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ON FINITE DIMENSIONALITY OF THE KERNEL OF ELLIPTIC OPERATORS GIVEN IN NON-DIVERGENT AND DIVERGENT FORM

Abstract

In the paper, the theorems on finite dimensionality of the kernel of some quasielliptic operators given in divergent and non-divergent forms are proved. The proof is carried out for the operators in the non-divergent form and then by means of Fourier transformation in the sense of distributions. The proof scheme is carried over the case of the operators in divergent form.

Some classes of quasielliptic equations in R^n were considered in [1], [2] and etc. We'll consider a class of such equations that differ from the appropriate equations in [1], [2].

The case of a quasielliptic equation in the domain not coinciding with R^n, was considered in [3].

Consider in R^n the equation

$$Lu = \sum_{(\alpha,\lambda)\leq 1} a_\alpha(x) D^\alpha u = f(x), \tag{1}$$

where $x = (x_1,...,x_n) \in R^n$, $D^\alpha = \dfrac{\partial^{|\alpha|}}{\partial x_1^{\alpha_1}...\partial x_n^{\alpha_n}}$, $\alpha = (\alpha_1,...,\alpha_n)$, $|\alpha| =$ $= \alpha_1+...+\alpha_n$, $\lambda = (\lambda_1,...,\lambda_n)$, $\lambda_j > 0$, $j = \overline{1,\ j}$, λ_j^{-1} is a integer, $(\alpha, \lambda) =$ $= \alpha_1\lambda_1 + ... + \alpha_n\lambda_n$, $f(x) \in L_2(R^n)$.

It is assumed that the coefficients of equation (1) $a_\alpha(x)$ are bounded measurable functions in R^n, moreover $a_\alpha(x)$ are continuous for $(\alpha, \lambda) = 1$ and there exists $\lim\limits_{x\to\infty} a_\alpha(x)$ of all $a_\alpha(x)$. Therewith

$$\sum_{(\alpha,\lambda)\leq 1} a_\alpha(\infty)(i\xi)^\alpha \neq 0, \quad \xi \in R^n$$

and

$$\left| \sum_{(\alpha,\lambda)=1} a_\alpha(x)(i\xi)^\alpha \right| > c > 0$$

for $\sum\limits_{i=1}^{n} |\xi_i|^{\frac{1}{\lambda_i}} = 1$, $\xi = (\xi_1,...,\xi_n)$, $\xi^\alpha = \xi_1^{\alpha_1}...\xi_n^{\alpha_n}$.

Define $W^\lambda(R^n)$ as a completion of $\overset{\circ}{C}{}^\infty(R^n)$ by the norm

$$\|u\|^2_{W^\lambda(R^n)} = \int_{R^n} \sum_{(\alpha,\lambda)\leq 1} |D^\alpha u|^2 \, dx, \quad dx = dx_1...dx_n.$$

Obviously, the operator L brings about continuous mapping of $W^{\lambda}(R^n)$ in $L_2(R^n)$. The main goal of the paper is to show that for this mapping $\dim KerL < \infty$.

At first we prove some lemmas.

Lemma 1. I*f* $u(x) \in W^{\lambda}(R^n)$ *has a compact support, then*

$$\|D^{\alpha}u\|_{L_2(R^n)} \leq C\,\|L_0u\|_{L_2(R^n)}\,, \qquad (\alpha,\lambda)=1, \tag{2}$$

where $L_0 = \sum\limits_{(\alpha,\lambda)=1} \widehat{a}_{\alpha}D^{\alpha}$, $\widehat{a}_{\alpha} = const$, $\sum\limits_{(\alpha,\lambda)=1} \widehat{a}_{\alpha}(i\xi)^{\alpha} \neq 0$, *for* $\xi \in R^n$, $\xi \neq 0$.

This statement follows immediately after the Fourier transformation with respect to x.

Lemma 2. *If* $u(x) \equiv 0$ *for* $\sum\limits_{1}^{n} x_i^2 h^{2\lambda_i} \geq 1$, $u(x) \in W^{\lambda}(R^n)$, *then*

$$\int\limits_{R^n} u^2dx \leq c\cdot h^2 \cdot \int\limits_{R^n} \sum_{(\alpha,\lambda)=1} |D^{\alpha}u|^2\,dx, \tag{3}$$

where $c = const$ *and is independent of* h *and* u.

