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EXISTENCE OF GLOBAL SOLUTION OF CAUCHY PROBLEM FOR A CLASS SYSTEM OF SEMI-LINEAR HYPERBOLIC EQUATIONS WITH DAMPING OF FOURTH ORDER

Abstract

A theorem on the existence of global solution of the Cauchy problem for a class of a system of semi-linear equations containing partial derivatives of fourth order with respect to some variables and second order partial derivatives with respect to some other variables is proved.

The existence of global solution to the Cauchy problem for semi-linear hyperbolic equations with damping was investigated in [11-13]. The global solvability of the Cauchy problem for hyperbolic equations of higher order was studied in [6,9,10,14,16], for the system of semi-linear hyperbolic equations in [11-13].

In [7] the Cauchy problem for the system of semi-linear equations with a weak connection is considered and the conditions for the growth of a non-linear part providing the existence of global solution are found. In the present paper, the Cauchy problem for the system of two semi-linear hyperbolic equations with weak connection, with dissipations and a quasi-elliptic part is investigated. Different derivatives of second and fourth orders with respect different variables participate in the quasi-elliptic part of the equations under consideration.

Problem statement and main result. In the domain $[0,\infty)\times R_n$ consider the Cauchy problem

$$\left.\begin{array}{l} u_{1tt}+u_{1t}+\Delta^2_{I_1}u_1-\Delta_{J_1}u_1=f_1\left(u_1,u_2\right) \\ u_{2tt}+u_{2t}+\Delta^2_{I_2}u_2-\Delta_{J_2}u_2=f_2\left(u_1,u_2\right) \end{array}\right\}, \tag{1}$$

$$u_k\left(0,x\right)=\varphi_k\left(x\right),\quad u_{kt}\left(0,x\right)=\psi_k\left(x\right),\quad x\in R_n,\quad k=1,2, \tag{2}$$

where $\Delta_{Ik}=\sum\limits_{i\in I_k}\frac{\partial^2}{\partial x_i^2}$, $\Delta_{Jk}=\sum\limits_{i\in J_k}\frac{\partial^2}{\partial x_j^2}$, $I_k\subset N_n=\{1,...,n\}$, $J_k=N_n\backslash I$, $k=1,2$.

Denote by $m_r=\overline{\overline{J_r}}$, $r=1,2$ the number of the elements of J_r by $n_r=\overline{\overline{I_r}}=n-m_r$ the number of the elements of I_r. For definiteness suppose

$$m_1\geq m_2.$$

The system of type (1) is met by investigating the vibrations of deformed systems under mobile loads (see [18]).

Assume that the following conditions are fulfilled:

1) Let $n+m_r\leq 4$;

2) $f_1\left(\cdot\right)$ and $f_2\left(\cdot\right)$ are continuously-differentiable functions on R_2;

3) For any $(u,v)\in R_2$ the following estimation is fulfilled:

$$\left|f_r\left(u,v\right)\right|\leq c\left|u\right|^{p_r}\left|v\right|^{q_r}, \tag{3}$$

where

$$p_r\geq 0,\quad q_r\geq 0,\quad p_r+q_r\geq 2, \tag{4}$$

$$\frac{n+m_1}{4}p_r+\frac{n+m_2}{4}q_r>1+\Psi(q_r)\,,\quad r=1,2. \tag{5}$$

Here

$$\Psi(s)=\begin{cases}\frac{m_1-m_2}{4}, & s\geq 2,\\ \frac{(m_1-m_2)(2-s)}{8}, & 0\leq s<2.\end{cases}$$

Denote by $W_{2,k}^{2s,s}$, $k=1,2$ functional spaces with the finite norm:

$$\|u\|^2_{W_{2,k}^{2s,s}}=\left\{\|u\|^2_{L_2(R_n)}+\sum_{i\in I_k}\left\|D^{2s}_{x_i}u\right\|^2+\sum_{i\in J_k}\left\|D^{s}_{x_j}u\right\|^2\right\}^{\frac12}.$$

Let U^k_δ be a ball of radius $\delta>0$ centered at the zero in the space $\left[W_{2,k}^{2,1}\cap L_m(R_n)\right]\times[L_2(R_n)\cap L_m(R_n)]$, i.e.

$$U^k_\delta=\left\{(u,\nu):\|u\|_{W_{2,k}^{2,1}}+\|u\|_{L_m(R_n)}+\|\nu\|_{L_2(R_n)}+\|\nu\|_{L_m(R_n)}<\delta\right\}.$$

We prove the following main theorem.

