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DIFFRACTION BY AN IMPEDANCE STRIP: A NEW PRESENTATION BASED ON PHYSICAL OPTICS APPROACH

Abstract

Diffraction of plane electomagnetic waves by an infinitely long strip having the same impedance on both faces is investigated. The solution for impedance surface with arbitrary value of the surface impedance is constructed as a linear combination of known solutions for perfectly electric conducting (PEC) *and perfect magnetic conducting* (PMC) *surfaces. Solutions for PEC and PMC strip of the same width are found numerically. The coefficients in the presentation for the radiation pattern for the impedance strip take into account losses which depend on the surface impedance. These coefficients are evaluated analytically in physical optics approximation when all solutions* (PEC,PMC and impedance strip) *can be obtained analytically. For other values of the wave length the resulting fields are examined numerically. It is necessary to solve once the diffraction problem by PEC and PMC strips numerically for a given strip width, after that the solution for an impedance strip is obtained as a simple superposition of both solutions for any value of the impedance. The method of solving considering diffraction problem is based on presenting the diffracted field in terms of the induced electric and magnetic current densities. The problem is formulated as simultaneous integral equations. Obtained integral equations allow to derive the high frequency asymptotic expressions of the far field radiation pattern.Utilizing the Fourier transform to the integral equations the unkown current density functions can be expanded into the infinite series containing the Chebyshev polynomials. Finally, the problem is reduced to infinite systems of linear algebraic equations satisfied by the expansion coefficients. Radiation pattern, radar cross section are plotted for different values of the impedance, the wave length and the incidence angle showing the comparison of the constructed solution and exact solution for impedance strip. It is showed that presented expression allows to obtain the solution for impedance strip with good accuracy for wide range of values of impedance and wave length.*

1.Introduction A problem on wave diffraction by plane strip with impedance boundary conditions (IBC) was considered by many authors $[1-3]$. In these papers asymptotic methods that are valid in so called shortwave area when the lenght of incident wave is significantly less than geometric sizes of the strip, are developed.

However, a resonance case when the sizes of the strip are comparable with the lenght of incident wave, is of great interest. In this case, asymptotic methods don't function. In the paper [4], a numerical-analytic method allowing to solve the considered problem with any preassigned accuracy, is suggested. Notice that application of IBC allows not to consider the area interior to scatterer [1] that leads to simplification of calculations. The value of impedance is usually connected with losses (absorption interior to the strip).

In the present paper, it is suggested to obtain the solution of the diffraction problem by the strip for an arbitrary value of impedance by means of composition of known solutions for perfectly electric conducting (PEC) and perfectly magnetic conducting (PMC) strips with certain coefficients. Analytic expressions for these coefficients are derived from the so called physical optics (PO) approximations comparing analytic solutions for PEC and PMC impedance strip. The values of the coefficients themselves in the representation are determined by impedance, sizes of the strip and angle of incidence. Thus, for obtaining the solution for impedance strip under the given wave lenght it is necessary to solve once the diffraction problem by PEC and PMS for the given wave lenght and put them together with desired coefficients. Pecularitics of geometric of the strip (strip's width, edge effect) will be considered by exact solutions for PEC and PMC strips but absorption connected with impedance is considered by appropriate coefficients. Accuracy of such a construction of the solution is numerically verified for different values of wave length and impedance. Advantage of such an approach is that it is not necessary to solve again the problem for a new value of impedance, it suffices to use the solutions for a problem with perfect boundary conditions.

2. Solution method. Consider a two-dimensional problem on wave diffraction by a strip with boundary condition (BC) in the form

$$\frac{\partial}{\partial y}E_z \pm \frac{ik\eta_0}{\eta}E_z = 0,\ y \to \pm 0, \tag{1}$$

where η is surface impedance of the strip, η_0 is free space impedance. The value of impedance is considered the same from the both sides of the strip $(y \to \pm 0)$.

Assume that the incident field is E - polarized plane wave. Infinitely thin strip of width $2a$ is arranged in the plane $y = 0$ and is infinite along the axis z. The incident field $\overrightarrow{E^i} = \overrightarrow{z}U_0(x,y) = \overrightarrow{z}E_z^0(x,y)$ is described by the expression $U_0(x,y) = e^{-ik\left(x\alpha_0 + y\sqrt{1-\alpha_0^2}\right)}$, where $\alpha_0 = \cos\theta_0$, θ_0 is an incidence angle, $k = \dfrac{2\pi}{\lambda}$ is a wave number. Dependence on time is assumed $e^{-i\omega t}$ and is omitted in all formulae. Denote the scattered field by the function $\overrightarrow{E^r} = \overrightarrow{z}U_r(x,y)$. Total field $\overrightarrow{E} = \overrightarrow{z}U(x,y)$ is a sum of incident field and scattered field, i.e. $U(x,y) = U_0(x,y) + U_r(x,y)$. The function $U(x,y)$ describing the solution of the problem should satisfy the following conditions:

- Helmholtz equation in all the space, except the strip:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + k^2 U = 0;$$

- conditions on radiation at infinity for the scattered field $U_r(x,y)$ [1, 5]:

$$\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial U_r}{\partial r} - iU_r\right) = 0,\ r = \sqrt{x^2+y^2};$$

- Makesner condition $[1,5]$ on strip's ribs $(y=0,\ x\to\pm a)$;
- boundary conditions on the surface of the strip (1).