Proof. Let $h=1$. Then inequality (3) coincides with the Friedrichs inequality [4]:

$$\int\limits_{|x|\leq 1} u^2dx \leq c \int\limits_{|x|\leq 1} \sum_{(\alpha,\lambda)=1} |D^{\alpha}u|^2\,dx \tag{4}$$

Having made change of variables $x_i = h^{\lambda_i}y_i$ in (4), we get (3).

Lemma 3. *If* $u(x) \equiv 0$ *for* $\sum\limits_{1}^{n} x_i^2 h^{2\lambda_i} \geq 1$, $u(x) \in W^{\lambda}(R^n)$, *then*

$$\int\limits_{R^n} \left|D^{\beta}u\right|^2 dx \leq c\cdot h^{(\alpha,\lambda)-1} \int\limits_{R^n} \sum_{(\alpha,\lambda)=1} \left|D^2u\right|^2 dx \tag{5}$$

for $(\beta,\lambda) \leq 1$.

The proof of this lemma is similar to the proof of lemma 2.

Lemma 4. *There exists a* $\rho = const$ *such that if* $u(x) \in W^2(R^n)$, $u(x) \equiv 0$ *for* $\sum\limits_{1}^{n} x_i^2 h^{2\lambda_i} \geq 1$, $h = \rho^{-1}$, *then*

$$\|u(x)\|_{W^{\lambda}(R^n)} \leq c\,\|Lu\|_{L_2(R^n)} \tag{6}$$

Proof. Represent Lu in the form:

$$Lu = \sum_{(\alpha,\lambda)=1} a_{\alpha}(0)D^{\alpha}u + \sum_{(\alpha,\lambda)=1} \left(a_{\alpha}(x) - a_{\alpha}(0)\right)D^{\alpha}u + \sum_{(\alpha,\lambda)<1} a_{\alpha}(x)D^{\alpha}u$$

From lemma 1 it follows that

$$\begin{gathered} \sum_{(\alpha,\lambda)=1} \|D^{\alpha}u\|_{L_2(R^n)} \leq c\,\|Lu\|_{L_2(R^n)} + \\ +c\sum_{(\alpha,\lambda)=1} \|\left(a_{\alpha}(x) - a_{\alpha}(0)\right)D^{\alpha}u\|_{L_2(R^n)} + \\ +c\sum_{(\alpha,\lambda)<1} \|a_{\alpha}(x)D^{\alpha}u\|_{L_2(R^n)}\,. \end{gathered} \tag{7}$$

From (7) and continuity of $a_\alpha(x)$ for $(\alpha,\lambda)=1$ it follows that

$$\sum_{(\alpha,\lambda)=1}\|D^\alpha u\|_{L_2(R^n)} \le c\,\|Lu\|_{L_2(R^n)} + c\cdot\varepsilon(\rho)\sum_{(\alpha,\lambda)=1}\|D^\alpha u\|_{L_2(R^n)} + \\ + \sum_{(\alpha,\lambda)<1}\|a_\alpha(x)D^\alpha u\|_{L_2(R^n)} \tag{8}$$

where $\varepsilon(\rho)\to 0$ as $\rho\to 0$. From (8) for small ρ we get:

$$\sum_{(\alpha,\lambda)=1}\|D^\alpha u\|_{L_2(R^n)} \le 2c\,\|Lu\|_{L_2(R^n)} + c_1\sum_{(\alpha,\lambda)<1}\|D^\alpha u\|_{L_2(R^n)}\,. \tag{9}$$

From (9), using (5) for sufficiantly large h, we get (6).

Lemma 5. *If $u(x)$ has a compact support, $u(x)\in W^\lambda(R^n)$, then*

$$\|u(x)\|_{W^\lambda(R^n)} \le c\,\|Lu\|_{L_2(R^n)} + c\sum_{(\alpha,\lambda)<1}\|D^\alpha u\|_{L_2(R^n)}\,. \tag{10}$$

Proof. Let $Supp\,u(x)=K,\ x_0\in K$. From lemma 4 it follows that there exists the U_{x_0} vicinity of the point x_0 such that

$$\|\theta(x)u(x)\|_{W^\lambda(R^n)} \le c\,\|L(\theta u)\|_{L_2(R^n)} \tag{11}$$

if only $Supp\theta(x)\subset U_{x_0},\ \theta(x)\in \overset{\circ}{C}{}^{\infty}(R^n)$. Choose from the covering K with vicinities U_{x_0} the final subcovering $U_{x_1},...,U_{x_m}$ and consider the partitioning of the unit $\sum\limits_1^m \theta_i(x)=1,\ x\in K$ subject to the covering $\{U_{x_1},...,U_{x_m}\}$.

