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IMPACT BY A ROUGH BLUNT WEDGE ON AN ELASTIC FILAMENT WITH REGARD TO ITS FAILURE UNDER SUBSONIC CONDITIONS

Abstract

In the paper, a problem on break of flexible elastic filament under normal impact on it by a plane frond edge and with regard to external medium pressure for subsonic notion condition is investigated.

In the paper, a problem on break flexible elastic filament under lateral impact on it by a plane front wedge with regard to external medium pressure for subsonic motion condition is investigated. The similar self-model problem is considered in the paper [2].

Notice that break of the filament with regard to external medium pressure under normal impact by a sharp wedge was studied in the papers [3, 4].

1. Let a normal impact with constant velocity V by a plane front rigid symmetric wedge be delivered on an infinitely long linear unstressed flexible elastic filament. It is accepted that the filament is retained against the wedge cheek of constant pressure P, and P is normal to the filament (fig.1).

Fig 1.

Since the subsonic motion condition is considered, the velocity of A and A_1 is less than the velocity of the elastic wave and filament. Denote the front of the wedge by $2L$, i.e. $|BB_1| = 2L$. Under impact, four elastic waves whose fronts are the points M_1, C_1, C_2, M_2 and two strong distortion waves A, A_1 (fig 1) originate in the filament. Since the wedge is symmetric, the filament's behavior in the domain M_2, A_1, B_1C_2 and M_1, ABC_1O is the same. In the domain C_2O and C_1O to the moment $t = \dfrac{L}{a}$ (a is velocity of elastic wave) the filament is in the rest state with respect to the wedge. Here $a = \sqrt{\dfrac{E}{\rho}}$ is elastic wave velocity in the filament. Interaction of wedge's rough surface and the filament is described by Coulomb's dry friction law.

Equation of motion of linear-elastic filament in the outline area in dimensionless form will be [1 – 4]

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x_1^2} - \mu P \frac{\partial u}{\partial x} \nu_0; \tag{1.1}$$

$$\nu_0 = sign\upsilon; \quad \upsilon = \frac{\partial u}{\partial t}, \quad x_1 = x - L;$$

Fig 2.

Here μ is a friction coefficient, x is a Lagrange coordinate of the filament particle, u is displacement of the filament section of the wedge surface after impact, t is time.

Wave motion scheme in the filament after impact in the plane (x, t) is shown in figure 2 $\left(0 \leq t < \dfrac{L}{a}\right)$.

After elastic waves reflection there arise new areas in the filament. Denote them by 1, 2, 3, 4 and etc. By these indices we'll supply the parameters of corresponding areas.

2. Since the filament behavior with respect to the point O is symmetric, we'll consider the problem in the right hand side of the filament $OBAM_1$.

Conditions on strong distortion waves (at the climbing point A) in the dimensionless form will be [1, 2]

$$\frac{b - \upsilon_1}{1 + \varepsilon_1} = \frac{b \sec\gamma - \upsilon_2}{1 + \varepsilon_2} = z; \tag{2.1}$$

$$z\left(\upsilon_2 - \upsilon_1 \cos\gamma - M \sin\gamma\right) = \sigma_1 \cos\gamma - \sigma_2 - F_{Tp}; \tag{2.2}$$

$$z\left(M \cos\gamma - \upsilon_2 \sin\gamma\right) = \sigma_1 \sin\gamma + Q; \tag{2.3}$$

$$b = Mctg\gamma; \qquad M = Va^{-1}. \tag{2.4}$$

Here υ_1, υ_2 are filament particles velocities in areas 1 and 2, respectively; $\varepsilon_1, \varepsilon_2$ are strains in the areas 1 and 2; σ_1, σ_2 are stresses in the areas 1 and 2; F_{Tp}, Q are concentrated forces at the breakpoint A; z is dimensionless velocity of strong distortion wave.