Theorem 1. *Let supposition (3) and conditions 1-3 be fulfilled. Then there exists such $\delta>0$ that for any $((\varphi_1,\psi_1),(\varphi_2,\psi_2))\in U^1_\delta\times U^2_\delta$ problem (1), (2) has a unique solution $(u_1,u_2)\in C\left([0,\infty);W_{2,1}^{2,1}\times W_{2,2}^{2,1}\right)\cap([0,\infty);L_2(R_n)\times L_2(R_n))$ and for (u_1,u_2) the following estimations are valid:*

$$\|u_k(\cdot,t)\|_{L_2(R_n)}\leq C(\delta)(1+t)^{-\frac{n+m_k}{8}},$$

$$\sum_{i\in I_k}\left\|D^2_{x_i}u_k(\cdot,t)\right\|_{L_2(R_n)}+\sum_{i\in J_k}\|D_{xj}u_k(\cdot,t)\|_{L_2(R_n)}\leq C(\delta)(1+t)^{-\frac{n+m_k+4}{8}},$$

$$\|u_{k_t}(t,\cdot)\|_{L_2(R_n)}\leq C(\delta)(1+t)^{-\gamma_k},$$

where $\gamma_k=\min\left\{\frac{n+m_k+8}{8},\frac{(p_k-1)(n+m_k)}{8}\right\}$, $k=1,2$.

When $p_1=0$, $q_2=0$ problem (1), (2) was investigated in [19].

Auxiliary results. By denoting

$$w=\begin{pmatrix}u_1\\u_2\end{pmatrix},\quad A=\begin{pmatrix}-\Delta^2_{I_1}+\Delta_{J_1}+1 & 0\\ 0 & -\Delta^2_{I_2}+\Delta_{J_2}+1\end{pmatrix},\quad D(A)=W_{2,1}^{4,2}\times W_{2,2}^{4,2},$$

$$B=\begin{pmatrix}1&0\\0&1\end{pmatrix},\quad D(B)=L_2(R_n)\times L_2(R_n)\,,\qquad F(w)=\begin{pmatrix}f_1(u_1,u_2)\\ f_2(u_1,u_2)\end{pmatrix}$$

we can write problem (1), (2) as the Cauchy problem for the operator-differential equation

$$\ddot w+B\dot w+Aw=F(w)\,, \tag{6}$$

$$w(0)=w_0,\quad \dot w_t(0)=w_1 \tag{7}$$

in Hilbert space $H=L_2(R_n)\times L_2(R_n)$, where $\dot w=\begin{pmatrix}u_{1t}\\u_{2t}\end{pmatrix}$, $\ddot w=\begin{pmatrix}u_{1tt}\\u_{2tt}\end{pmatrix}$, $w_0=\begin{pmatrix}\varphi_1(x)\\\varphi_2(x)\end{pmatrix}$, $w_1=\begin{pmatrix}\psi_1(x)\\\psi_2(x)\end{pmatrix}$.

Obviously, A is a self-adjoint positive-definite operator. Using the imbedding theorem and conditions 1-2, we can prove that the nonlinear operator $F(w)$ acting from $H = D\left(A^{1/2}\right) = W_{2,1}^{2,1} \times W_{2,2}^{2,1}$ to H satisfies the Lipschits local condition.

Using the theorem on solvability of the Cauchy problem for nonlinear differential equations in Hilbert space (see [8]), we get the following theorem.