From (11) we have

$$\|u\|_{W^\lambda(R^n)} = \left\|\sum_i \theta_i u\right\|_{W^\lambda(R^n)} \le \left\|\sum_i \theta_i u\right\|_{W^\lambda(R^n)} \le$$

$$\le \sum_i \|L(\theta_i u)\|_{L_2(R^n)} \le c_1\,\|Lu\|_{L_2(R^n)} + c_1\sum_{(\alpha,\lambda)\le 1}\|D^\alpha u\|_{L_2(R^n)}\,.$$

Lemma 5 is proved.

Lemma 6. *Let Ω be a finite domain, $\overline{\Omega'}\subset\Omega,\ u(x)\in W^\lambda(\Omega)$, then*

$$\|u\|_{W^\lambda(\Omega')} \le c\,\|Lu\|_{L_2(\Omega)} + c\sum_{(\alpha,\lambda)<1}\|D^\alpha u\|_{L_2(\Omega)} \tag{12}$$

For the proof we must consider the function $\sigma(x)\in C^\infty(R^n)$ $\sigma(x)\equiv 1$ for $x\in\Omega'$, $\sigma(x)\equiv 0$ for $x\overline{\in}\Omega$ and to the function $u_1(x)=\sigma(x)$ apply the statement of lemma 5.

Lemma 7. *If $f(x)\equiv 0$ for $|x|>1$, $u(x)\in W^\lambda(R^n)$ $u(x)$ is the solution of equation (1), then there exists a $\delta=const>0$ such that* $\exp(\delta|x|)\,u\in W^\lambda(R^n)$ *and*

$$\|\exp(\delta|x|)\,u\|^2_{W^\lambda(R^n)} \le c\,\|f\|^2_{L_2(R^n)} + c\int_\Omega\sum_{(\alpha,\lambda)<1}|D^\alpha u|^2\,dx, \tag{13}$$

where Ω is some compact independent of f.

Proof. The operator $L_\infty = \sum\limits_{(\alpha,\lambda)\leq 1} a_\alpha(\infty)D^\alpha$ brings about isomorphism of the spaces $W^\lambda(R^n)$ and $L_2(R^n)$. Let $\left\|L_\infty^{-1}\right\|_{L_2(R^n)\to W^\lambda(R^n)} = k_0$. Let for $|x| > R \quad |a_\alpha(x) - a_\alpha(\infty)| \leq \delta(R), \quad \delta(R) \to 0$ as $R \to \infty$.

Extend the coefficients of $a_\alpha(x)$ from the domain $|x| > R$ on R^n having assumed $a_\alpha(x) = a_\alpha(\infty)$ for $|x| < R$ and denote the new coefficients by $a_\alpha^*(x)$, the operator $\sum\limits_{(\alpha,\lambda)\leq 1} a_\alpha^* D^\alpha$ by L_R.

If R is sufficiantly large, the operator L_R brings about isomorphism of $W^\lambda(R^n)$ and $L_2(R^n)$, moreover $\left\|L_R^{-1}\right\|_{L_2(R^n)\to W^\lambda(R^n)} \leq \dfrac{k_0}{2}$.

Consider the function

$$\exp\left(\delta\omega(x)\right) u = \upsilon,$$

where $\omega(x) = |x|$ for $1 < |x| < N$, $N = const$, $\quad N > R+1$, $\omega\left(x\right) = N+1$ for $|x| > N+1$, $\omega\left(x\right) \in C^\infty\left(R^n\right)$, $|D^\alpha\omega| < c_\alpha$, for $N < |x| < N+1$, $c_\alpha = const$ is independent of N. The function υ satisfies the equation:

$$L\upsilon + \delta L'\upsilon = 0, \qquad |x| > R$$

$$L_\infty\upsilon + \delta L''\upsilon = \exp\left(\delta\omega(x)\right) f + \sum_{(\alpha,\lambda)\leq 1} c_\alpha(x) D^\alpha u = f_1, \quad |x| < R,$$

