MATHEMATICS

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CONSTRUCTION OF TRANSFORMATION OPERATOR WITH CONDITION AT INFINITY FOR STARK'S EQUATION

Abstract

The existence of a transformation operator taking the solution of the unperturbed equation to the solution of the perturbed equation is proved in this paper. The proof of this fact is based on constructive Riemann function.

Problem statement and formulation of results.

Consider the equation

$$y'' + [x + p(x)]\, y = \lambda y \tag{1}$$

on a semi-axis $0 \le x < \infty$, where $y(x,\lambda)$ is an unikown function. Here we give a condition on the existence of B. Ya. Levin type transformation operator [1].

Theorem 1. *Let the following conditions be fulfilled:*

a) the function $p(x)$ is continuously differentiable on a semi-axis $0 \le x < \infty$;

b) there exists an integral of the form $\rho_4(x) = \int\limits_x^\infty (1+\alpha)^4 e^{\sqrt{2}\alpha^{\frac{3}{2}}} |p(\alpha)|\, d\alpha$.

Then for any λ with $\operatorname{Im}\lambda > 0$, equation (1) has the solution $\psi(x,\lambda)$ with the condition

$$\lim_{x\to\infty} \psi(x,\lambda)\, \psi_0^{-1}(x,\lambda) = 1 \tag{2}$$

and there exists a kernel $K(x,t)$ such that

$$\psi(x,\lambda) = \psi_0(x,\lambda) + \int\limits_x^\infty K(x,t)\, \psi_0(t,\lambda)\, dt, \tag{3}$$

moreover the kernel $K(x,t)$ and its first order derivatives satisfy the inequalities

$$1)\, |K(x,t)| \le \frac{1}{2}\rho_0\left(\frac{x+t}{2}\right) e^{\rho_1\left(\frac{x+t}{2}\right)}, \tag{4}$$

where

$$\rho_j(x) = \int\limits_x^\infty (1+\alpha)^j\, e^{\sqrt{2}\alpha^{\frac{2}{3}}} |p(\alpha)|\, d\alpha, j = 0,1; \tag{5}$$

3) *$K(x,t)$ satisfies the differential equation*

$$K''_{xx} - K''_{tt} + (x-t)\, K = K(x,t)\, p(t) \tag{6}$$

and the conditions

$$K(x,t) = \frac{1}{2}\int\limits_x^\infty p(\alpha)\, d\alpha \tag{7}$$

$$K(x,t) = 0 \text{ for } x > t,\ \lim_{t+x\to\infty} K(x,t) = \lim_{t+x\to\infty} \frac{\partial K}{\partial t} = 0 \tag{8}$$

Proof. Reduce problem (6)-(8) to an integral equation. To this end we reduce equation (5) to the canonical form. For that we make up the characteristic equation $\frac{1}{4}\left(dt^2 - dx^2\right) = 0$. This equation admits two different integrals

$$\frac{t-x}{2} = C_1, \quad \frac{t+x}{2} = C_2$$

Consequently, we have to introduce new variables ξ and η by the formulae $\dfrac{t+x}{2} = \xi$, $\dfrac{t-x}{2} = \eta$, $x = \xi - \eta$, $t = \xi + \eta$.

It is obvious that

$$\frac{\partial^2 K}{\partial x^2} = \frac{1}{4}\frac{\partial^2 K}{\partial \xi^2} - \frac{1}{2}\frac{\partial^2 K}{\partial \xi \partial \eta} + \frac{1}{4}\frac{\partial^2 K}{\partial \eta^2},$$

$$\frac{\partial^2 K}{\partial t^2} = \frac{1}{4}\frac{\partial^2 K}{\partial \xi^2} + \frac{1}{2}\frac{\partial^2 K}{\partial \xi \partial \eta} + \frac{1}{4}\frac{\partial^2 K}{\partial \eta^2}.$$