Theorem 2. *Let conditions 1-2 be fulfilled. Then there exists such $T' > 0$ that for any $w_0 \in D\left(A^{1/2}\right)$, $w_1 \in H$ problem (6), (7) has a unique solution $w(\cdot) \in C([0,T');H_1) \cap C^1([0,T');H)$.*

If $T_0 = \sup T'$ i.e. if T_0 is the length of maximal interval of the existence of the solution $w(\cdot) \in C([0,T_0);H_1) \cap C^1([0,T_0);H)$, then

1) either $T_0 = +\infty$;

2) or $\limsup\limits_{t \to T_0 - 0} \left[\|w(t)\|_{H_1} + \|\dot{w}(t)\|\right] = +\infty$.

From theorem 2 it follows that if a priori estimation

$$\sum_{k=1}^{2} \left[\|u_k(t,\cdot)\|_{W_{2,k}^{2,1}} + \|u_{k_t}(t,\cdot)\|_{L_2(R_n)}\right] \le C, \quad t \in [0,T_0), \tag{8}$$

is fulfilled, then $T_0 = +\infty$, i.e. in this case there exists a global solution.

Proof of theorem 1. The following representation holds for the solution of problem (1), (2)

$$u_k(x,t) = u_{k0}(\cdot x,t) * \varphi_k(x) + u_{k1}(x,t) * \psi_k(x) +$$

$$+\int_0^t u_{k1}(x,t-\tau) * f_k(u_1(x,t),u_2(x,t))\,d\tau, \quad k = 1,2, \tag{9}$$

where $u_k(\cdot,t) = F^{-1}(\overline{u_k}(\cdot,t))$, $k = 1,2, F^{-1}$ is the Fourier inverse transformation, $\overline{u}_{k0}(\cdot,t)$ and $\overline{u}_{k1}(\cdot,t)$ is the solution of the following problems:

$$\left.\begin{array}{c} \overline{u}_{k0tt} + \overline{u}_{k0t} + \sum\limits_{i \in I_k} \xi_i^4 \overline{u}_{k0} + \sum\limits_{i \in J_k} \xi_j^2 \overline{u}_{k0} = 0, \quad t > 0, \quad \xi \in R_n, \\ \overline{u}_{k0}(0,\xi) = 1, \ \overline{u}_{k0t}(0,\xi) = 0, \quad \xi \in R^n, \quad k = 1,2; \end{array}\right\} \tag{10}$$

$$\left.\begin{array}{c} \overline{u}_{k1tt} + \overline{u}_{k1t} + \sum\limits_{i \in I_k} \xi_i^4 \overline{u}_{k1} + \sum\limits_{i \in J_k} \xi_j^2 \overline{u}_{k1} = 0, \quad t > 0, \quad \xi \in R_n, \\ \overline{u}_{k1}(0,\xi) = 1, \ \overline{u}_{k1t}(0,\xi) = 0, \quad \xi \in R_n, \quad k = 1,2. \end{array}\right\} \tag{11}$$

If by $\overline{l^r} = (l_1^r, l_2^r, ..., l_n^r)$ we denote a vector in R_n with the coordinates $l_k^r = 2$ for $k \in I_r$, $l_k^r = 1$ for $k \in J_r$, then we get $\left|\frac{1}{l^r}\right| = \frac{1}{l_1^r} + ... + \frac{1}{l_1^r} = \frac{n+m_r}{2}$, where $r,k = 1,2$. Therefore, from the results of the paper [9] it follows that for $u_{k0}(\cdot,t)$, $u_{k1}(\cdot,t)$, $k = 1,2$ the following estimations are valid:

$$\|u_{k0}(t,\cdot) * \varphi_k(\cdot)\|_{L_2(R_n)} \le$$

$$\le c(1+t)^{-\frac{n+m_k}{8}} \left[\|\varphi_k(\cdot)\|_{L_1(R_n)} + \|\varphi_k(\cdot)\|_{L_2(R_n)}\right]; \tag{12}$$

$$\sum_{i \in I_k} \left\|D_{x_i}^2 (u_{k0}(t,\cdot) * \varphi_k(\cdot))\right\|_{L_2(R_n)} +$$

$$+\sum_{i\in J_k}\left\|D_{x_j}\left(u_{k0}\left(t,\cdot\right)*\varphi_k\left(\cdot\right)\right)\right\|_{L_2(R_n)}\leq$$

