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Naila M. NAMAZOVA

ON SOME BOUNDARY VALUE PROBLEMS FOR A FOURTH ORDER DIFFERENTIAL EQUATION WITH MULTIPLE CHARACTERISTICS

Abstract

In the paper, for a fourth order differential equation polynomially dependent on the spectral parameter we find fundamental systems of Birkhoff type solutions. The spectrum and expansion in eigen and adjoint functions are studied for definite boundary conditions.

Consider the following differential equation

$$\frac{d^4y}{dx^4}+P_1(x,\lambda)\frac{d^3y}{dx^3}+P_2(x,\lambda)\frac{d^2y}{dx^2}+P_3(x,\lambda)\frac{dy}{dx}+P_4(x,\lambda)y=0, \tag{1}$$

with the boundary conditions

$$U_i(y)=\alpha_i y^{(k)}(0)+\beta_i y^{(k_j)}(1)+\sum_{j=0}^{k_i-1}\alpha_{ij}y^{(j)}(0)+\beta_{ij}y^{(j)}(1)=0, \tag{2}$$

$$3\geq k_1\geq k_2\geq k_3\geq k_4\geq 0,\quad k_1>k_3,\ k_2>k_4.$$

$P_i(x,\lambda)=\sum\limits_{k=0}^{i}P_{ik}\lambda^k, i=\overline{1,4}.P_{ii}$ are constant numbers, $P_{ik}(x)$ are sufficiently smooth functions on [0,1].

Call the equation

$$\theta^4+P_{11}\theta^3+P_{22}\theta^2+P_{33}\theta+P_{44}=0 \tag{3}$$

a characteristic equation for (1).

In the papers [1,2], problem (1)-(2) was studied under the suppositions that characteristic equation (3) has two different roots each with multiplicity two, or one root has multiplicity three, the other one is prime. In these papers some classes of boundary conditions for which theorems on multiple expansion in eigen and adjoint functions were obtained, are distinguished.

Therewith, the coefficients of equation (1) allowing to get representations of Birkhaff and non-Birkhoff type fundamental systems of solutions of equations were considered. In the first case these solutions have an ordinary exponential asymptotics containing only entire powers of the spectral parameter, in the second case the solutions have addends in fractional powers of the parameter both in the index of the exponent and in the multiplier at the exponent. In the papers [3,4], problem (1)-(2) was considered under the assumption that the characteristic equation has one fourfold root, and the expansions in eigen functions were obtained for the functions from the domain of definition of the appropriate operator for the case of fundamental solutions of non-Birkhoff type.

In the present paper we consider the case for one fourfold characteristic equation and investigate fundamental systems of solutions of Birkhoff type. In spite of the fact that in the paper [4], the solution of the most general form was obtained, the solutions of Birkhoff type should be separately considered. Here we meet serious difficulties both of computing and theoretical character. This is revealed immediately while writing the asymptotics in parameter of fundamental systems of solutions.

Be means of the substitution

$$\frac{d^k y}{dx^k} = \lambda^k y_{k+1},\ k = \overline{0,3} \tag{4}$$

differential equation $(0,1)$ may be reduced to the form

$$\frac{dy}{dx} = A\left(x,\lambda\right) y, \tag{5}$$

where the matrix $A\left(x,\lambda\right)$ has the form: $A\left(x,\lambda\right) = \sum\limits_{i=-1}^{3} a^{(-i)}/\lambda^i$,

$$a^{(1)} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -P_{44} & -P_{33} & -P_{22} & -P_{11} \end{pmatrix},$$

$$a^{(-i)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -P_{4,3-i} & -P_{4,2-i} & -P_{4,1-i} & -P_{4,-i} \end{pmatrix}, \quad i = \overline{0,3}$$

$P_{kl} = 0$ for $l < 0$, $k = \overline{1,4}$.

