MECHANICS

Ismail A. AMIRASLANOV, Nigar I. AMIRASLANOVA

CONTACT FILTRATION UNDER A DAM. ANALYTIC SOLUTION OF BOUNDARY VALUE PROBLEM.

Abstract

Theory of contact filtration generated under a dam is investigated on an example. The stated problem is solved analytically.

Very often the contact filtration holds along the foundation of a dam when there is a thin water layer between the soil and the dam. Examine this phenomenon on an example.

Let the soil occupy the half-space $y < 0$, the impermeable dam (apron) the layer $y > 0$, $|x| < \bar{\ell}$, water under pressure P_+ the domain $y > 0$, $x < -\ell$, water under pressure P_- the domain $y > 0$, $x > +\ell$. For definiteness we'll assume that $P_+ > P_-$; therewith the water leaks under the dam from left to right (fig. 1).

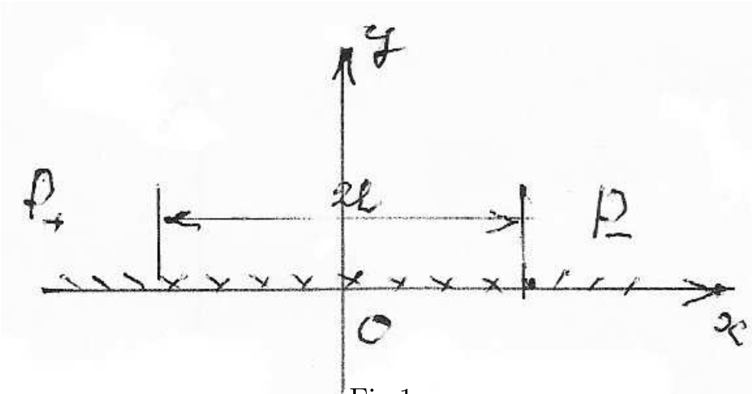

Fig.1.

The problem is assumed to be plane. The main equations of the plane problem, by means of the complex potential $f(z)$ according to [1] may be written as follows:

$$\upsilon_x + i\upsilon_y = f'(z), \quad P = -\frac{\rho g}{k} \operatorname{Re} f(z) - \rho g y. \tag{1}$$

Here P is fluid's pressure, k is filtration coefficient, y is directed to an opposite way to the gravity force direction.

The boundary conditions of the problem under consideration have the following form. For

$$\begin{array}{c} |x| < \ell, \;\; y = 0 \quad \upsilon_y = 0 \quad \text{for} \quad |x| > \ell \;\; \frac{\partial P}{\partial x} = 0 \\ (\upsilon_x + i\upsilon_y \to 0 \quad \text{as} \quad z = x + iy \to \infty) \end{array} . \tag{2}$$

Hence, on the basis of (2), for the complex potential we have the following boundary value problem. For $Jmz = 0 \quad |\operatorname{Re} z| < \ell \quad Jmf'(z) = 0$

$$\text{for} \quad |\operatorname{Re} z| > \ell \qquad \operatorname{Re} f'(z) = 0, \qquad f'(z) \to 0 \quad \text{as} \quad z \to \infty. \tag{3}$$

The solution of this boundary value problem is

$$f'(z) = \frac{c}{\sqrt{z^2 - \ell^2}}, \quad \left(\sqrt{z^2 + \ell^2} \to z \quad \text{as} \quad z \to \infty\right). \tag{4}$$

The real constant c is determined from the additional condition

$$\text{for} \quad y = 0 \quad \int_{-e}^{e} \frac{dP}{dx} dx = \Delta p \qquad (\Delta p = P_+ - P_-). \tag{5}$$

Substituting in (5) solution (4), according (1) we get

$$c = -\frac{k\Delta p}{\pi \rho g}. \tag{6}$$

Near the singular point $z = \ell$ solution (4), (6) behaves as

$$f'(z) = \frac{K}{\sqrt{2\pi\varepsilon}}, \qquad \upsilon_x = -\frac{K \sin\frac{\varphi}{2}}{\sqrt{2\pi|\varepsilon|}}$$

$$\upsilon_y = -\frac{K \cos\frac{\varphi}{2}}{\sqrt{2\pi|\varepsilon|}} \quad \left(\varepsilon = z - \ell = |\varepsilon| \ell^{i\varphi}, \quad \varepsilon << \ell\right). \tag{7}$$