Assuming

$$U(\xi,\eta) = U\left(\frac{t+x}{2}, \frac{t-x}{2}\right) = K(x,t) = K(\xi-\eta, \xi+\eta),$$

for this function we get the following equation

$$L(U) \equiv \frac{\partial^2 U(\xi,\eta)}{\partial \xi \partial \eta} + 2\eta U(\xi,\eta) = U(\xi,\eta)\, p(\xi,\eta) \tag{9}$$

and the boundary conditions

$$U(\xi,0) = \frac{1}{2}\int_{\xi}^{\infty} p(x)\,dx, \tag{10}$$

$$\lim_{\xi\to\infty} u(\xi,\eta) = 0 \text{ for a fixed } \eta > 0. \tag{11}$$

Introduce the Riemann function [2] $R(\xi,\eta;\xi_0,\eta_0)$ of the equation $L[U] = f(\xi,\eta)$, where

$$f(\xi,\eta) = \frac{1}{2}U(\xi,\eta)\,p(\xi,\eta), \tag{12}$$

i.e. the function satisfying the equation

$$L^*[R] \equiv \frac{\partial^2 R}{\partial \xi \partial \eta} + 2\eta R \begin{cases} 0 < \eta < \eta_0 \\ \xi_0 < \xi < \infty \\ 0 < \eta < \xi \end{cases}$$

$$R(\xi,\eta;\xi_0,\eta_0)\big|_{\xi=\xi_0} = 1 \quad 0 \le \eta \le \eta_0$$

$$R(\xi,\eta;\xi_0,\eta_0)\big|_{\eta=\eta_0} = 1 \quad \xi_0 \le \xi \le \infty.$$

Now, apply the Riemann method [2] to equation (9).

We assume that the right hand side of equation (9) is temporarily known. It is abvious that

$$2\{RL[U] - UL^*[R]\} = \frac{\partial X}{\partial \eta} + \frac{\partial Y}{\partial \xi}, \tag{13}$$

where

$$X \equiv R\frac{\partial U}{\partial \xi} - U\frac{\partial R}{\partial \xi}, \quad Y \equiv R\frac{\partial U}{\partial \eta} - U\frac{\partial R}{\partial \eta}.$$

Integrating the both hand sides of identity (13) with respect to the domain D_k and using the known formula [3], we get

$$\iint\limits_{D_k} \frac{\partial X}{\partial \eta} + \frac{\partial Y}{\partial \xi} d\xi d\eta = \int\limits_{l} \left(X d\xi - Y d\eta \right),$$

where l is a contour of the domain D_k

$$l = AM + MP + PQ + QA.$$

So,

$$2\iint\limits_{D_k} RL\left[U\right] = \left(\int\limits_{AM} + \int\limits_{MP} + \int\limits_{PQ} + \int\limits_{QA} \right) \left(X d\xi - Y d\eta \right).$$

Along AM we get

$$\int\limits_{AM} X d\xi = +2\iint\limits_{AM} \left(R\frac{\partial U}{\partial \xi} - U\frac{\partial U}{\partial \xi} \right) d\xi.$$

We can rewrite the integrand function in the form

$$R\frac{\partial U}{\partial \xi} - U\frac{\partial U}{\partial \xi} = \frac{\partial}{\partial \xi}\left(UR \right) - 2U\frac{\partial R}{\partial \xi}$$

then,

$$\int\limits_{AM} \left(R\frac{\partial U}{\partial \xi} - U\frac{\partial U}{\partial \xi} \right) d\xi = \left(UR \right)_M - \left(UR \right)_A - 2\int\limits_{AM} U\frac{\partial R}{\partial \xi} d\xi, \tag{14}$$

where for example, $(UR)_M$ is the value of the product UR at the point M. In the same way, integration with respect to MP will give the following result

$$-\int\limits_{MP} Y d\eta = -\int \left[\frac{\partial}{\partial \eta}\left(UR \right) - 2U\frac{\partial R}{\partial \eta} \right] d\eta =$$

$$= -\left[\left(UR \right)_P - \left(UR \right)_M - 2\int\limits_{AM} U\frac{\partial R}{\partial \eta} d\eta \right] = \left(UR \right)_M - \left(UR \right)_P + 2\int\limits_{AM} U\frac{\partial R}{\partial \eta} d\eta. \tag{15}$$

Now, consider the integrals along the straight line PQ

$$\int\limits_{PQ} X d\xi = \int\limits_{PQ} \left(R\frac{\partial U}{\partial \xi} - U\frac{\partial U}{\partial \xi} \right) d\xi =$$

$$= -\left[(UR)_Q - (UR)_P - 2\int_{PQ} R\frac{\partial RU}{\partial \xi}d\xi\right] = (UR)_P - (UR)_Q + 2\int_{PQ} R\frac{\partial RU}{\partial \xi}d\xi. \quad (16)$$