Make the change

$$y\left(x,\lambda\right) = \widetilde{m} z\left(x,\lambda\right). \tag{6}$$

Let $\widetilde{m}$ be such that $\widetilde{m}^{-1} a^{(1)} \widetilde{m} = J$, where J is the Jordan matrix for the fourfold non-zero root θ_1. Then by direct calculation we find that

$$\widetilde{m} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \theta_1 & 1 & 0 & 0 \\ \theta_1^2 & 2\theta_1 & 1 & 0 \\ \theta_1^3 & 3\theta_1^2 & 3\theta_1 & 0 \end{pmatrix}.$$

Substituting (6) in (5), for $z\left(x,\lambda\right)$ we find:

$$\frac{dy}{dx} = \left[\lambda J + \sum_{i=0}^{3} \widetilde{m}^{-1} a^{(-i)}\left(x\right) \widetilde{m}\right] z\left(x,\lambda\right). \tag{7}$$

We look for the solution $z\left(x,\lambda\right)$ in the form:

$$z\left(x,\lambda\right) = \left[\sum_{y=0}^{\infty} \lambda^{-y} g_{ij}^{(x)}\left(x\right) e^{\theta\lambda x}\right]_{i,j=1}^{4}. \tag{8}$$

Substituting (8) in (7), allowing term by term differentiation of the series and equating between themselves the coefficients at the same powers of λ, using formula (7) from [5, p. 6] and assuming the conditions

$$\sum_{i=1}^{4} P_{i,i-1}(x)\,\theta_1^{4-i}=0,\quad \sum_{i=2}^{4} P_{i,i-2}(x)\,\theta_1^{4-i}=0,$$

$$\sum_{i=3}^{4} P_{i,i-3}(x)\,\theta_1^{4-i}=0,\quad \forall x\in[0,1]\,, \tag{9}$$

to be fulfilled, we easily get the groups of equations that give an algorithm for finding $g_{ij}^{(y)}(x)$, $i,j=\overline{1,4}$. Passing from $z(x,\lambda)$ to $y(x,\lambda)$, we get formal solutions.

Note that in [5], the principal matrix of the system (5) is assumed to be "already" reduced to the canonical form.

Here we'll get rid of the condition of infinite differentiability of the coefficients of equation (1) for finding an asymptotics of the formal solution having changed it by Tamarkin conditions.

So, using the method stated in Tamarkin's work [6], we get:

Theorem. *Let $P_{il}(x)\in C^{6-i+l}[0,1]$, $l<i$, and conditions (9) be fulfilled. Then at each of half-planes $\Phi_{\pm}=\{\lambda:\operatorname{Re}\theta_1\lambda\geq 0\,(+)\,,\operatorname{Re}\theta_2\lambda\leq 0\,(-)\}$ differential equation (1) has f.s.s. allowing as $|\lambda|\to\infty$ the asymptotic representations*

$$y_i(x,\lambda)=$$

$$=\left[g_{i0}^{(0)}(x)+\frac{1}{\lambda}g_{i0}^{(1)}(x)+\frac{1}{\lambda^2}g_{i0}^{(2)}(x)+\frac{1}{\lambda^3}g_{i0}^{(3)}(x)+O\left(\frac{1}{\lambda^4}\right)\right]e^{\theta\lambda x}, \tag{10}$$

where $g_{i0}^{(0)}(x)$ and $g_{i0}^{(j)}(x)$, $j=\overline{1,3}$ are fundamental solutions of homogeneous and particular solutions of inhomogeneous differential equations of fourth order with coefficients expressed by $P_{il}^{(\mu)}(x)$, where μ denotes a derivative $\mu<4$.

Study the structure of Birkhoff expansions of f.s.s. (10) for the case $\theta_1=i$. In this case conditions (9) accept the following forms:

$$P_{43}=P_{32}=P_{21}=P_{10}=0,\quad P_{20}=P_{31}=P_{42}=0,$$

$$P_{30}(x)\,i+P_{40}(x)=0,\quad \forall x\in[0,1]\,,$$

and equation (1) turns into the equation

$$y^{IV}-4i\lambda y^{III}-6\lambda^2 y^{II}+\left(4i\lambda^3+P_{30}(x)\right)y^{I}+$$

$$+\left(\lambda^4+P_{41}(x)\,\lambda+P_{40}(x)\right)y=0. \tag{11}$$

Suppose that boundary conditions (2) are given in the form:

