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OSCILLATIONS OF A NONHOMOGENEOUS DIFFERENT MODULUS BEAM WITH A LOAD MOVING ON IT SITUATED ON NONHOMOGENEOUS VISCOELASTIC FOUNDATION

Abstract

In the paper it is supposed that the material of the beam is inhomogeneous in height and length of the beam. The equation of motion is a fourth order differentiable equation with variable coefficients.

Influence of environment created by a non-homogeneous viscoelastic foundation is simulated by the viscoelastic variant of the Winkler scheme. The solution is constructed on the base of combination of approximate analytic methods. The numerical analysi is conducted for concrete values of typical parameters.

As is known, at present, natural and composite materials having non-homogeneous properties and differently resisting to tension and compression [1,3] are widely used in civil engineering, machine-building and a number of other fields of engineering.

In the present paper, it is supposed that the beam is continuously non-homogeneous in length and height and differently resists to compression-tension and lies an a viscoelastic foundation. It is accepted that the modulus of elasticity and specific density depend on spatial coordinates in the following way:

at tension

$$E^{+} = E_0^{+} f_1(x) \cdot f_2(z)$$

at compression

$$E^{-} = E_0^{-} f_1(x) \cdot f_2(z)$$

$$\rho = \rho_0 \psi_1(x) \cdot f_2(z) \tag{1}$$

E_0^{+}, E_0^{-}, ρ_0 corresponds to a homogeneous different modulus medium, the function $f_1(x)$ with its derivatives to the second order is a continuous function, $f_2(z)$, $\psi_2(x)$, $\psi_1(x)$ are continuous functions.

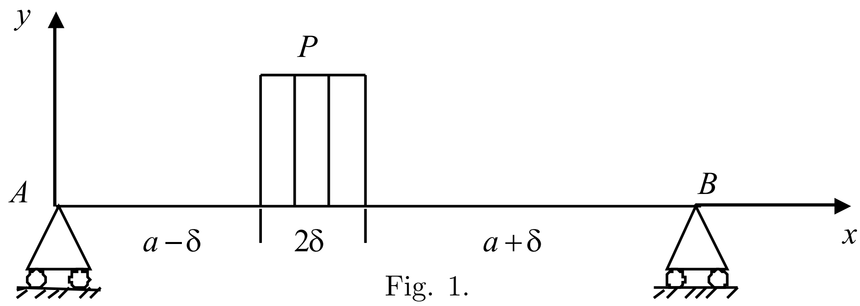


Fig. 1.

Let's consider a problem on motion of a load of weight P that moves on a continuously non-homodeneous beam differently resistung to tension-compression and lying on a visco-elastic foundation.

It is supposed that the load along the beam moves with considerable velocity, and the load's mass is very small compared with the beam's mass.Denote by V the velocity of motion, P is the weight of the moving load. Assume that for $t = 0$ the load enters into the beam, and at time t it will be at the distance $a = Vt$ (fig.1).

In order to compose the motion equation it is necessary to determine the value of the bending moment through the curvature of the central line (the cross section of the beam has two symmetry axes).

As the beam's material is of different modulus, the neutral line doesn't coinside with the central line. Teh stress distribution on the cross section changes by the principle

$$\sigma^{+} = E_0^{+} f_1(x) f_2(z) \cdot (l + \Omega z)$$

in contrative zones

$$\sigma^{-} = E_0^{-} f_1(x) f_2(z) \cdot (l + \Omega z) \tag{2}$$

Here l is relative deformation of the central line, Ω is curvature. Between l, Ω the boundaries of the central line with the bending moment are connected with the following relations:

$$\int_{-h}^{+h} \sigma ds = \int_{S_1} \sigma^{+} ds + \int_{S_2} \sigma^{-} ds = 0 \tag{3}$$

$$M = \int_{S} z\sigma ds = \int_{S_1} z\sigma^{+} dz + \int_{S_2} z\sigma^{-} dz. \tag{4}$$