$$\leq c\left(1+t\right)^{-\frac{n+m_k+4}{8}}\left[\left\|\varphi_k\left(\cdot\right)\right\|_{L_1(R_n)}+\left\|\varphi_k\left(\cdot\right)\right\|_{W_{2,k}^{2,1}(R_n)}\right]\ ; \tag{13}$$

$$\left\|D_t\left(u_{k0}\left(t,\cdot\right)*\varphi_k\left(\cdot\right)\right)\right\|_{L_2(R_n)}\leq$$

$$\leq c\left(1+t\right)^{-\frac{n+m_k+8}{8}}\left[\left\|\varphi_k\left(\cdot\right)\right\|_{L_1(R_n)}+\left\|\varphi_k\left(\cdot\right)\right\|_{W_{2,k}^{2,1}(R_n)}\right]\ ; \tag{14}$$

$$\left\|u_{k0}\left(t,\cdot\right)*\varphi_k\left(\cdot\right)\right\|_{L_2(R_n)}\leq$$

$$\leq c\left(1+t\right)^{-\frac{n+m_k}{8}}\left[\left\|\varphi_k\left(\cdot\right)\right\|_{L_1(R_n)}+\left\|\varphi_k\left(\cdot\right)\right\|_{L_2(R_N)}\right]\ ; \tag{15}$$

$$\sum_{i\in I_k}\left\|D_{x_i}^2\left(u_{k1}\left(t,\cdot\right)*\psi_k\left(\cdot\right)\right)\right\|_{L_2(R_n)}+\sum_{i\in J_k}\left\|D_{x_j}\left(u_{k1}\left(t,\cdot\right)*\psi_k\left(\cdot\right)\right)\right\|_{L_2(R_n)}\leq$$

$$\leq c\left(1+t\right)^{-\frac{n+m_k+4}{8}}\left[\left\|\psi_k\left(\cdot\right)\right\|_{L_1(R_n)}+\left\|\psi_k\left(\cdot\right)\right\|_{L_2(R_n)}\right]; \tag{16}$$

$$\left\|D_t\left(u_{k1}\left(t,\cdot\right)*\psi_k\left(\cdot\right)\right)\right\|_{L_2(R_n)}\leq$$

$$\leq c\left(1+t\right)^{-\frac{n+m_k+8}{8}}\left[\left\|\psi_k\left(\cdot\right)\right\|_{L_1(R_n)}+\left\|\psi_k\left(\cdot\right)\right\|_{L_2(R_n)}\right]\ . \tag{17}$$

Using (12)-(17), from (9) we get the following estimations

$$\left\|u_k\left(\cdot,t\right)\right\|_{L_2(R_n)}\leq c\left(1+t\right)^{-\frac{n+m_k}{8}}E_k\left(\varphi_k,\psi_k\right)+$$

$$+\int_0^t\left(1+t-\tau\right)^{-\frac{n+m_k}{8}}G_k\left(\tau\right)d\tau; \tag{18}$$

$$\sum_{i\in I_k}\left\|D_{x_i}^2u_k\left(\cdot,t\right)\right\|_{L_2(R_n)}+\sum_{i\in J_k}\left\|D_{x_j}u_k\left(\cdot,t\right)\right\|_{L_2(R_n)}\leq$$

$$\leq c\left(1+t\right)^{-\frac{n+m_k+4}{8}}E_k\left(\varphi_k,\psi_k\right)+c\cdot\int_0^t\left(1+t-\tau\right)^{-\frac{n+m_k+4}{8}}G_k\left(\tau\right)d\tau, \tag{19}$$

where

$$G_k\left(\tau\right)=\left\|f_k\left(u_1\left(\cdot,\tau\right),u_2\left(\cdot,\tau\right)\right)\right\|_{L_1(R_n)}+\left\|f_k\left(u_1\left(\cdot,\tau\right),u_2\left(\cdot,\tau\right)\right)\right\|_{L_2(R_n)},k=1,2, \tag{20}$$

$$E_k\left(\varphi_k,\psi_k\right)=\left\|\varphi_k\right\|_{W_{2,k}^{2,1}}+\left\|\psi_k\right\|_{L_2(R_n)}+\left\|\varphi_k\right\|_{L_1(R_n)}+\left\|\psi_k\right\|_{L_1(R_n)}. \tag{21}$$