Along the straight line QA we have

$$-\int_{QA} X d\xi = \int_{QA}\left(R\frac{\partial U}{\partial \eta} - U\frac{\partial U}{\partial \eta}\right)d\eta = (UR)_Q - (UR)_A + 2\int_{QA} R\frac{\partial U}{\partial \xi}d\xi. \quad (17)$$

Considering formulae (14), (15), (16), (17), we get:

$$\int_{\xi_0}^{\xi} d\xi \int_0^{\eta_0} R(\xi,\eta;\xi_0,\eta_0) f(\xi,\eta)\, d\eta - \int_{\eta_0}^{\eta} R(\xi,0;\xi_0,\eta_0)\frac{\partial U(\xi,0)}{\partial \xi}d\xi -$$

$$-\int_0^{\eta_0} U\left(\widetilde{\xi},\eta\right)\frac{\partial R\left(\widetilde{\xi},\eta;\xi_0,\eta_0\right)}{\partial \eta}d\eta - U(\xi_0,\eta_0)\, R(\xi_0,\eta_0;\xi_0,\eta_0) =$$

$$= -U\left(\widetilde{\xi},\eta_0\right) R\left(\widetilde{\xi}_0,\eta_0;\xi_0,\eta_0\right) \qquad \left(\xi_0 < \widetilde{\xi} < \xi\right).$$

Passing to limit as $\xi \to \infty$ and taking into account the condition $\lim\limits_{\xi\to\infty} u(\xi,\eta) = 0$ we get that the limit from the right exists and equals 0. Consequently, there exists a limit from the left as well. Therefore, we have

$$U(\xi_0,\eta_0) = -\int_{\xi_0}^{\infty} R(\xi,0;\xi_0,\eta_0)\frac{\partial U(\xi,0)}{\partial \xi}d\xi + \int_{\xi_0}^{\infty} d\xi \int_0^{\eta_0} R(\xi,\eta;\xi_0,\eta_0) f(\xi,\eta)\, d\eta.$$

Taking into account formulae (10), (12), we get

$$U(\xi_0,\eta_0) = \frac{1}{2}\int_{\xi_0}^{\infty} R(\xi,0;\xi_0,\eta_0)\, p(\xi)\, d\xi +$$

$$+\frac{1}{2}\int_{\xi_0}^{\infty} d\xi \int_0^{\eta_0} U(\xi,\eta)\, P(\xi+\eta)\, R(\xi,\eta;\xi_0,\eta_0)\, d\eta. \quad (18)$$

Thus, for solving problem (9)-(11) it suffices to solve integral equation (18) with respect to $U(\xi_0,\eta_0)$.

To investigate integral equation (18) we'll need the obvious form of the Riemann function $(\xi,\eta;\xi_0,\eta_0)$ that is a solution of the problem:

$$\frac{\partial^2 R}{\partial \xi \partial \eta} + \eta R = 0 \begin{cases} 0 < \eta < \eta_0 \\ \xi_0 < \xi < \infty \\ 0 < \eta < \xi \end{cases} \quad (19)$$

$$R(\xi,\eta;\xi_0,\eta_0)\big|_{\xi=\xi_0} = 1 \quad 0 \le \eta \le \eta_0 \quad (20)$$

$$R(\xi,\eta;\xi_0,\eta_0)\big|_{\eta=\eta_0} = 1 \quad \xi_0 \le \xi \le \infty. \quad (21)$$

Introduce new variables by the formulae

$$\xi = \xi_1, \qquad \eta = \sqrt{\eta_1},$$

then,

$$R(\xi, \eta; \xi_0, \eta_0) = R\left(\xi_1, \sqrt{\eta_1}; \xi_1^0, \sqrt{\eta_1^0}\right), \quad \frac{\partial R}{d\xi} = \frac{\partial R}{d\xi_1}\frac{d\xi_1}{d\xi_1} = \frac{\partial R}{d\xi_1}$$

$$\frac{\partial^2 R}{d\xi \partial \eta} = \frac{\partial^2 R}{d\xi_1 \partial \eta_1}\frac{d\eta_1}{d\eta} = \frac{\partial^2 R}{d\xi_1 \partial \eta_1}\left(\frac{d\eta_1}{d\eta}\right)^{-1} = \frac{\partial^2 R}{d\xi_1 \partial \eta_1} 2\sqrt{\eta_1}.$$