Notice that on a strip with IBC for the case of E-polarized incident wave there exists electric and magnetic currents flowing in the following direction: electric current along the strip $\overrightarrow{j}^{e}-\overrightarrow{z}j_z^e$ and magnetic current across $\overrightarrow{j}^{m}-\overrightarrow{x}j_x^m$. Surface currents are determined as a jump of field components when passing through the boundary of the strip:

$$j_z^e=-\frac{1}{ik}\left(\frac{\partial E_z\left(x,+0\right)}{\partial y}-\frac{\partial E_z\left(x,-0\right)}{\partial y}\right),$$

$$j_x^m=-\left(E_z\left(x,+0\right)-E_z\left(x,-0\right)\right).$$

Applying the well known solution methods $[1,4,5]$ we'll find a scattered field in the form of the sum of two potentials-simple and double layers

$$E_z^{2,s}\left(x,y\right)\equiv-\frac{i}{4}\int\limits_{-a}^{a}\left[f_e\left(x'\right)+f_m\left(x'\right)\frac{\partial}{\partial y}\right]G\left(x-x',y\right)dx',\qquad(2)$$

where $G\left(x-x',y\right)$ is Green's two-dimensional function of free space.

Unknown densities of potentials $f_e\left(x'\right),f_m\left(x'\right)$ in representation (2) have physical sense. They correspond to densities of surface electric and magnetic currents, respectively.

Moreover, the Makesner conditions [5] on the ribs $(\varsigma\to\pm0)$ with respect to the functions $f_e\left(x'\right),f_m\left(x'\right)$ are formulated in the following way:

$$\widetilde{f}_e\left(\xi\right)=O\left(\xi^{-1/2}\right),\ \widetilde{f}_m\left(\xi\right)=O\left(\xi^{1/2}\right).$$

Notice that in addition to (2), there exist another forms by means of which scattered field is found. For instance, it is possible to find it a simple layer in the form of one potential or a double layer one potential. Each of representations reduce to different integral equations (IE) for finding densities of potentials $[6]$.

Solution method of diffraction problem by an impedance strip was stated in the paper [4] in detail. We reduce terminal IE where the unknown functions are the Fourier transforms $F_e\left(\alpha\right)$, $F_m\left(\alpha\right)$ with respect to the unknown functions $f_e\left(x\right)$ and $f_m\left(x\right)$, respectively:

$$-\frac{\eta}{\varepsilon}F_e\left(\beta\right)=4i\frac{\sin\varepsilon\left(\beta+\alpha_0\right)}{\varepsilon\left(\beta+\alpha_0\right)}+\frac{1}{\varepsilon\pi}\int\limits_{-\infty}^{\infty}F_e\left(\alpha\right)\frac{\sin\varepsilon\left(\alpha-\beta\right)}{\left(\alpha-\beta\right)}\left(1-\alpha^2\right)^{-1/2}d\alpha,\qquad(3)$$

$$\frac{1}{\eta}F_m\left(\beta\right)=4\sqrt{1-\alpha_0^2}\frac{\sin\varepsilon\left(\beta+\alpha_0\right)}{\varepsilon\left(\beta+\alpha_0\right)}-$$

$$-\frac{1}{\pi}\int\limits_{-\infty}^{\infty}F_m\left(\alpha\right)\frac{\sin\varepsilon\left(\alpha-\beta\right)}{\left(\alpha-\beta\right)}\left(1-\alpha^2\right)^{-1/2}d\alpha,\qquad(4)$$

where $\varsigma = x/a, \varepsilon = ka$,

$$F_e(\alpha) = \int_{-1}^{1} \widetilde{f}_e(\xi) e^{-i\varepsilon\alpha\xi} d\xi, \;\; \widetilde{f}_e(\xi) \equiv a f_e(a\xi),$$

$$F_m(\alpha) = \int_{-1}^{1} \widetilde{f}_m(\xi) e^{-i\varepsilon\alpha\xi} d\xi, \;\; \widetilde{f}_m(\xi) \equiv f_m(a\xi).$$

Proceeding from IE $(3), (4)$ in the PO approximation ($\varepsilon = ka \to \infty$) on the basis of representation

$$\lim_{\varepsilon\to\infty} \frac{\sin\varepsilon(t-\beta)}{t-\beta} = \pi\delta(t-\beta)$$

one can get asymptotic expressions $[4, 7]$ for the desired Fourier transforms of the form:

$$F_e(\beta) = -4i\frac{\sqrt{1-\beta^2}}{1+\eta\sqrt{1-\beta^2}}\frac{\sin\varepsilon(\beta+\alpha_0)}{(\beta+\alpha_0)}, \tag{5}$$

$$F_m(\beta) = 4\sqrt{1-\alpha_0^2}\frac{\eta}{1+\eta\sqrt{1-\beta^2}}\frac{\sin\varepsilon(\beta+\alpha_0)}{\varepsilon(\beta+\alpha_0)}.$$