$$U_1(y)\equiv y(0)=0,\quad U_2(y)\equiv y(1)=0,$$

$$U_3(y)\equiv y^{I}(0)-y^{I}(1)=0,\quad U_4(y)\equiv y^{II}(0)=0.$$

According to [7, p. 259], a unique solution of equation (10) with the inhomogeneous right side $f(x)$, satisfying these boundary conditions is represented in the form

$$y(x,\lambda,f)=\int_0^1 G(x,\xi,\lambda)\,f(\xi) \tag{12}$$

where

$$G(x,\xi,\lambda)=\frac{\Delta(x,\xi,\lambda)}{\Delta(\lambda)}, \tag{13}$$

$$\Delta(\lambda)=\begin{vmatrix} U_1(y_1) & U_1(y_2) & U_1(y_3) & U_1(y_4) \\ \multicolumn{4}{c}{\cdots\cdots\cdots\cdots\cdots\cdots} \\ U_4(y_1) & U_4(y_2) & U_4(y_3) & U_4(y_4) \end{vmatrix}$$

$y(x,\lambda)$, $k=\overline{1,4}$ are fundamental systems of solutions of homogeneous equation (11),

$$\Delta(x,\xi,\lambda)=\begin{vmatrix} g(x,\xi,\lambda) & y_1(x,\lambda) & y_2(x,\lambda) & y_3(x,\lambda) & y_4(x,\lambda) \\ U_1(g)_x & . & . & . & . \\ U_2(g)_x & . & . & \Delta(\lambda) & . \\ U_3(g)_x & . & . & . & . \\ U_4(g)_x & . & . & . & . \end{vmatrix},$$

$g(x,\xi,\lambda)=\pm\frac{\sum\limits_{k=1}^{4}W_{4k}(\xi,\lambda)y_k(x,\lambda)}{2W(\xi,\lambda)}$, +if $0\le\xi\le x$, -if $0\le x\le\xi$; $W(\xi,\lambda)$ is a Wronskain determinant from $y_k(x,\lambda)$, $k=\overline{1,4}$; $W_{4k}(\xi,\lambda)$ is an algebraic complement of the element $(4,k)$ of the determinant $W(\xi,\lambda)$.

The calculations show that

$$W(\xi,\lambda)=12^{4i\lambda x}\left[1+O\left(\frac{1}{\lambda}\right)\right],$$

$$\Delta(\lambda)=2e^{i\lambda}\left[1+O\left(\frac{1}{\lambda}\right)-\left[4+O\left(\frac{1}{\lambda}\right)\right]e^{i\lambda}\right],$$

and the principal part of expansion in spectral parameter $g(x,\xi,\lambda)$ is the function $\pm\frac{1}{2}\frac{(x-\xi)^3}{3!}e^{i\lambda(x-\xi)}$, where the sign "+" is taken for $\xi\le x$, the sign "-" for $\xi>x$. The appropriate addends for $U_i(g)_x$, $i=\overline{1,4}$ will be:

$$-\frac{1}{2}\frac{(-\xi)^3}{3!}e^{i\lambda\xi};\quad \frac{1}{2}\frac{(1-\xi)^3}{3!}e^{i\lambda(1-\xi)},$$

$$-\frac{1}{2}\left[\frac{\xi^2}{2}e^{-i\lambda\xi}+\frac{(-\xi)^3}{3!}e^{-i\lambda\xi}\right]-\frac{1}{2}\left[\frac{3(1-\xi)^3}{2}e^{i\lambda(1-\xi)}+\frac{(1-\xi)^3}{3!}e^{i\lambda(1-\xi)}\right];$$

$$-\frac{1}{2}\left[\frac{\lambda^2}{!}\xi^3+i\lambda\xi^2-\xi\right]e^{-i\lambda\xi}.$$

Consider the sector $\operatorname{Re} i\lambda\ge 0$, i.e. the lower half-plane. For obtaining asymptotic representation of the Green function (13) in this sector, at first we get them for its numerator. To this and, we preliminarily transform the determinant $\Delta(x,\xi,\lambda)$ so that the elements of the first column denoted by $g_0(x,\xi,\lambda)$, $g_1(\xi,\lambda)$, $g_2(\xi,\lambda)$, $g_3(\xi,\lambda)$, $g_4(\xi,\lambda)$ of the transformed determinant $\Delta_0(x,\xi,\lambda)$ in the considered sector couldn't increase for $|\lambda|\to\infty$.

