to it are proportional to algebraic complements of the elements of some row of the determinant of the system (1.11). For C_{1k}, C_{2k} we have:

$$C_{1k}=\left[\mu_k\rho_2\sqrt{\frac{g_1}{g_0}}Y'_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)-\left(1+\frac{n}{2}\right)Y_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)\right]B_k, \tag{1.13}$$

$$C_{2k}=-\left[\mu_k\rho_2\sqrt{\frac{g_1}{g_0}}J'_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)-\left(1+\frac{n}{2}\right)J_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)\right]B_k,$$

where B_k are arbitrary constants.

Substituting (1.13) in (1.9), (1.10) and summing over all roots, we get the homogeneous solution in the following form:

$$u_\varphi\left(\rho,\xi\right)=\sum_{k=1}^{\infty}\rho^{-\frac{n}{2}}\left[\mu_k\rho_2\sqrt{\frac{g_1}{g_0}}L^{(0;1)}_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho;\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)-\right.$$

$$\left.-\left(1+\frac{n}{2}\right)L^{(0;0)}_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho;\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)\right]m_k\left(\xi\right), \tag{1.14}$$

$$\sigma_{\rho\varphi}=\sum_{k=1}^{\infty}g_0\rho^{\frac{n}{2}-1}\left[\mu_k^2\frac{g_1}{g_2}\rho\rho_2\cdot L^{(1;1)}_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho;\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)-\right.$$

$$-\mu_k\sqrt{\frac{g_1}{g_0}}\left(1+\frac{n}{2}\right)\left(\rho L^{(1;0)}_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho;\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)-\right.$$

$$-\rho_2 L^{(0;1)}_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho;\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)\Big)+$$

$$+\left(1+\frac{n}{2}\right)^2 L^{(0;0)}_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho;\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)\Big] m_k(\xi); \tag{1.15}$$

$$\sigma_{\varphi\xi}=\sum_{k=1}^{\infty}\rho^{\frac{n}{2}}g_1\left[\mu_k\rho_2\sqrt{\frac{g_1}{g_0}}\cdot L^{(0;1)}_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho;\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)-\right.$$

$$\left.-\left(1+\frac{n}{2}\right)L^{(0;0)}_{1+\frac{n}{2}}\left(\mu_k\sqrt{\frac{g_1}{g_0}}\rho;\mu_k\sqrt{\frac{g_1}{g_0}}\rho_2\right)\right] m'_k(\xi); \tag{1.16}$$

where

$$L^{(i;j)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho;\mu\sqrt{\frac{g_1}{g_0}}\rho_2\right)=J^{(i)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right)Y^{(j)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_2\right)-$$

$$-J^{(j)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho_2\right)Y^{(i)}_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\rho\right);\ i,j=0;1;$$

$m_k(\xi)=D_{1k}e^{\mu_k\xi}+D_{2k}e^{-\mu_k\xi}$; D_{1k},D_{2k} are arbitrary constants.

Represent (1.7), (1.8) in the form

$$A\upsilon=\mu^2\upsilon, \tag{1.17}$$

where

$$A\upsilon=\left\{-\frac{g_0}{g_1}\left[\frac{d^2\upsilon}{d\rho^2}+\frac{(n+1)}{\rho}\frac{d\upsilon}{d\rho}-\frac{(n+1)}{\rho^2}\upsilon\right],\ g_0\rho^n\left(\frac{d\upsilon}{d\rho}-\frac{\upsilon}{\rho}\right)\Big|_{\rho=\rho_s}=0\right\}.$$

It is easy to prove that A is a self- adjoint operator in the Hilbert space $L_2(\rho_1,\rho_2)$ with the weight ρ^{n+1} . Consequently, all the eigen values $\lambda_k(A)=\mu_k^2$ are real, and the eigen functious are orthonormed, complete and form a basis in the space $L_2(\rho_1,\rho_2)$ [6]:

$$(\upsilon_k;\upsilon_p)=\int_{\rho_1}^{\rho_2}\upsilon_k(\rho)\,\overline{\upsilon}_p(\rho)\,\rho^{n+1}d\rho=\delta_{kp}. \tag{1.18}$$

Substitute (1.16) in boundary conditions (1.3),

$$\sum_{k=1}^{\infty}g_1\rho^n\upsilon_k(\rho)\,m'_k(\xi)\Big|_{\xi=\pm l}=f^{\pm}(\rho). \tag{1.19}$$

Multiplying expression (1.19) by $\rho\overline{\upsilon}_p(\rho)$ and integrating within $[\rho_1,\rho_2]$, allowing for (1.18) we get

$$m'_k(\xi)\big|_{\xi=\pm l}=t_k^{\pm},$$

i.e.

$$\left(\mu_k e^{\mu_k\xi}D_{1k}-\mu_k e^{-\mu_k\xi}D_{2k}\right)_{\xi=\pm l}=t_k^{\pm}, \tag{1.20}$$

where

$$t_k^{\pm}=\frac{1}{g_1}\int_{\rho_1}^{\rho_2}\rho f^{\pm}(\rho)\,\overline{\upsilon}_k(\rho)\,d\rho.$$