Therefore, equation (19) is rewritten in the form:

$$0 = \frac{\partial^2 R(\xi, \eta; \xi_0, \eta_0)}{\partial \xi \partial \eta} + \eta R = \frac{\partial^2 R\left(\xi_1, \sqrt{\eta_1}; \xi_1^0, \sqrt{\eta_1^0}\right)}{\partial \xi_1 \partial \eta_1} 2\sqrt{\eta_1} + \sqrt{\eta_1} \cdot R.$$

Thus, we reduced problem (19)-(21) to the form:

$$\frac{\partial^2 R}{d\xi_1 \partial \eta_1} + \frac{1}{2} R\left(\xi_1, \sqrt{\eta_1}; \xi_1^0, \sqrt{\eta_1^0}\right) = 0 \tag{19'}$$

$$R|_{\xi_1 = \xi_1^0} = 1 \tag{20'}$$

$$R|_{\eta_1 = \eta_1^0} = 1. \tag{21'}$$

We'll look for the solution of equation (19′) in the form:

$$R = G[w(\xi_1, \eta_1)],$$

where

$$w = (\xi_1, \eta_1) = \sqrt{2\left(\xi_1 - \xi_1^0\right)\left(\eta_1 - \eta_1^0\right)} = \lambda,$$

$G(w)$ is a differentiable function.

It is obvious that

$$\frac{\partial R}{\partial \xi_1} = \frac{\partial G}{\partial w}\frac{\partial w}{\partial \xi_1};$$

$$\frac{\partial^2 R}{\partial \xi_1 \partial \eta_1} = \frac{\partial^2 G}{\partial w^2}\frac{\partial w}{\partial \xi_1}\frac{\partial w}{\partial \eta_1} + \frac{\partial G}{\partial w}\frac{\partial^2 w}{\partial \xi_1 \partial \eta_1} = \frac{2}{4\left[2\left(\xi_1 - \xi_1^0\right)\left(\eta_1 - \eta_1^0\right)\right]^{\frac{1}{2}}} = \frac{1}{2\lambda}.$$

Then, for $G(\lambda)$ we get the equation

$$G''(\lambda) + \frac{1}{\lambda} G'(\lambda) + G(\lambda) = 0.$$

It is known that the Bessel function

$$J_0(\lambda) = \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2}\left(\frac{\lambda}{2}\right)^{2k}$$

is a special solution of this equation.

It is obvious from the last one that having taken $R = J_0(\lambda)$, we'll get such a solution of (19′)that reduces on the characteristics $\xi_1 = \xi_1^0$, $\eta_1 = \eta_1^0$ to a unit, since in this case $\lambda = 0$.

Thus,

$$R\left(\xi_1, \sqrt{\eta_1}; \xi_1^0, \sqrt{\eta_1^0}\right) = J_0\sqrt{2\left(\xi_1 - \xi_1^0\right)\left(\eta_1 - \eta_1^0\right)}.$$

Taking into account change of variables $\xi = \xi_1$, $\eta = \sqrt{\eta_1}$, we get

$$R\left(\xi, \eta; \xi_0, \eta_0\right) = J_0\sqrt{2\left(\xi - \xi_0\right)\left(\eta^2 - \eta_0^2\right)}.$$

Now, investigate integral equation (18) by the sequential approximations method

$$U\left(\xi_0, \eta_0\right) = \frac{1}{2}\int_{\xi_0}^{\infty} R\left(\xi, 0; \xi_0, \eta_0\right) p\left(\xi\right) d\xi,$$

$$U_n\left(\xi_0, \eta_0\right) = \frac{1}{2}\int_{\xi_0}^{\infty} d\xi \int_0^{\eta_0} U_{n-1}\left(\xi, \eta\right) P\left(\xi + \eta\right) R\left(\xi, \eta; \xi_0, \eta_0\right) d\eta.$$

The following estimation for the Bessel function is known [2]:

$$\left|J_v\left(x\right)\right| \leq \left|\frac{x^v}{2}\right| \frac{e^{\left|J_m x\right|}}{\nu + 1} \tag{22}$$

for any $\nu > -\dfrac{1}{2}$.