Multiply the elements of the columns $\Delta(x,\xi,\lambda)$ with the numbers 2,3,4,5 by $-\dfrac{W_{41}(\xi,\lambda)}{W(\xi,\lambda)}$, $-\dfrac{W_{42}(\xi,\lambda)}{W(\xi,\lambda)}$, $-\dfrac{W_{43}(\xi,\lambda)}{W(\xi,\lambda)}$, $-\dfrac{W_{44}(\xi,\lambda)}{W(\xi,\lambda)}$ and put together with the elements of the first column. Then we get:

$$g_0(x,\xi,\lambda)=0,\quad for\ \ \xi<x,\quad g_0(x,\xi,\lambda)=-\frac{(x-\xi)^3}{3!}e^{i\lambda(x-\xi)},\quad \xi\geq x,$$

$g_1(\xi,\lambda)=\dfrac{\xi^2}{12}e^{-i\lambda\xi}-e^{i\lambda(1-\xi)}$,..., further expanding $\Delta_0(x,\xi,\lambda)$ in the elements of the first row and by direct calculation taking into account that for algebraic complements of the elements $\Delta(\lambda)$ the following equalities are valid

$$\Delta_{21}(\lambda)=\Delta_{31}(\lambda)=\Delta_{41}(\lambda)=0,\quad \Delta_{12}(\lambda)=e^{2i\lambda}\left[i\lambda^3-\lambda^2-4i\lambda-6+2i\lambda e^{-i\lambda}\right],$$

$$\Delta_{22}(\lambda)=(-6+2i\lambda)e^{i\lambda},\ \Delta_{32}(\lambda)=2e^{i\lambda},\quad \Delta_{42}(\lambda)=e^{2i\lambda}[1-2i\lambda],$$

$$\Delta_{13}(\lambda)=e^{i\lambda}\left[-2i\lambda^3-6i\lambda^2-\lambda e^{-i\lambda}\right],\ \Delta_{23}(\lambda)=\left(-6i\lambda+2\lambda^2\right)e^{i\lambda},$$

$$\Delta_{33}(\lambda)=-2i\lambda e^{i\lambda},\ \Delta_{43}(\lambda)=e^{2i\lambda}\left[4-e^{-i\lambda}\right],$$

$$\Delta_{14}(\lambda)=e^{2i\lambda}\left[-2i\lambda^3+\lambda^2-8i\lambda-2+\left(2-2i\lambda-\lambda^2\right)e^{-i\lambda}\right],$$

$$\Delta_{24}(\lambda)=e^{i\lambda}\left[2\lambda^2-6i\lambda-2+2e^{-i\lambda}\right],\ \Delta_{34}(\lambda)=e^{i\lambda}[2-2i\lambda],$$

$$\Delta_{44}(\lambda)=e^{i\lambda}\left[3+i\lambda-e^{-i\lambda}\right]$$

(here the calculations are carried out for $P_{30}=P_{41}=0$, and this doesn't influence on final result), after simple transformations we get that for $\xi\geq x$ $\dfrac{\Delta_0(x,\xi,\lambda)}{\Delta(\lambda)}$ increases exponentially. Therefore, by integrating the resolvent in closely expanding contours in the plane of spectral parameter, one can't obtain a formula of expansion of eigen and adjoint functions. Both in the case of different roots and in the case of two different (each with multiplicity two) roots of the principal characteristic polynomial corresponding to equation (1), these boundary conditions are regular, and the Green function of appropriate spectral problems decreases with definite growth as $|\lambda|\to\infty$. Such situations hold also for boundary conditions $y(0)=0$, $y(1)=0$, $y'(0)-y'(1)=0$, $y''(1)=0$.