Here

$$K = \frac{k\Delta p}{pg\sqrt{\pi\ell}}. \tag{8}$$

By means of (7) we calculate the invariant characteristics of the singular point

$$\Gamma_1 = \frac{\rho}{2\varepsilon^2} K^2, \quad \Gamma_2 = \Gamma_3 = 0. \tag{9}$$

In the present problem, according to (8) we have

$$\Gamma_1 = \frac{k^2 (\Delta\rho)^2}{2\rho\varepsilon^2 g^2 \pi\ell}. \tag{10}$$

The quantity Γ_1 is a configuration force of filtration flow acting on soil at the singular point $z = \ell$ and causing all possible critical phenomena near this point (for instance, origin of contact filtration, local cavities in the soil or vice versa bucking of the soil, break of water stream under a dam).

It is natural that occurance of such critical phenomena falling outside the frames of the considered filtration, will be characterized with appropriate critical values of the quantity Γ_1. Denote by Γ_C a constant characterizing the contact filtration initiation so that for $\Gamma_1 < \Gamma_c$ there is no water under the dam, and for $\Gamma_I > \Gamma_c$ it is form a water layer is generated under the dam and contact filtration occurs.

Hence, by means of (10) we find critical pressure differential in the dam

$$(\Delta p)_c = \frac{\varepsilon g}{k}\sqrt{2\pi\rho\Gamma_c\ell}. \tag{11}$$

The solution constructed above is suitable only for $\Delta p < (\Delta p)_c$. For $\Delta p > (\Delta p)_c$ it is necessary to take into account the filtration variation condition because of local wash-out of soil particles. The suggested theory of break of water under a dam refers only the problems having singular points. Such points with trivial exceptions will be for instance the extreme points of an apron (at arbitrary curvilinear outlines of an apron and channel's bottom). According to the principle of microscope [2], a filtration field in the vicinity of these points is always described by formula (7), and the intensity coefficient of the field K will be some function of geometrical and physical parameters of the appropriate boundary value problem of filtration theory.

The end of a sheet piling has the similar singularity, however, physically this case is less interesting since in this case the soil particles can't carried away by the flow and therefore a comparatively stable nonlinear filtration area in which the structure of soil particles differ from the initial one, is generated near the sheet piling end.

Accept the following natural physical assumption: wash-out of soil particles at some point of its surface is determined by filtration velocity at this point. From this assumption it follows that the wash-out of particles begins always at singular points where the filtration velocity in infinitely great. It is obvious that according to this general assumption, the initiation of wash-out of particles near the singular point is characterized by some critical value of field intensity factor at this point $K > K_C$ (for $K < K_C$ the particles wash-out doesn't happen). The quantity K_C depends on the strength of coupling of soil particles one with another, on the form of their sizes and physical properties of liquid, but is independent of macro parameters of the problem (the dam's weight, pressure differential, geometry of the apron and etc.). Therefore, for the given pair of soil-liquid, the quantity K_C may be determined experimentally (for example on a model).

According to universal model (9), the constants Γ_C and K_C are connected in the following way

$$2\varepsilon^2\Gamma_C = \rho K_C^2 \tag{12}$$

Thus, the theory K_C based on natural physical assumptions and theory Γ_C following from the general theory of motion of singularities of physical field reduces to identical results.

The issues of force interaction of filtrational flow and soil skeleton are of great value. It should be underlined that the process of breaking off a separate particle from the soil surface weakly depends on macrostresses in the skeleton, it may happen also at very great compression macrostresses. Physically, it completely differs from the process of macro-failure of the soil skeleton.