where

$$L'\upsilon = \sum_{(\alpha,\lambda)<1} b_\alpha(x) D^\alpha \upsilon, \quad |b_\alpha(x)| \leq \beta = const,$$

$$L''\upsilon = \sum_{(\alpha,\lambda)<1} b'_\alpha(x) D^\alpha \upsilon, \quad \left|b'_\alpha(x)\right| \leq \beta = const,$$

$$|c_\alpha(x)| \leq c = const.$$

Thus, $L_R\upsilon + \delta L^\circ\upsilon = f_1$, where L° is an operator with bounded coefficients. Since L_R is uniquely invertible $L_2(R^n) \to W^\lambda(R^n)$, then for sufficiantly small δ the operator $L_R + \delta L^\circ$ is also uniquely invertible and υ is determined in a unique way, and

$$\begin{gathered}\|\upsilon\|_{W^\lambda(R^n)} \leq c\left(\left\|\exp\left(\delta\omega(x)\right) f\right\|_{L_2(R^n)} + \|u\|_{W^\lambda(R^n)}\right) \leq \\ \leq c_1 \|f\|_{L_2(R^n)} + c_1 \|u\|_{W^\lambda(|x|<R)}.\end{gathered} \tag{14}$$

From (14) it follows that

$$\left\|\exp\left(\delta\omega(x)\right) u\right\|_{W^\lambda(R^n)} \leq c_2\left(\|f\|_{L_2(R^n)} + \|u\|_{W^\lambda(|x|<R)}\right) \tag{15}$$

Since the constant c_1 is independent of N, from (15) we have

$$\left\|\exp\left(\delta(x)\right) u\right\|_{W^\lambda(R^n)} \leq c_1 \|f\|_{L_\alpha(R^n)} + \|u\|_{W^\lambda(|x|,R)}. \tag{16}$$

Having applied lemma 6 for estimating $\|u\|_{W^\lambda(|x|,<R)}$, we get the necessary estimation (13).

Lemma 8. *For $f=0$ the space of solutions of equation (1) from $W^{\lambda}(R^n)$ is finite dimensional.*

Proof. Consider any set of the solutions normed by the condition

$$\|u_m\|_{W^{\lambda}(R^n)}=1.$$

It is enough to prove compactness of the family $\{u_m(x)\}$ in $W^{\lambda}(R^n)$.

From (16) it follows that whatever were ε, there will be found T such that

$$\int\limits_{|x_i|>T}\sum_{(\alpha,\lambda)\le 1}|D^{\alpha}u_m|^2\,dx\le\varepsilon,$$

where T is independent of m.

From the imbedding theorem [5] and lemma 5 it follows that the family $\{u_m\}$ is compact in $W^{\lambda}(Q_T)$, where $Q_T=\{x\colon |x|<T\}$. Let $u^m(x)$ be $\varepsilon-$net, of the set $\{u_m\}$ in $W^{\lambda}(Q_T)$. Then the sequence $u^{m_0}(x)=\theta\left(\dfrac{x}{T}\right)u^m(x)$, where $\theta(x)\equiv 1$ for $|x_i|\le 1$, $\theta(x)\equiv 0$ for $|x_i|>2$ will be the 2ε-net of the set $\{u_m\}$ in $W^{\lambda}(R^n)$. Existence of the ε-net provides compactness.

Theorem. *For $f=0$ the space of the solutions of equation (1) from $L_2(R^n)$ is finite dimensional.*

The statement of this item is some amplification of the statement of lemma 8. Its proof is similar to the proof of the statement of lemma 8, but only inequality (12) must be used in a more exact form:

$$\|u\|_{W^{\lambda}(\Omega')}\le c\,\|Lu\|_{L_2(\Omega)}+c\,\|u\|_{L_2(\Omega)}\,. \tag{17}$$

We don't give the proof of inequality (17) because of its bulky form. Finite dimensionality of the space of solutions of equation (1) belonging to $L_2\left(R^n\right)$ follows from (17).