From the first two equations we can get the following relation:

$$e = \mu \cdot \Omega \tag{5}$$

Here we introduce the denotation

$$\mu = -\frac{\int_{S_1} f_2(z)\, b(z)\, dz + \alpha \int_{S_2} f_2(z)\, b(z)\, dz}{\int_{S_1} zb(z) \cdot f_2(z)\, dz + \alpha \int_{S_2} zb(z) \cdot f_2(z)\, dz} \tag{6}$$

where S_1 and S_2 are the zones of tensile and compressive stresses on the beam's cross section. Removing (4) and taking into account (6), we get the following expression for the bending moment:

$$M = J_0 E_0^{+} K(\rho, \alpha) f_1(x) \frac{\partial^2 w}{\partial x^2}. \tag{7}$$

Here we accept the following denotation:

$$\rho = z_0 \cdot h^{-1}; \quad \alpha = \frac{E_0^{-}}{E_0^{+}} \tag{8}$$

The equation of motion of a beam with regard to visco-elastic resistance of the material will take the following form:

$$\frac{\partial^2}{\partial x^2}\left[f_1(x)\frac{\partial^2 w}{\partial x^2}\right] + \overline{k}_1(x) + \overline{k}_2(x)\frac{\partial^2 w}{\partial t^2} +$$

$$+m^2\psi_1(x)\frac{\partial^2 w}{\partial t^2}=\left(E^+J_0K\right)^{-1}\cdot P(x,t). \qquad (9)$$

Here we introduce the denotation:

$$\overline{k}_1(x)=k_1(x)\left(E^+J_0K\right)^{-1}; \qquad \overline{k}_2(x)=k_2(x)\left(E^+J_0K\right)^{-1}$$

$$\rho=\frac{\rho_0}{2h}\int\limits_{-1}^{+1}\psi_2(\rho)\,d\rho; \qquad m^2=\overline{\rho}\left(E_0J_0K\right)^{-1} \qquad (10)$$

$E_0^+\cdot J_0$ is the rigidity of the homogeneous beam identically resisting to tension and compression.

In order to represent $P(x,t)$ in the expanded form expand it in Fourier series in sines assuming that the load is uniformly distributed on the small area of the bar's length from $a-\delta$ to $a+\delta$. Then we get:

$$\text{for } a-\delta<x;\ P(x,t)=0$$

$$\text{for } \ a-\delta\le a+\delta;\ P(x,t)=\frac{p_0}{2\delta},$$

$$\text{for } \ a+\delta<x;\ P(x,t)=0 \qquad (11)$$

The coefficients A_m of the Fourier series of the expansion of the function $P(x,t)$

$$P(x,t)=\sum_{m=1}^{\infty}A_m(t)\sin\frac{m\pi x}{l} \qquad (12)$$

are the limiting values

$$A_m=\frac{2P_0}{m\pi\delta}\sin\frac{m\pi t}{l}\cdot\sin\frac{m\pi t}{l} \qquad (13)$$

as $\delta\to 0$ that gives

$$A_m=\frac{2P_0}{l}\sin\frac{m\pi a}{l} \qquad (14)$$

Here it is taken into account:

$$\lim_{\delta\to 0}\frac{\sin\dfrac{m\pi\delta}{l}}{\dfrac{m\pi\delta}{l}}=1 \qquad (15)$$

Taking into account (14) in (13) and taking into attention $v\cdot t=a$, we get

$$P(x,t)=\sum_{m=1}^{\infty}\frac{2P_0}{l}\cdot\sin\frac{m\pi v}{l}\cdot\sin\frac{m\pi x}{l} \qquad (16)$$

Having been restricted by one member of the series, with regard to viscoelastic ressistance [3] we get the equation of motion in the following form:

$$A_1(x)\frac{\partial^2 w}{\partial x^4}+A_2(x)\frac{\partial^3 w}{\partial x^3}+A_3(x)\frac{\partial^2 w}{\partial x^2}+\overline{k}_1(x)\,w+$$

$$+\overline{k}_2(x)\frac{\partial^2 w}{\partial t^2}+m_0^2\psi_1(x)\frac{\partial^2 w}{\partial t^2}=\overline{P}\sin\beta t\cdot\sin\frac{m\pi x}{l} \qquad (17)$$

where the following denotation are introduced:

$$A_1(x)=f_1(x);\quad A_2(x)=\frac{df_1}{dx};\quad A_3(x)=\frac{d^2f_1}{dx^2}$$

$$m_0^2=\overline{p}(E_0J_0K)^{-1};\quad \overline{p}=\frac{2P_0}{l\left(E_0^+JK\right)};\quad \left(\beta=\frac{m\pi}{l}V\right) \qquad (18)$$

Equation (17) is complicated and definition of its exact solution is difficult. Therefore, by solving the problem we'll use the combined approximate-analytic method whose essence is the following. In step I assuming that the motion happens acoording to harmonic law, the method of separation of variables is applied, in step 2 the Bubnov-Galerkin method of orthogonalization is used.