Notice that when the inequality

$$F_{Tp} < \mu_* Q, \tag{2.5}$$

is fulfilled on the strong distortion wave, this implies the condition [2]

$$\upsilon_2 = 0. \tag{2.6}$$

For constructing the solutions in the subsonic conditions area $M < tg\gamma$, solution for supersonic conditions on the sound line $M = tg\gamma$ should be kept in mind and natural requirement of continnity of changes determined in the parameters problem while passing through this line to subsonic area should be taken into account. On the indicated line, under supersonic condition in the area $\gamma < 2\gamma_*$, it holds the inequality $[2-4]$ $F_{Tp} < \mu_* Q$, therefore, it is natural to require to observe this inequality also in some vicinity of the sound line in the subsonic area. Furthermore, we have to construct the solution of the problem under subsonic condition that continuously passes to the solution of the problem on pointwise impact on an elastic filament for which on the strong distortion wave for $F = Q = 0$, $P = 0$, $L = 0$, the condition

$$\varepsilon_1 = \varepsilon_2 \tag{2.7}$$

should be taken into account.

Also at the point B we have a kinematic condition in the form

$$\upsilon_3 (x_1, t)|_{x_1=0} = \ \upsilon_2 (x_1, t))|_{x_1=0} \cos\gamma. \tag{2.8}$$

On the fronts M_1 and C_1 we have conditions of displacement fucntion continuity in the form

$$u_1 (x_1, t) = 0 \text{ for } x_1 - t = 0; \tag{2.9}$$

$$u_3 (x_1, t) = 0 \text{ for } x_1 + t = 0. \tag{2.10}$$

Notice that the we'll supply the filament motion parameters $u (x_1, t)$, $\varepsilon (x_1, t)$, $\sigma (x_1, t)$, $\upsilon (x_1, t)$ arising in the areas 1,2,3 by the corresponding indices.

The motion equation of the linear-elastic filament in the area before the climbing point A in the dimensionless form will be

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x_1^2}. \tag{2.11}$$

To get the solution of the problem, we represent the function $u (x_1, t)$ and trajectory of the strong distortion wave $x_* (t)$ in the form of expansion in the parameter μ (μ is assumed to be small $\mu > 1$). And we are restricted by two terms in the expansion.

We represent the trajectory of strong distortion wave in the form (dimensionless form)

$$x_1 = x_* (t) = z_0 t + \mu z_1 t^2. \tag{2.12}$$

We represent the solution of equation (1.1) in the areas 2,3 in the form

$$u_2 (x_1, t) = \left(1 + \varepsilon_2^{(0)}\right) x_1 + \upsilon_2^{(0)} t + \mu \times$$

$$\times\left[a_2\left(t-x_1\right)^2+b_2\left(t+x_1\right)^2+\frac{P\left(1+\varepsilon_2^{(0)}\right)}{4}\left(x_1^2-t^2\right)\nu_0\right] \qquad (2.13)$$

$$u_3\left(x_1,t\right)=\varepsilon_3^{(0)}x_1+\upsilon_3^{(0)}t+\mu\left[a_3\left(t-x_1\right)^2+b_3\left(t+x_1\right)^2+P\frac{\varepsilon_3^{(0)}}{4}\left(x_1^2-t^2\right)\nu_0\right], \qquad (2.14)$$

respectively.

Represent the solution of equation (2.11) in the area 1 as follows

$$u_1\left(x_1,t\right)=\varepsilon_1^{(0)}x_1+\upsilon_1^{(0)}t+\mu\left[a_1\left(t-x_1\right)^2+b_1\left(t+x_1\right)^2\right]. \qquad (2.15)$$

Here $\varepsilon_1^{(0)}$, $\upsilon_1^{(0)}$, $\varepsilon_2^{(0)}$, $\upsilon_2^{(0)}$, $\varepsilon_3^{(0)}$, $\upsilon_3^{(0)}$, z_0 correspond to the self-model solution $(P=0)$ [2]. The coefficients $z_1, a_1, b_1, a_2, b_2, a_3, b_3$ and $\varepsilon_1^{(0)}$, $\upsilon_1^{(0)}$, $\varepsilon_2^{(0)}$, $\upsilon_2^{(0)}$, $\varepsilon_3^{(0)}$, $\upsilon_3^{(0)}$, z_0 must be determined.