After solving (1.20) we determine the unknown constants D_{1k} and D_{2k}:

$$D_{1k}=\frac{t_k^+e^{\mu_k l}-t_k^-e^{-\mu_k l}}{2\mu_k sh\left(2\mu_k l\right)},\ D_{2k}=\frac{t_k^+e^{-\mu_k l}-t_k^-e^{\mu_k l}}{2\mu_k sh\left(2\mu_k l\right)}.$$

2. Assume that the cylinder of a small thickness.Study the asymptotic behavior of the solution of the above studied problem. The left hand side of transcendental equation (1.12) as an entire function of the parameter μ has a denumerable set of zeros with concentration point at infinity .

Assume

$$\rho_1=1-\varepsilon;\ \rho_2=1+\varepsilon \tag{2.1}$$

where $\varepsilon=\frac{r_2-r_1}{2r_0}$ is a small parameter characterizing the cylinder thickness.

Substituting (2.1) in ((1.12), we get:

$$\Delta\left(\mu,\rho_1,\rho_2\right)=D\left(\mu;\varepsilon\right)=0. \tag{2.2}$$

The function $D\left(\mu;\varepsilon\right)$ as $\varepsilon\to 0$ has two groups of zeros with the following asymptotic properties:

a) the first group consists of double zero $\mu=0$;

b) the second group consists of a denumerable set of zeros that are of order $O\left(\varepsilon^{-1}\right)$.

Give the sheme of the proof of these properties.

Represent $D\left(\mu;\varepsilon\right)$ in the form:

$$D\left(\mu;\varepsilon\right)=\frac{4\varepsilon}{\pi}\mu^2\left\{\frac{g_1}{g_0}+\left[-\frac{2}{3}\mu^2\frac{g_1^2}{g_0^2}+\frac{2}{3}\frac{g_1}{g_0}\left(1+\frac{n}{2}\right)^2+\frac{4}{3}\frac{g_1}{g_0}\left(1+\frac{n}{2}\right)\right]\varepsilon^2+\right.$$

$$+\left[\frac{2}{15}\frac{g_1^3}{g_0^3}\mu^4+\frac{g_1^2}{g_0^2}\left(\frac{2}{15}-\frac{4}{15}\left(1+\frac{n}{2}\right)^2-\frac{8}{15}\left(1+\frac{n}{2}\right)\right)\mu^2+\frac{g_1}{g_0}\left(\frac{2}{15}\left(1+\frac{n}{2}\right)^4+\right.\right.$$

$$\left.\left.+\frac{8}{15}\left(1+\frac{n}{2}\right)^3+\frac{4}{5}\left(1+\frac{n}{2}\right)^2+\frac{8}{15}\left(1+\frac{n}{2}\right)\right)\right]\varepsilon^4+.....\Bigg\}=0, \tag{2.3}$$

Note that

$$D\left(\mu;\varepsilon\right)=\mu^2 D_0\left(\mu;\varepsilon\right)$$

and $\lim\limits_{\mu\to 0}D_0\left(\mu;\varepsilon\right)\neq 0.$

Thus, we get that $\mu=0$ is a double zero of $D\left(\mu;\varepsilon\right)$.

Show that all the zeros of $D_0\left(\mu;\varepsilon\right)$ unboundedly increase as $\varepsilon\to 0$. Assume the contrary. Suppose $\mu\to\mu_k^*$ as $\varepsilon\to 0$. Then the limit relation $D_0\left(\mu_k;\varepsilon\right)=\varepsilon D_*\left(\mu_k^*\right)$ is valid as $\varepsilon\to 0$. The limiting points of the zeros set μ_k as $\varepsilon\to 0$ are determined from the equation $D_*\left(\mu_k^*\right)=\frac{4g_1}{\pi g_0}=0$. Consequently the assumption on the existence of bounded as $\varepsilon\to 0$ zeros is not valid.

Determine the character of tendency of $\mu_k\to\infty$ as $\varepsilon\to 0$. As $\varepsilon\to 0$ the following cases are possible:

1) $\varepsilon\mu_k\to 0$, 2) $\varepsilon\mu_k\to const$ 3) $\varepsilon\mu_k\to\infty$.

Similar to the method in [5] we can show that cases 1) and 3) are impossible here.

For constructing the asymptotic of the zeros of the second group, we look for them in the form :

$$\mu_k = \frac{\delta_k}{\varepsilon} + O(\varepsilon) \tag{2.4}$$

Substituting (2.4) in (1.12) and taking into account the asymptotic expansions of functions $J_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\right)$, $Y_{1+\frac{n}{2}}\left(\mu\sqrt{\frac{g_1}{g_0}}\right)$ for large values of the argument, [3] for δ_k we get:

$$\sin\left(2\sqrt{\frac{g_1}{g_0}}\delta_k\right) = 0, \tag{2.5}$$

i.e. $\delta_k = \frac{\pi k}{2\sqrt{\frac{g_1}{g_0}}}$.