Using conditions 2 and the Holder inequality, we get

$$\left\|f_k\left(u_1\left(\cdot,\tau\right),u_2\left(\cdot,\tau\right)\right)\right\|_{L_1(R_n)}\leq$$

$$\leq\left(\int_{R_n}\left|u_1\left(x,\tau\right)\right|^{p_k\rho_k}dx\cdot\right)^{1/\rho_k}\cdot\left(\int_{R_n}\left|u_2\left(x,\tau\right)\right|^{q_k\rho_k'}dx\cdot\right)^{1/\rho_k'}, \tag{22}$$

where

$$\rho_k>1,\quad \rho_k'>1,\quad \frac{1}{\rho_k}+\frac{1}{\rho_k'}=1,\quad k=1,2. \tag{23}$$

Using the multiplicative inequality (see [17]), from (22), (23) we get

$$\|f_k(u_1(\cdot,\tau),u_2(\cdot,\tau))\|_{L_1(R_n)} \le c\,\|u_1(\cdot,\tau)\|_{L_1(R_n)}^{\gamma_{0,k}p_k}\cdot\prod_{i\in I_k}\left\|D_{x_j}^2u_1(\cdot,\tau)\right\|_{L_2(R_n)}^{\gamma_{i,k}p_k}\times$$

$$\times\prod_{i\in J_k}\left\|D_{x_j}u_1(\cdot,\tau)\right\|_{L_2(R_n)}^{\gamma_{j,k}p_k}\cdot\|u_2(\cdot,\tau)\|_{L_1(R_n)}^{\gamma_{0,k}q_k}\times$$

$$\times\prod_{i\in I_k}\left\|D_{x_i}^2u_2(\cdot,\tau)\right\|_{L_2(R_n)}^{\gamma_{i,k}q_k}\cdot\prod_{i\in J_k}\left\|D_{x_j}u_2(\cdot,\tau)\right\|_{L_2(R_n)}^{\gamma_{i,k}q_k},\qquad(24)$$

where

$$\begin{gathered}\gamma_{0,r}=1-\left(\tfrac12-\tfrac{1}{\rho_kp_k}\right)\tfrac{n+m_k}{2},\quad \gamma_{i,r}=\tfrac12\left(\tfrac12-\tfrac{1}{\rho_kp_k}\right),\quad i\in I_k,\\ \gamma_{j,r}=\tfrac12-\tfrac{1}{\rho_kp_k},\quad i\in J_k,\quad \gamma'_{0,r}=1-\tfrac12\left(1-\tfrac{1}{\rho'_kq_k}\right)\tfrac{n+m_k}{2},\\ \gamma'_{i,r}=\tfrac12\left(1-\tfrac{1}{\rho'_kq_k}\right),\quad i\in I_k,\quad \gamma'_{j,r}=\tfrac12-\tfrac{1}{\rho'_kp_k},\quad i\in J_k,\\ r,k=1,2,\quad r\ne k.\end{gathered}\qquad(25)$$

Similarly, we have the following estimations

$$\|f_k(u_1(\cdot,\tau),u_2(\cdot,\tau))\|_{L_2(R_n)} \le c\,\|u_1(\cdot,\tau)\|_{L_2(R_n)}^{\eta_{0,k}p_k}\cdot\prod_{i\in I_k}\left\|D_{x_i}^2u_1(\cdot,\tau)\right\|_{L_2(R_n)}^{\eta_{i,k}p_k}\times$$

$$\times\prod_{j\in J_k}\left\|D_{x_j}u_1(\cdot,\tau)\right\|_{L_2(R_n)}^{\eta_{j,k}p_k}\cdot\|u_2(\cdot,\tau)\|_{L_2(R_n)}^{\eta'_{0,k}q_k}\times$$