Substituting formulae (2.12) – (2.15) in conditions (2.1), (2.6), (2.7)-(2.10) and equating the coefficients at the same degrees of μ, determine $\varepsilon_1^{(0)}$, $\varepsilon_2^{(0)}$, $\varepsilon_3^{(0)}$, z_0, z_1, $\upsilon_1^{(0)}, \upsilon_2^{(0)}, \upsilon_3^{(0)}$, a_1, b_1, a_2, b_2, a_3, b_3 in the form

$$\varepsilon_1^{(0)}=\varepsilon_2^{(0)}=M\left(\sec\gamma-1\right)ctg\gamma;\;\; z_0=b\left[b+\left(1-b\right)\cos\gamma\right]^{-1};$$
$$\upsilon_2^{(0)}=0; \upsilon_3^{(0)}=0;\;\; \varepsilon_3^{(0)}=0;\;\; a_1=0;\;\; z_1=0$$
$$a_3=0; b_1=0; \upsilon_1^{(0)}=-\varepsilon_1^{(0)}; \qquad (2.16)$$
$$a_2=\frac{P}{8}\left(1+\varepsilon_2^{(0)}\right)\frac{1+z_0}{1-z_0};\; b_2=\frac{P}{8}\left(1+\varepsilon_2^{(0)}\right)\frac{1-z_0}{1+z_0};$$
$$b_3=\frac{P}{2}\left(1+\varepsilon_1^{(0)}\right)\frac{z_0^2}{1-z_0^2}\cos\gamma;$$

Then stresses and velocity of the filament's particle in the areas 1,2,3 allowing for (2.16) in dimensionless form will be

$$\begin{cases}\sigma_1=\sigma_1^{(0)}=\varepsilon_1^{(0)}=Mctg\gamma\left(\sec\gamma-1\right); \sigma_1^{(0)}=\sigma_2^{(0)};\\ \upsilon_1=\upsilon_1^{(0)}=\varepsilon_1^{(0)}=-Mctg\gamma\left(\sec\gamma-1\right);\end{cases} \qquad (2.17)$$

$$\begin{cases}\sigma_2\left(x_1,t\right)=\sigma_2^{(0)}+\mu P_0\left[\frac{1-z_0}{1+z_0}\left(t+x_1\right)-\frac{1+z_0}{1-z_0}\left(t-x_1\right)+2x_1\right];\\ \upsilon_2\left(x_1,t\right)=\mu P_0\left[\frac{1+z_0}{1-z_0}\left(t-x_1\right)+\frac{1-z_0}{1+z_0}\left(t+x_1\right)-2t\right];\\ P_0=P\left(1+\varepsilon_2^{(0)}\right)\cdot 4^{-1};\end{cases} \qquad (2.18)$$

$$\begin{cases}\sigma_3\left(x_1,t\right)=\varepsilon_3\left(x_1,t\right)=\mu P\left(1+\varepsilon_2^{(0)}\right)\frac{z_0^2}{1-z_0}\left(t+x_1\right)\cos\gamma;\\ \upsilon_3\left(x_1,t\right)=\mu P\left(1+\varepsilon_2^{(0)}\right)\frac{z_0^2}{1-z_0}\left(t+x_1\right)\cos\gamma;\end{cases} \qquad (2.19)$$

respectively.