Unlike the isotropic shell for fixed values of "k" and for large values $\sqrt{\frac{g_1}{g_0}}$ (strong anisotropy) the variability index of stress state δ_k tends to zero. In this case, some boundary layer solutions have no damping properties and may cover the domain engaged by a shell.

The displacement and stresses corresponding to the root $\mu^2 = 0$ are determine by the following formulas:

$$u_\varphi^{(1)}(\rho,\xi) = A_0\rho\xi, \tag{2.6}$$

$$\sigma_{\varphi\xi}^{(1)} = g_1 A_0 \rho^{n+1},\ \sigma_{\rho\varphi}^{(1)} = 0. \tag{2.7}$$

Assuming $\rho = 1 + \varepsilon\eta\,(-1 \le \eta \le 1)$, the solutions corresponding to the second group of zeros may be represented in the form:

$$\begin{aligned}
u_\varphi^{(2)} &= \sum_{k=1}^{\infty}\left[\sqrt{\frac{g_1}{g_0}}\delta_k \cos\left(\sqrt{\frac{g_1}{g_0}}\delta_k(1-\eta)\right) + O(\varepsilon)\right] m_k(\xi),\\
\sigma_{\rho\varphi}^{(2)} &= \frac{g_1}{\varepsilon}\sum_{k=1}^{\infty}\left[\delta_k^2 \sin\left(\sqrt{\frac{g_1}{g_0}}\delta_k(1-\eta)\right) + O(\varepsilon)\right] m_k(\xi),\\
\sigma_{\varphi\xi}^{(2)} &= g_1\sum_{k=1}^{\infty}\left[\sqrt{\frac{g_1}{g_0}}\delta_k \cos\left(\sqrt{\frac{g_1}{g_0}}\delta_k(1-\eta)\right) + O(\varepsilon)\right] m_k'(\xi).
\end{aligned} \tag{2.8}$$

Show the character of the constructed solutions. Represent the displacement in the form:

$$u_\varphi(\rho,\xi) = A_0\rho\xi + \sum_{k=1}^{\infty}\upsilon_k(\rho)\, m_k(\xi). \tag{2.9}$$

The displacements determined by the second group of solutions are contained in the second addend.

For stresses we have :

$$\sigma_{\varphi\xi} = g_1\rho^{n+1}A_0 + g_1\sum_{k=1}^{\infty}\rho^n\upsilon_k(\rho)\, m_k'(\xi)\ , \tag{2.10}$$

$$\sigma_{\rho\varphi} = g_0\sum_{k=1}^{\infty}\rho^n\left(\upsilon_k'(\rho) - \frac{\upsilon_k(\rho)}{\rho}\right) m_k(\xi). \tag{2.11}$$

For the torques M_{trq} of stresses acting in the section $\xi = const$, we have

$$M_{kp} = 2\pi \int_{\rho_1}^{\rho_2} \sigma_{\varphi\xi} \rho^2 d\rho, \tag{2.12}$$

Substitute (2.10) in (2.12)

$$M_{trq} = \frac{2\pi g_1 A_0}{n+4} \left(\rho_2^{n+4} - \rho_1^{n+4}\right) + 2\pi g_1 \sum_{k=1}^{\infty} \left(\int_{-1}^{1} \rho^{n+2} \upsilon_k(\rho)\, d\rho \right) m_k'(\xi). \tag{2.13}$$

Multiply the both parts of (1.7) by ρ^{n+2} and integrate the obtained expression in $[\rho_1, \rho_2]$:

$$\int_{\rho_1}^{\rho_2} \rho^{n+2} \upsilon_k''(\rho)\, d\rho + (n+1) \int_{\rho_1}^{\rho_2} \rho^{n+1} \upsilon_k'(\rho)\, d\rho +$$

$$+ \frac{g_1}{g_2} \mu_k^2 \int_{\rho_1}^{\rho_2} \rho^{n+2} \upsilon_k(\rho)\, d\rho - (n+1) \int_{\rho_1}^{\rho_2} .\upsilon_k(\rho)\, \rho^n d\rho = 0 \tag{2.14}$$

By means of integration by parts and using (1.8), from (2.14) we get:

$$\int_{\rho_1}^{\rho_2} \rho^{n+2} \upsilon_k(\rho)\, d\rho = 0 \tag{2.15}$$

Substitute (2.15) in (2.13)

$$M_{kp} = \frac{2\pi g_1}{n+4} \left(\rho_2^{n+4} - \rho_1^{n+4}\right) A_0. \tag{2.16}$$

Solution (2.6) determines the internal stress-strain state of the shell. The stress state determined by this solution is equivalent to torque M_{trq} of stresses acting in the section $\xi = const$.

The stress state corresponding to the second group of solutions is selfbalanced at each section $\xi = const$, and has the boundary layer character. The first terms of its asymptotic expansion are equivalent to Saint –Venant's edge effect in theory of nonhomogeneous plates [5].

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Received September 04, 2013; Revised Desember 25, 2013