In step I we accept

$$w=w_0(x)\sin\beta\cdot t \qquad (19)$$

Here $w_0(x)$ should satisfy the following boundary conditions. Substituting (19) in (17), we get:

$$A_1(x)\frac{d^4w_0}{dx^4}+A_2(x)\frac{d^3w_0}{dx^3}+A_3(x)\frac{d^2w_0}{dx^2}+\overline{k}_1(x)w_0-$$

$$-\beta^2\cdot\left(\overline{k}_2(x)-m_0^2\psi_1(x)\right)w_0=\overline{P}\cdot\sin\frac{m\pi}{l}x \qquad (20)$$

In order to find β^2 we use the Bubnov and Galerkin method accepting that

$$w_0=\sum_{i=1}^{n}C_i(x)\theta_i(x) \qquad (21)$$

C_i are unknown constants, each $\theta_i(x)$ satisfies the boundary conditions.

In this case the orror function will be:

$$\eta(x)=\sum_{i=1}^{n}C_i\left[A_1\frac{d^4\theta_i}{dx^4}+A_2\frac{d^3\theta}{dx^3}+A_3\frac{d^2\theta}{dx^2}+\overline{k}_1\theta_i-\right.$$

$$\left.-\left(\beta^2\left(\overline{k}_2(x)-m_0^2\psi_1(x)\right)\right)w_0-\overline{P}\sin\frac{m\pi}{l}x\right]\neq 0 \qquad (22)$$

Then based on the Bubnov-Galerkin method we can write

$$\int_0^1\eta(x)\cdot\theta_k(x)\,dx=0;\quad k=1,2,...,n \qquad (23)$$

For an arbitrary approximation, β^2 is determined from the equality to zero of the basic determinant of the system of homogeneous linear algebraic equations with respect to the coefficients C_i:

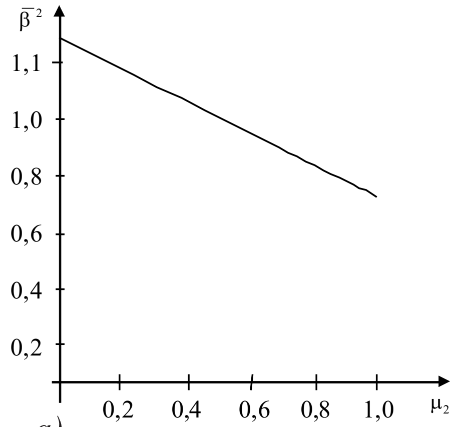
$$\left\|\beta^2\right\|=0 \qquad (24)$$

However, for practical calculation one can be satisfiied with the first approximation although for any approximation it is not difficult to find β^2.

For the first approximation we find:

$$\beta^2 = \frac{\int\limits_0^l \left[A_1(x) \frac{d^4\theta_1}{dx^4} + A_2(x) \frac{d^3\theta_1}{dx^3} + A_3(x) \frac{d^2\theta_1}{dx^2} + \overline{k}_1(x)\theta_1 \right] \theta_1(x)\, dx}{\int\limits_0^l \left[\overline{k}_2(x)\theta_1 + m_0^2 \psi_1(x) \right] \theta_1^2 dx} \tag{25}$$

From (25) for $\overline{k}_2(x) = 0$, $k_1 \neq 0$ we get the solution of the similar problem for non-homogeneous elastic resistance, $k_1 = 0$; $k_2 \neq 0$ is the solution for a pure viscous resistance.