Since $\xi > \xi_0$, $\eta < \eta_0$ we have

$$R\left(\xi, \eta; \xi_0, \eta_0\right) = J_0\left[i\sqrt{2\left(\xi - \xi_0\right)\left(\eta_0^2 - \eta^2\right)}\right].$$

Considering estimation (22), we have:

$$\left|U_0\left(\xi_0, \eta_0\right)\right| \leq \frac{1}{2}\int_{\xi_0}^{\infty} \left|R\left(\xi, 0; \xi_0, \eta_0\right)\right| \left|p\left(\xi\right)\right| d\xi \leq$$

$$\leq \frac{1}{2}\int_{\xi_0}^{\infty} \left|e^{\sqrt{2}\eta_0\sqrt{\xi - \xi_0}}\right| \left|p\left(\xi\right)\right| d\xi \leq \frac{1}{2}\int_{\xi_0}^{\infty} \left|e^{\sqrt{2}\eta_0\sqrt{\xi - \xi_0}}\right| \left|p\left(\xi\right)\right| d\xi.$$

Here we take into account that $\xi > \xi_0 > \eta_0 > \eta$.

Consequently, for the first approximation we get an estimation of the from

$$\left|U\left(\xi_0, \eta_0\right)\right| \leq \frac{1}{2}\rho\left(\xi_0\right), \tag{23}$$

where $\rho\left(\xi_0\right) = \frac{1}{2}\int_{\eta_0}^{\infty} \left|e^{\sqrt{2}\alpha_0^2}\right| \left|p\left(\alpha\right)\right| d\alpha$.

Estimate the second approximation

$$\left|U_1\left(\xi_0, \eta_0\right)\right| \leq \int_{\xi_0}^{\infty} d\xi \int_0^{\eta_0} \left|U_0\left(\xi, \eta\right)\right| \left|P\left(\xi + \eta\right)\right| R\left(\xi, \eta; \xi_0, \eta_0\right) d\eta \leq$$

$$\leq \frac{1}{4}\int\limits_{\xi_0}^{\infty} d\xi \int\limits_0^{\eta_0} \rho\left(\xi\right)\left|P\left(\xi+\eta\right)\right| e^{\sqrt{2\left(\xi-\xi_0\right)\left(\eta_0^2-\eta^2\right)}} d\eta \leq$$

$$\leq \frac{\rho\left(\xi_0\right)}{4}\int\limits_{\xi_0}^{\infty} d\xi \int\limits_0^{\eta_0} \left|P\left(\xi+\eta\right)\right| e^{\sqrt{2}\sqrt{\xi}\cdot\eta_0} d\eta \leq \frac{\rho\left(\xi_0\right)}{4}\int\limits_{\xi_0}^{\infty} e^{\sqrt{2}\xi^{\frac{3}{2}}} d\xi \int\limits_0^{\eta_0} \left|P\left(\xi+\eta\right)\right| d\eta.$$

Having made change of variables $\xi+\eta=\beta$, we get

$$\int\limits_0^{\eta_0} \left|P\left(\xi+\eta\right)\right| d\eta = \int\limits_{\xi}^{\xi+\eta_0} \left|P\left(\beta\right)\right| d\beta \leq \int\limits_{\xi_0}^{\infty} \left|P\left(\beta\right)\right| d\beta.$$

Hence we have

$$\int\limits_0^{\eta_0} \left|P\left(\xi+\eta\right)\right| d\eta \leq \sigma\left(\xi\right), \tag{24}$$

where $\sigma\left(\xi\right) = \int\limits_{\xi_0}^{\infty} \left|P\left(\beta\right)\right| d\beta.$

Therefore

$$\left|U_1\left(\xi_0,\eta_0\right)\right| \leq \frac{\rho\left(\xi_0\right)}{4}\int\limits_{\xi_0}^{\infty} e^{\sqrt{2}\xi^{\frac{3}{2}}}\sigma\left(\xi\right) d\xi \leq \frac{\rho\left(\xi_0\right)}{4}\widetilde{\rho}\left(\xi_0\right), \tag{25}$$

where $\widetilde{\rho}\left(\xi_0\right) = \int\limits_{\xi_0}^{\infty} e^{\sqrt{2}\alpha^2}\sigma\left(\alpha\right) d\alpha.$