$$\times\prod_{i\in I_k}\left\|D_{x_i}^2u_2(\cdot,\tau)\right\|_{L_2(R_n)}^{\eta'_{i,k}q_k}\cdot\prod_{j\in J_k}\left\|D_{x_j}u_2(\cdot,\tau)\right\|_{L_2(R_n)}^{\eta''_{j,k}q_k}.\qquad(26)$$

$$\begin{gathered}\eta_{0,r}=1-\left(1-\tfrac{1}{\hat\rho_kp_k}\right)\tfrac{n+m_k}{2},\quad \eta_{i,r}=\tfrac14\left(1-\tfrac{1}{\hat\rho_kp_k}\right),\quad i\in I_k,\\ \eta_{j,r}=\tfrac12\left(1-\tfrac{1}{\hat\rho_kp_k}\right),\quad j\in J_k,\quad \eta'_{0,r}=1-\left(1-\tfrac{1}{\hat\rho'_kq_k}\right)\tfrac{n+m_k}{4},\end{gathered}\qquad(27)$$

$$\eta'_{i,r}=\frac14\left(1-\frac{1}{\hat\rho'_kq_k}\right),\quad i\in I_k,\quad \eta'_{j,r}=\frac12\left(1-\frac{1}{\hat\rho'_kp_k}\right),\quad j\in J_k,$$

$$r,k=1,2,\quad r\ne k$$

where

$$\hat\rho_k>1,\quad \hat\rho'_k>1,\quad \frac{1}{\hat\rho_k}+\frac{1}{\hat\rho'_k}=1,\quad k=1,2.\qquad(28)$$

Denoting

$$X_k(t)=(1+\tau)^{\frac{n+m_k}{8}}\|u_k(\cdot,t)\|_{L_2(R_n)},\qquad(29)$$

$$Y_k(t)=(1+t)^{\frac{n+m_k+4}{8}}\left[\prod_{i\in I_k}\left\|D_{x_i}^2u_k(\cdot,t)\right\|_{L_2(R_n)}+\prod_{j\in J_k}\left\|D_{x_j}u_k(\cdot,t)\right\|_{L_2(R_n)}\right],\qquad(30)$$

from (12)-(17), (24), (26) and (29), (30) we get

$$X_k(t)\le(1+t)^{\frac{n+m_k}{8}}E_k(\varphi_k,\psi_k)+$$

$$(1+t)^{\frac{n+m_k}{8}}\int_0^t (1+t-\tau)^{-\frac{n+m_k}{8}}\left\{(1+t-\tau)^{-\gamma_k}\left[X_1(\tau)+Y_1(\tau)\right]^{p_k}\times\right.$$

$$\times\left[X_2(\tau)+Y_2(\tau)\right]^{q_k}+$$

$$\left.+(1+t-\tau)^{-\eta_k}\left[X_1(\tau)+Y_1(\tau)\right]^{2p_k}\left[X_2(\tau)+Y_2(\tau)\right]^{2q_k}\right\}d\tau; \tag{31}$$

$$Y_k(t)\le(1+t)^{\frac{n+m_k+4}{8}}E_k(\varphi_k,\psi_k)+$$

$$(1+t)^{\frac{n+m_k+4}{8}}\int_0^t (1+t-\tau)^{-\frac{n+m_k+4}{8}}\left\{(1+t-\tau)^{-\gamma_k}\left[X_1(\tau)+Y_1(\tau)\right]^{p_k}\times\right.$$

$$\times\left[X_2(\tau)+Y_2(\tau)\right]^{q_k}+$$

$$\left.+(1+t-\tau)^{-\eta_k}\left[X_1(\tau)+Y_1(\tau)\right]^{2p_k}\left[X_2(\tau)+Y_2(\tau)\right]^{2q_k}\right\}d\tau, \tag{32}$$

where

$$\gamma_k=\frac{n+m_1}{8}p_k\gamma_{0r}+\frac{n+m_1+4}{8}p_k\left[\sum_{i\in I_k}\gamma_{ik}+\sum_{j\in J_k}\gamma_{jk}\right]+$$

$$+\frac{n+m_2}{8}q_k\gamma'_{0r}+\frac{n+m_2+4}{8}q_k\left[\sum_{i\in I_k}\gamma'_{ik}+\sum_{j\in J_k}\gamma'_{jk}\right]$$