For $x_1 = 0$, i.e. at the nodes of the wedge the formulae (2.17)-(2.19) will take the form

$$\begin{cases} \sigma_2 = \sigma_2^{(0)} = \mu P \left(1 + \varepsilon_2^{(0)}\right) \dfrac{z_0}{1 - z_0^2} t; \\ \upsilon_2 = \mu P \left(1 + \varepsilon_2^{(0)}\right) \dfrac{z_0^2}{1 - z_0^2} t \end{cases} \tag{2.20}$$

$$\begin{cases} \sigma_3 = \mu P \left(1 + \varepsilon_2^{(0)}\right) \dfrac{z_0^2}{1 - z_0^2} t \cos\gamma; \\ \upsilon_3 = \mu P \left(1 + \varepsilon_2^{(0)}\right) \dfrac{z_0^2}{1 - z_0^2} t \cos\gamma; \end{cases} \tag{2.21}$$

It follows from formulae (2.20), (2.21) that in the area 1,2,3 the filament is stretched. The stresses $\sigma_2(0,t)$ at the nodes of the wedge, the decreasing linear function in time and the functions $\upsilon_2(0,t), \sigma_3(0,t), \upsilon_3(0,t)$ are positive and are linear functions in time.

3. Now, let's consider a problem allowing for possiblity of elastic filament breakage under impact for the subsonic motion condition. For that, it is necessary to determine max σ in the solution constructed above, and equating this quantity to breaking stress σ_{np}, to obtain a line on the plane (γ, M) restricting the conditions for which the solutions constructed above remain acceptable, and in the domain exterion to this boundary to construct the solution allowing for the filament's breakage.

Obviously, at the moment $t = \dfrac{L}{a}$ the elastic waves outgoing from the point B and B_1 (fig. 1) meet at the point O and at this place the stress is doubled, the particle's growth vanishes. Then, the stress at this point will be

$$\sigma = 2\sigma_3, \tag{3.1}$$

here σ_3 is expressed by formula (2.19).

The filament stress at the wedge nodes, allowing for (2.20),(2.21) will be

$$\sigma_1(x_1,t)|_{x_1=0} = \sigma_2^{(0)} - \mu P \left(1 + \varepsilon_2^{(0)}\right) \frac{z_0}{1 - z_0^2} (1 - z_0 \cos\gamma) t \tag{3.2}$$

It follows from expressions (3.1),(3.2) that max σ coincides with the value of σ and is determined by formula (3.2) at the initial stage of collision, i.e.

$$\max \sigma = \sigma_2^{(0)},$$

or

$$\sigma_2^{(0)} = \sigma_{np}. \tag{3.3}$$

Taking into account the expression $\sigma_2^{(0)}$ from (2.17) in formula (3.3), the failure condition will take the form

$$b(\sec\gamma - 1) = \sigma_{np}; \;\; b = M ctg\gamma;$$

or

$$M tg\frac{\gamma}{2} = \sigma_{np} \tag{3.4}$$

Notice that under impact by a sharp wedge on elastic filament, the failure condition at the impact point is of the form [4]

$$2Mtg\frac{\gamma}{2} = \sigma_{np};\ M = \frac{V}{a}.$$

It follows from formulae (3.4),(3.5) that for the given velocity of impact by a sharp wedge, the filament terminates at the impact point for critical angle $\gamma = \gamma_p$, but for just these data, under impact by a blunt wedge, the filament at the nodes B and B_1 don't terminate. But breakage may happen only when the velocity M by the blunt wedge is twice greater than the velocity of a sharp wedge and under the same velocities of impact, the opening angle of the blunt wedge is twice smaller than the opening angle of a sharp wedge.

The solution of the problem with breakage at the nodes of the wedge B and B_1 $(x_1 = 0)$ is similar to the problem on impact by a rough sharp wedge on elastic filament with regard to its breakage under subsonic condition [4]. Therefore, we'll not stop on the solution of this problem. However, under impact by the blunt wedge on elastic filament, after breakage in the nodes of the wedge B and B_1, the destructed part of the filament BB_1 is in the rest state and moves together with the wedge (fig.3)

Fig 3.

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