Taking into account (25), we estimate the third approximation

$$\left|U_2\left(\xi_0,\eta_0\right)\right| \leq \frac{1}{8}\int\limits_{\xi_0}^{\infty} d\xi e^{\sqrt{2}\sqrt{\xi}\eta_0} \int\limits_0^{\eta_0} \rho\left(\xi\right)\widetilde{\rho}\left(\xi\right)\left|P\left(\xi+\eta\right)\right| d\eta \leq$$

$$\leq \frac{\rho\left(\xi_0\right)}{8}\int\limits_{\xi_0}^{\infty} e^{\sqrt{2}\xi^{\frac{3}{2}}}\widetilde{\rho}\left(\xi\right) d\xi \int\limits_0^{\eta_0} \left|P\left(\xi+\eta\right)\right| d\eta$$

since $\xi>\xi_0>\eta_0>\eta$.

Taking into account (24), we get

$$\left|U_2\left(\xi_0,\eta_0\right)\right| \leq \frac{\rho\left(\xi_0\right)}{8}\int\limits_{\xi_0}^{\infty} e^{\sqrt{2}\xi^{\frac{3}{2}}}\sigma\left(\xi\right)\widetilde{\rho}\left(\xi\right) d\xi.$$

It is obvious that

$$d\widetilde{\rho}\left(\xi\right) = d\int\limits_{\xi_0}^{\infty} e^{\sqrt{2}\alpha^{\frac{3}{2}}}\sigma\left(\alpha\right) d\alpha = -e^{\sqrt{2}\alpha^{\frac{3}{2}}}\sigma\left(\xi\right) d\xi. \tag{26}$$

Therefore

$$|U_2(\xi_0,\eta_0)| \leq \frac{\rho(\xi_0)}{8}\frac{\widetilde{\rho}^2(\xi_0)}{2!}. \qquad (27)$$

The validity of the following estimation is proved by the mathematical induction method

$$|U_m(\xi_0,\eta_0)| \leq \frac{\rho(\xi_0)}{2^{n+1}}\frac{\widetilde{\rho}^n(\xi_0)}{2!}. \qquad (28)$$

Considering (23)-(28), we get the absolute and uniform convergence of the series

$$U(\xi_0,\eta_0) = \sum_{n=0}^{\infty} U_n(\xi_0,\eta_0)$$

in any finite domain of variables ξ and η subjected to the condition $0<\eta<\xi$ and for the sum $U(\xi_0,\eta_0)$ of this series the following estimation is valid

$$|U(\xi_0,\eta_0)| \leq \frac{1}{2}\rho(\xi_0)\, l^{\frac{\rho_1(\xi_0)}{2}}, \qquad (29)$$

where

$$\rho_j(\xi_0) = \int_{\xi_0}^{\infty} \alpha^j l^{\sqrt{2}\alpha^{\frac{3}{2}}} |p(\alpha)|\, d\alpha, \quad (j=0,1), \quad \rho_0(\xi_0) = \rho(\xi_0).$$

It directly follows from estimation (29) that

$$|K(x,t)| \leq \frac{1}{2}\rho\left(\frac{t+x}{2}\right)\exp\left[\rho_1, \frac{x+t}{2}\right] \qquad (30)$$

where

$$\rho_j(x) = \int_{x}^{\infty} \alpha^j l^{\sqrt{2}\alpha^{\frac{3}{2}}} |p(\alpha)|\, d\alpha, \quad (j=0,1), \quad \rho_0(x) = \rho(x).$$

References

[1]. Levin B. Ya. *The Fourier type transformation for second order differential equation* // DAN SSSR, 1956, vol. 106, No2, pp. 187-190 (Russian).

[2]. Koshlyakov N. S. and others. *Equations of mathematical physics* "Vysshaya shkola", 1970 (Russian).

[3]. Fikhtenholtz G.M. *Course of differential and integral calculus.* 1970, Vol. III, 656 p. (Russian)

[4]. Gasymov M.G., Mustafayev B. A. *The inverse scattering problem for a perturbed anharmonic equation on a semi-axis.* DAN SSSR, 1976, vol. 228, No 1, pp. 11-14 (Russian).

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