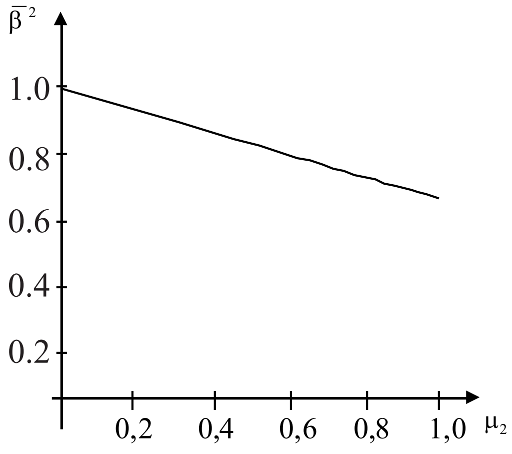


a)

$\mu_1 = 0,3$

μ_2	$\overline{\beta}^2 = \frac{1,15}{1+0,5\mu_2}$
0	1,15
0,2	1,04
0,4	0,95
0,6	0,88
0,8	0,82
1,0	0,76

tab. 1.



b)

$\mu_1 = 0$

μ_2	$\overline{\beta}^2 = \frac{1}{1+0,5\mu_2}$
0	1
0,2	0,90
0,4	0,83
0,6	0,76
0,8	0,71
1,0	0,66

tab. 2.

Fig. 2.

As it is seen from (25), in order to determine the value of β^2, the values of the functions $f_1(x), \psi_1(x), f_2(z), \psi_1(z)$ and also of $k_1(x)$ and $k_2(x)$ of approximating function $\theta_1(x)$ should be given.

Numerical analysis is carried out for the case

$$\theta_1 = \sin \alpha_n x \quad \alpha_n = \frac{m\pi}{l}; \quad f_2 = e^{\alpha_1 \eta}; \quad \psi_2 = e^{\alpha_2 n} \quad \left(\eta = z \cdot h^2\right)$$

$$\overline{x} = x \cdot l^{-1}; \quad f_1(x) = 1 + \mu_1 \overline{x}; \quad \psi_1(x) = 1 + \mu_2 \overline{x};$$
$$k_1 = 1 + c\overline{x}; \quad k_2 = 0 \tag{26}$$

Making a number of elementary transformations, we get:

$$\beta^2 = \frac{(1 + 0,5\mu_1)\,\alpha_n^4 + k_1(1 + 0,5c)}{m_0^2(1 + 0,5\mu_2)} \tag{27}$$

In the case when the beam is non-homogeneous only by modulus of elasticity, we have:

$$\beta_0^2 = \frac{(1 + 0,5\mu_1)\,\alpha_n^4 + k_1(1 + 0,5c)}{m_0^2}. \tag{28}$$

Then from (27) and (28) we get:

$$\overline{\beta}^2 = \left(\frac{\beta_i}{\beta_0}\right) = \frac{1}{1 + 0,5\mu_2}. \tag{29}$$

Ignoring the external resistance we get:

$$\beta_2^2 = \frac{p(\mu_1, \mu_2)}{m_0^2}\alpha_n^4; \quad \rho = \frac{1 + 0,5\mu_1}{1 + 0,5\mu_2} \tag{30}$$

The results of calculations are in fig.2

References

[1]. Tolokonnikov L.A. *On relation between stresses and strains in different-modulus media.* Inz. MTT, 1968, No 6, pp.108-110 (Russian)

[2]. Lomakin E.V., Rabotnov Yu.N. *Relation of theory of elasticity for an isotropic different-modulus body*, MTT, 1978, No 6, pp.19-54 (Russian)

[3]. Rzhanytsin A.R. *Civil Enginnering mechanics.* M. 1982, 399 p. (Russian)

[4]. H. Garnet, L. Levy. *Free vibrations of Reinforced elastic shells.* Conference of ACME, November 16-20, 1969, Los-Angeles. Caliphornia

[5]. Kolcin G.B. *Analysis of structural elements made of non-homogeneous materials.* Kishinyev, M., 1970, 172 p. (Russian)

[6]. Gordon V.A. *Eigen flexural vibrations of non-homogeneous bars.* Applied problems of strength and plasticity. Gorky University, 1985, pp.88-95. Analysis of structural elements (Russian)

[7]. Kravchuk A.S., Maiboroda V.P., Urzhumtsev Yu.S. *Mechanics of polymer and composite materials*, 1985, 365 p. (Russian)

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