Further, the similar matters will be considered for the equation in divergent form, i.e.

$$\sum_{(\alpha,\lambda)\le 1,\ (\beta,\alpha)\le 1}D^{\alpha}a_{\alpha\beta}(x)D^{\beta}u=\sum_{(\alpha,\lambda)=1}D^{\alpha}f \tag{18}$$

The continuity of $a_{\alpha\beta}(x)$ for $(\alpha,\lambda)=(\beta,\lambda)=1$ is required. The results for equation (1) are found valid for equation (18) if $a_{\alpha\beta}(x)$ are bounded measurable functions in R_n, $a_{\alpha\beta}(x)=a_{\beta\alpha}(x)$, $a_{\alpha\beta}(x)$ are continuous if $(\alpha,\lambda)=(\beta,\lambda)=1$. There exits $\lim\limits_{|x|\to\infty}a_{\alpha\beta}(x)=a_{\alpha\beta}(\infty)$ of all the coefficients $a_{\alpha\beta}(x)$. Therewith

$$\sum_{(\alpha,\lambda)\le 1,\ (\beta,\lambda)\le 1}a_{\alpha\beta}(\infty)\left(i\xi\right)^{\alpha+\beta}\ne 0,\quad \forall\xi\in R^n\backslash\left\{0\right\}, \tag{19}$$

$$\sum_{(\alpha,\lambda)=1,\ (\beta,\lambda)=1}a_{\alpha\beta}(x)\left(i\xi\right)^{\alpha+\beta}\ne 0,\quad \forall\xi\in R^n\backslash\left\{0\right\},\quad x\in R^n \tag{20}$$

The generalized solution of equation (19) is the function $u(x) \in W^{\lambda}(R^n)$ such that

$$\sum_{(\alpha,\lambda)\leq 1,\ (\beta,\lambda)\leq 1} \int_{R^n} (-1)^{|\alpha|} a_{\alpha\beta}(x) D^{\beta} u D^{\alpha} \vartheta dx = \\ = \sum_{(\alpha,\lambda)=1} \int_{R^n} (-1)^{|\alpha|} f_{\alpha} D^{\alpha} \vartheta dx \tag{21}$$

whatever were $\vartheta(x) \in W^{\lambda}(R^n)$ having a compact support. Quasielliptic equation in the form (18) was investigated by Giusti [2]. The local smoothness property of solutions was investigated in this paper. In the paper [3], the estimates of the derivatives of the solution in L_p norms were obtained.

Mainly, the considerations remain if equation has the form (18). For example, we formulate an obvious analogue of lemma 1 that is easily proved by the Fourier transformation method.

Let $a_{\alpha\beta}(x) \equiv a_{\beta\alpha}(x) = const$ for $(\alpha,\lambda) = (\beta,\lambda) = 1$, $a_{\alpha\beta} = 0$ if $(\alpha,\lambda) < 1$ or $(\beta,\lambda) < 1$, (3) be fulfilled, $u(x) \in W^{\lambda}(R^n)$ have a compact support and be a generalized solution of equation (18) in the sense of identity (21). Then

$$\sum_{(\alpha,\lambda)=1} \|D^{\alpha} u\|_{L_2(R^n)} \leq c \sum_{(\alpha,\lambda)=1} \|f_{\alpha}\|_{L_2(R^n)},$$

where c is a constant independent of u. Prove this.

Let u be a generalized solution of our equation (18), i.e. $u \in W^{\lambda}(R^n)$ and satisfy relation (21). Consider $u(\psi) = \int_{R^n} u(x)\psi(x)dx$. This is a distribution. From the fact that $u(x)$ is a generalized solution we get

$$\sum_{(\alpha,\lambda)=1,\ (\beta,\lambda)=1} (-1)^{|\alpha|+|\beta|} \int_{R^n} a_{\alpha\beta} u D^{\alpha+\beta} \psi \, dx = \sum_{(\alpha,\lambda)=1} \int_{R^n} (-1)^{|\alpha|} f_a D^a \psi \, dx. \tag{$*$}$$

By definition of the derivative of the distribution

$$D^{\alpha+\beta} u(\psi) = (-1)^{|\alpha|+|\beta|} \int_{R^n} u D^{\alpha+\beta} \psi \, dx.$$

Then $(*)$ means that

$$\sum_{(\alpha,\lambda)=1,\ (\beta,\lambda)=1} a_{\alpha\beta} D^{\alpha+\beta} u(\psi) = \sum_{(\alpha,\lambda)=1} (-1)^{|\alpha|} \int_{R^n} f_{\alpha} D^{\alpha} \psi \, dx$$

or

$$\sum_{(\alpha,\lambda)=1,\ (\beta,\lambda)=1} a_{\alpha\beta} D^{\alpha+\beta} u \equiv \sum_{(\alpha,\lambda)=1} D^{\alpha} F_{\alpha}, \tag{**}$$

where F_{α} is a distribution of $F_{\alpha}(\psi) = \int_{R^n} f_{\alpha} \psi(x) dx$.