$$\eta_k=\frac{n+m_1}{8}p_k\eta_{0r}+\frac{n+m_1+4}{8}p_k\left[\sum_{i\in I_k}\eta_{ik}+\sum_{j\in J_k}\eta_{jk}\right]+$$

$$+\frac{n+m_2}{8}q_k\eta'_k\gamma_{0r}+\frac{n+m_2+4}{8}q_k\left[\sum_{i\in I_k}\eta'_{ik}+\sum_{j\in J_k}\eta''_{jk}\right].$$

Hence, taking into account (25), (27) we get

$$\gamma_k=\frac{n+m_1}{8}p_k+\frac{n+m_2}{8}q_k-\left[\frac{n+m_1}{4\rho_1 p_k}q_k+\frac{n+m_2}{4\rho'_k q_k}q_k\right],$$

$$\eta_k=\frac{n+m_1}{4}p_k+\frac{n+m_2}{4}q_k-\left[\frac{n+m_1}{2\hat{\rho}_k p_k}q_k+\frac{n+m_2}{2\hat{\rho}'_k q_k}q_k\right],\quad k=1,2.$$

Introducing the denotation

$$Z(t)=\sup_{\tau\in[0,t]} ess\sum_{k=1}^{2}\left[X_k(\tau)+Y_k(\tau)\right], \tag{33}$$

from (29)-(33) we get

$$Z(t)\le c_1E_0+\left[c_2Z^p(t)+c_3Z^{2p}(t)\right]\times$$

$$\times\sum_{k=1}^{2}\left\{(1+t)^{\frac{n+m_k}{8}}\int_0^t(1+t-\tau)^{-\frac{n+m_k}{8}}\left\{(1+\tau)^{-\gamma_k}+(1+\tau)^{-\eta_k}\right\}d\tau+\right.$$

$$+\left(1+t\right)^{-\frac{n+m_k+4}{8}}\times$$

$$\left.\times\int_0^t\left(1+t-\tau\right)^{-\frac{n+m_k+4}{8}}\left\{\left(1+\tau\right)^{\gamma_k}+\left(1+\tau\right)^{-\eta_k}\right\}d\tau\right\},t\in\left[0,T_0\right), \qquad (34)$$

where

$$E_0=\sum_{k=1}^{2}E_k\left(\varphi_k,\psi_k\right).$$

Subject to conditions (4), (5)

$$\gamma_k>1,\quad \eta_k>1\quad k=1,2,$$

therefore

$$\sum_{k=1}^{2}\left\{\left(1+t\right)^{-\frac{n+m_k}{8}}\int_0^t\left(1+t-\tau\right)^{-\frac{n+m_k}{8}}\left\{\left(1+\tau\right)^{\gamma_k}+\left(1+\tau\right)^{-\eta_k}\right\}d\tau\le\right.$$

$$\le c<+\infty,\qquad t\in\left[0,T_0\right) \qquad (35)$$

$$\sum_{k=1}^{2}\left(1+t\right)^{-\frac{n+m_k+4}{8}}\int_0^t\left(1+t-\tau\right)^{-\frac{n+m_k+4}{8}}\left\{\left(1+\tau\right)^{\gamma_k}+\left(1+\tau\right)^{-\eta_k}\right\}d\tau\le$$

$$\le c<+\infty,\qquad t\in\left[0,T_0\right), \qquad (36)$$

(see [20]).

From (34)-(36) we get the inequality

$$Z\left(t\right)\le c_1E_0+c_2Z^p\left(t\right)+c_2MZ^p\left(t\right),\quad t\in\left[0,T_0\right),$$

Hence it follows that for rather small E_0 it is fulfilled the inequality

$$Z\left(t\right)\le M_1\quad t\in\left[0,T_0\right). \qquad (37)$$

From (29), (30), (33) and (37) it follows that a priori estimation (8) is fulfilled, therefore $T_0=+\infty$.

From (29)-(33), (37) it also follows that estimations (3)-(5) are fulfilled.

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