Now, suppose that boundary conditions (2) are given in the forms:

$$y(0)-y(1)=0,\quad y'(0)-y'(1)=0,\quad y''(1)=0,\quad y'''(1)=1. \tag{14}$$

The characteristic determinant $\Delta(\lambda)$, whose eigen values are zeros, after certain calculations will have the following representation

$$\Delta(\lambda)=2\lambda^5e^{2i\lambda}\left[i+\lambda\left(-2+O\left(\frac{1}{\lambda}\right)\right)e^{i\lambda}+\left(11i+O\left(\frac{1}{\lambda}\right)\right)e^{2i\lambda}\right]. \tag{15}$$

Make the change $i\lambda = z$ and consider the function

$$g(z) = N_1' + \left(P_{11}'z + P_{10}'\right)e^z + P_{20}'e^{2z}, \quad where \;\; N_1'P_{11}', \;\; P_{10}', \; P_{20}' \neq 0.$$

Construct on the plane z two curvilinear strips V_1, V_2 determined by the inequalities $V_1 : |\mathrm{Re}\,(z + \ln z)| \le c_1$, $V_2 : |\mathrm{Re}\,(z - \ln z)| \le c_1$. In the strip V_1 it holds:

$$\mathrm{Re}\,(z + \ln z) = c, \quad -c_1 \le c \le c_1,$$

$$|ze^z| = \left|ze^{z+\ln z - \ln z}\right| = \left|ze^{-\ln z}\right| c_1' = c_1' \left|z'^{-1}\right| = c_1'; \quad c_1' = e^c;$$

$$\left|e^{2z}\right| = \left|e^{2z+2\ln z - 2\ln z}\right| = e^{2c_1}z^{-2} = O\left(\frac{1}{z^2}\right).$$

The zeros $g(z)$ in the sector V_1 asymptotically coincide with the zeros $N_1' + P_{11}'ze^z = 0$. Denote $z + \ln z = s$, then $N_1' + P_{11}'e^s = 0$. Solving this equation, we find $s = \ln|w_1| + i\,(\arg w_1 + 2r\pi)$, $r = 0, \pm 1, \pm 2, ...$, where $w_1 = -\dfrac{N_1'}{P_1'}$. Then $z_1 + iz_2 + \ln|z_1 + iz_2| + i\,(\arg(z_1 + iz_2)) = \ln|w_1| + i\,(\arg w_1 + 2r\pi)$. Hence

$$z_1 + \ln|z_1 + iz_2| = \ln|w_1|,$$

$$z_2 + (\arg(z_1 + iz_2)) = \arg w_1 + 2r\pi,$$

$$\arg(z_1 + iz_2) \to \pm\frac{\pi}{2},$$

$$z_2 = \left(2r\pi + \arg w_1 \mp \frac{\pi}{2}\right) + o(1) \quad |z_1 + iz_2| = |z_2|\,(1 + o(1)) \Rightarrow$$

$$\mathrm{Im}\,\lambda = -\left(\ln|w_1| - \ln\left|2r\pi + \arg w_1 \mp \frac{\pi}{2}\right|\right) + o(1)$$

$$\mathrm{Re}\,\lambda = \left(2r\pi + \arg w_1 \mp \frac{\pi}{2}\right) + o(1)$$

$$\lambda = 2r\pi + \arg w_1 \mp \frac{\pi}{2} - i\left(\ln|w_1| - \ln\left|2r\pi + \arg w_1 \mp \frac{\pi}{2}\right|\right) + o(1).$$

In the strip V_2 $\quad |ze^z| = \left|ze^{z+\ln z-\ln z}\right| = \left|ze^c e^{\ln z}\right| = z^2c_1 \quad \left|e^{2z}\right| = \left|e^{2z+2\ln z+2\ln z}\right| = O\left(z^2\right)$. The zeros $g(z)$ in the strip V_2 asymptotically coincide with the zeros

$$P_{11}'ze^z + P_{20}'e^{2z} = 0 \Rightarrow e^{2z}\left[P_{20} + P_{11}'ze^{-z}\right] = 0 \Rightarrow P_{20} - P_{11}'e^{-z+\ln z} = 0.$$

Denote $z - \ln z = s$, then $P_{20} - P_{11}'e^{-s} = 0$ and we find expressions for eigen values.