From (**) we get

$$\sum_{(\alpha,\lambda)=1,\ (\beta,\lambda)=1} a_{\alpha\beta}(i\xi)^{\alpha+\beta}\widetilde{u} = \sum_{(\alpha,\lambda)=1} (i\xi)^{\alpha}\widetilde{F}_{\alpha}.$$

Hence

$$\widetilde{u} = \sum_{(\alpha,\lambda)=1} \frac{(i\xi)^{\alpha}}{\sum\limits_{(\alpha,\lambda)=1,\ (\beta,\lambda)=1} a_{\alpha\beta}\,(i\xi)^{\alpha+\beta}}\widetilde{F}_{\alpha}. \qquad (***)$$

Denote $|\xi_1|^{\beta_1}\cdot|\xi_2|^{\beta_2}...|\xi_n|^{\beta_n} = |\xi|^{\beta}$.

Multiply the both hand sides of (***) by $\sum\limits_{(\beta,\lambda)=1} |\xi|^{\beta}$:

$$\left|\sum_{(\beta,\lambda)=1} |\xi|^{\beta}\cdot\widetilde{u}\right| = \sum_{(\alpha,\lambda)=1} \frac{(i\xi)^{\alpha}\sum\limits_{(\beta,\lambda)=1}|\xi|^{\beta}}{\sum\limits_{\substack{(\alpha,\lambda)=1\\(\beta,\lambda)=1}} a_{\alpha\beta}\,(i\xi)^{\alpha+\beta}}\widetilde{F}_{\alpha}.$$

It is easy to prove that the function $\Phi(\xi) = \dfrac{(i\xi)^{\alpha}\sum\limits_{(\beta,\lambda)=1}|\xi|^{\beta}}{\sum\limits_{\substack{(\alpha,\lambda)=1\\(\beta,\lambda)=1}} a_{\alpha\beta}\cdot(i\xi)^{\alpha+\beta}}$ is bounded in R^n.

Therefore

$$\left\|\sum_{(\beta,\lambda)=1} |\xi|^{\beta}\,\widetilde{u}\right\|^2_{L_2(R^n)} \leq c\sum_{(\alpha,\lambda)=1}\left\|\widetilde{F}_{\alpha}\right\|_{L_2(R^n)}$$

Since

$$\left|\sum_{(\beta,\lambda)=1}(i\xi)^{\beta}\,\widetilde{u}\right| \leq \sum_{(\beta,\lambda)=1}|\xi|^{\beta}\,|\widetilde{u}|$$

then

$$\left|(i\xi)^{\beta}\,\widetilde{u}\right|^2 \leq c\sum_{(\alpha,\lambda)=1}\left|\widetilde{F}_{\alpha}\right|^2, \quad (\beta,\lambda)=1,$$

$$\left\|D^{\widetilde{\beta}}u\right\|^2_{L_2(R^n)} \leq c\sum_{(\alpha,\lambda)=1}\left|\widetilde{F}_{\alpha}\right|^2_{L_2(R^n)}$$

or

$$\left\|D^{\widetilde{\beta}}u\right\|_{L_2(R^n)} \leq c\sqrt{\sum_{(\alpha,\lambda)=1}\left\|\widetilde{F}_{\alpha}\right\|^2_{L_2(R^n)}} \leq c\sum_{(\alpha,\lambda)=1}\left|\widetilde{F}_{\alpha}\right|_{L_2(R^n)}$$

hence

$$\sum_{(\alpha,\lambda)=1}\|D^{\alpha}u\|_{L_2(R^n)} \leq c\sum_{(\alpha,\lambda)=1}\|F_{\alpha}\|_{L_2(R^n)},$$

where c is a constant independent of u.

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