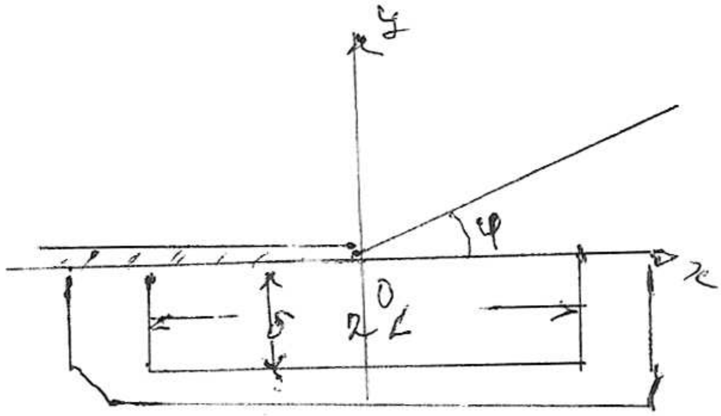

Fig.2.

Break of water under a dam, obviously begins from wash-out of soil particles near the right singularities of the point (for $x = \ell$ on fig. 2).

Here, the further development of the process is not considered. However, if it is assumed that the lateral size of the space formed under the dam near the point

$z=\ell$ because of soil wash-out is negligibly small compared with its length, and pressure change in this space is also negligibly small, then the character of space development may be estimated by means of relation (11), where the quantity ℓ should be assumed as a variable parameter. Obviously, on the base of this relation, having been initiated the space development will be unstable since irreversible decrease will be accompanied with decrease of Δp. Apparently, the velocity $\frac{d\ell}{dt}$ will depend on the process of transportation of particles in the space.

Make same calculations proving formula (9). In the present case the contour $\sum$ in [1] is any closed contour in lower semi-plane $y<0$ that covers the point $z=\ell$ (denoted by 0 in fig. 2), whose ends lie on a real axis and the whole contour is arranged in the zone of action of asymptotics (7) and behaves as

$$\frac{2\varepsilon^2}{\rho}\Gamma_1=\int\limits_{\sum}-\left(\left(\upsilon_x^2+\upsilon_y^2\right)n_z+2\upsilon_n\upsilon_x\right)d\sum.$$

Invariance of Γ_1 relative to the indicated non-closed contour $\sum$ follows owing to the fact that on the real axis $n_x=0$ and in addition on the axis x we have $\upsilon_y=0$ for $x<0$ and $\upsilon_x=0$ for $x>0$.

We use a narrow rectangular contour $\sum$ (fig. 2. ($\frac{\pi}{L}\to 0$ as $\ell\to\infty$)). From the previous one, from (7) and from $n_x=0$ on $\sum$ we have:

$$\frac{2\varepsilon^2}{\rho}\Gamma_1=2\int\limits_{-L}^{L}\upsilon_x\upsilon_y\Big|_{y=-\delta}dx=\frac{K^2}{\pi}\int\limits_{-L}^{L}\frac{\sin\varphi}{\sqrt{x^2+\delta^2}}dx=\frac{K^2}{\pi}\int\limits_{-L}^{L}\frac{\delta}{x^2+\delta^2}dx=$$

$$=\frac{K^2}{\pi}\int\limits_{-\infty}^{\infty}\frac{dt}{t^2+1}=K^2$$

$$\sin\varphi=\frac{y}{\varepsilon}\qquad \varepsilon^2=x^2+\delta^2.$$

Thus, the first formula of (9) is proved. It is proved similarly that $\Gamma_2=\Gamma_3=0$

References.

[1]. Amiraslanov I.A. *On a problem of potential theory.* Doklady AN Azerb SSR, 1984, vol. XL, No 9, pp. 22-25 (Russian).

[2]. Cherepanov G.P. *On the Theory of Fluidiration.* Parts I and II. I nd Eng. Chem. Fundam., vo. 11, No 2, 1972.

[3]. Amiraslanov I.A., Cherepanov G.P. *Some new problems of filtration theory. PMTD* 1982, No 4, pp. 79-85 (Russian).

[4]. Amiraslanov I.A., Abdullayeva L.A., Amiraslanova N.I. *Development of channels in compacted layers.* Doklady of AN Azerb. Republic. 2001, Baku (Russian).

[5]. Polybarinova-Kochina P.Ya. *Theory of underground water motion.* M. Nauka, 1977, 664 p. (Russian).

Ismail A. Amiraslanov, Nigar I. Amiraslanova
Azerbaijan Technological University,
8, Saglamlig str., AZ 2000, Ganja, Azerbaijan
Tel.: (99412) 539 47 20 (off.)

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