Denote the domain between the strips V_1, V_2 by U_1, the domain left from the strip V_1 by U_2 and the domain right from the strip V_2 by U_2 determined by the inequalities:

$$U_1 : \mathrm{Re}\,(z + \ln z) > c_1, \quad \mathrm{Re}\,(z - \ln z) < -c_1,$$

$$U_0 : \mathrm{Re}\,(z + \ln z) > c_1, \quad U_2 : \mathrm{Re}\,(z - \ln z) < -c_1.$$

We easily get:

Lemma. *There exist positive constants c_1 and c_2 such that no zero for which $|z| \ge c_2$, lie in domains $U_k\,(k = 0, 1, 2)$ and interior to domains U_k for $|z| \ge c_2$ one term remains dominating, namely the term of appropriate e^z, e^{2z}, e^{4z}.*

Theorem. *Boundary value problem (1), (14) has eigen values that are described by the asymptotic formulae*

$$\lambda_r = 2r\pi + \arg\left(\frac{-N_1'}{P_{11}'}\right) \mp \frac{\pi}{2} -$$

$$-i\left(\ln\left|\frac{-N_1^0}{P_{11}'}\right| - \ln\left|2r\pi + \arg\left(\frac{-N_1'}{P_{11}'}\right) \mp \frac{\pi}{2}\right|\right) + o\left(1\right), \quad r = 0, \pm 1, \pm 2....$$

$$\lambda_n = 2n\pi + \arg\left(\frac{P_{20}'}{P_{11}'}\right) \pm \frac{\pi}{2} -$$

$$-i\left(\ln\left|\frac{P_{20}'}{P_{11}'}\right| + \ln\left|2n\pi + \arg\left(\frac{P_{20}'}{P_{11}'}\right) \pm \frac{\pi}{2}\right|\right) + o\left(1\right), \quad n = 0, \pm 1, \pm 2....$$

Applying the above-mentioned procedure of transformation of the numerator of the Green function, we establish

Theorem. *If in the complex plane λ we reject the interiors of small circles k_ν, with the centers in eigen values, then for the Green function of problem (1), (14) we have the estimation*

$$G\left(x, \xi, \lambda\right) = O\left(1\right), \quad \left|\lambda\right| \to \infty. \tag{17}$$

This theorem is valid also in the case when conditions (2) are of periodic type.

Introduce the system of functions $\{\Phi_0, \Phi_1, \Phi_2, \Phi_3\}$ and compose the expression

$$F_0\left(\xi, \lambda\right) = \sum_{m=1}^{4}\sum_{k=0}^{4-m} P_{4-k,m}\frac{d^k}{dx^k}\left(\sum_{\nu=0}^{m-1}\lambda^\nu \Phi_{m-1-\nu}\left(\xi\right)\right).$$

By the contour integral method [7], from estimation (17) of the Green function we get the following theorem on fourfold expandability of functions not contained in the domain of definition of the studied operator in uniformly converging on [0,1] series in eigen functions of problem (1), (14).

Theorem. *Let the function $\Phi_k\left(x\right), \quad k = \overline{0,3}$ have continuous derivatives to order 6 and vanish at the ends of the interval [0,1] to order 5, inclusively. Then the following expansion formula holds:*

$$\frac{1}{2\pi y - 1}\sum_\nu \int_{c_\nu} \lambda^s d\lambda \int_0^1 G\left(x, \xi, \lambda\right) F\left(\xi, \lambda\right) d\xi = \Phi_s\left(x\right), \quad x = \overline{0,3}, \tag{18}$$

that uniformly converges for all $x \in [0,1]$, where c_ν is a simple closed contour surrounding only one pole λ_ν of the integrand function and the sum over ν extended on all the strips of the Green function.

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Naila M. Namazova
Nakhchivan State University,
University campus, AZ 7000, Nakhchivan, Azerbaijan
Tel.: (99412) 539 47 20 (off).

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