On the basis of the obtained analytic expressions for the Fourier transforms, for the scattered field $E_z^s(x,y)$ we can write analytic representation in the for field $(kr \to \infty)$. They will be of the form $[4-6]$:

$$E_z^s(r,\varphi) \approx A(kr)\,\Phi^{imp}(\varphi), \;\; kr \to \infty,$$

where $A(kr) = \sqrt{\dfrac{2}{\pi \hat{e} r}} e^{ikr - i\frac{\pi}{4}}$ is an amplitude, and $\Phi^{imp}(\varphi)$ is radiation pattern (RP).

The function $\Phi^{imp}(\varphi)$ is represented in the form of the sum

$$\Phi^{imp}(\varphi) = \Phi_e^{imp}(\varphi) + \Phi_m^{imp}(\varphi), \tag{6}$$

where

$$\Phi_e^{imp}(\varphi) = \frac{1}{4}F_e(\cos\varphi)\,\Phi_m^{imp}(\varphi) = \frac{1}{4}\varepsilon\sin\varphi F_m(\cos\varphi).$$

Thus, in PO approximation we have the following expressions:

1. for PEC ($\eta = 0$):

$$\Phi_e^{pec}(\varphi) = -i\sin\varphi\frac{\sin ka(\cos\varphi+\cos\theta_0)}{(\cos\varphi+\cos\theta_0)}; \tag{7}$$

2. for PMC ($\eta = i\infty$):

$$\Phi_m^{pmc}(\varphi) = i\sin\theta_0\frac{\sin ka(\cos\varphi+\cos\theta_0)}{(\cos\varphi+\cos\theta_0)}; \tag{8}$$

3. For an impedance strip:

$$\Phi^{imp}(\varphi) = -i\sin\varphi \frac{(1-\eta\sin\theta_0)}{1+\eta\sin\varphi} \frac{\sin ka(\cos\varphi+\cos\theta_0)}{(\cos\varphi+\cos\theta_0)}. \quad (9)$$

Comparing these expressions, we easily get the relation

$$\Phi^{imp}(\varphi,\eta) = \frac{1}{1+\eta\sin\varphi}\Phi_e^{pec}(\varphi) + \frac{\eta\sin\varphi}{1+\eta\sin\varphi}\Phi_m^{pmc}(\varphi).$$

$$\Phi^{imp}(\varphi,\eta) = A\Phi_e^{pec}(\varphi) + B\Phi_m^{pmc}(\varphi) \quad (10)$$

Proceeding from this fact we have the coefficients $A \equiv \dfrac{1}{1+\eta\sin\varphi}$, $B \equiv \dfrac{\eta\sin\varphi}{1+\eta\sin\varphi}$, that are independent of a frequency parameter ka. Note that $A+B=1$ and it is independent of incident angle.

As is seen from (10), in PO appropximation, we can get solution for IBC by superposition of the solutions for PEC and PMC with analytic coefficients A, B.

Further, for arbitrary value of a frequency parameter ka, we assume that formula (10) is valid i.e. we construct the solution for IBC in the form of the same sum (5), however in this case $\Phi_e^{pec}(\varphi)\,\Phi_m^{pmc}(\varphi)$ determine RP that are numerically obtained on the basis of strict solution [4, 7]. This is novelty of the present method.

Digitization of IE (3), (4) and corresponding system of the linear algebraic equations ($SLAE$) is described in detail in [4, 7] for IBC. Thus, for the given value of the wavelength the solution may be obtained by any preassigned accuracy.

3. Numerical results. Compare physical characteristics for the solution by an impedance strip on the basis of numerical solution [4] and the solution obtained in the form of the sum (10) where only the solutions for PEC and PMC are numerically found. We'll consider the following quantities:

- scattering diameter (SD) $\dfrac{\sigma_{2D}}{\lambda}$ is expressed by RP $\Phi^{imp}(\varphi)$ [1, 5, 6, 7]:

$$\frac{\sigma_{2D}}{\lambda}(\varphi) = \frac{2}{\pi}|\Phi(\varphi)|^2.$$

- backward scattering diameter (BSD) σ_{2D}^{mono} is determined as a value of scattering diameter under observation angle equal to incidence angle $\varphi = \theta_0$ [1, 5]:

$$\sigma_{2D}^{mono} = \frac{\sigma_{2D}}{\lambda}(\theta_0) = \frac{2}{\pi}\left|\Phi^{imp}(\theta_0)\right|^2.$$

In PO approximation we have the expressions

$$\frac{\sigma_{2D}}{\lambda} = \frac{2}{\pi}\sin^2\varphi\left(\frac{\sin\theta_0}{\sin\varphi}\right)^{2\nu}\left\{\frac{\sin\varepsilon(\cos\varphi+\cos\theta_0)}{\cos\varphi+\cos\theta_0}\right\}^2, \ \varepsilon = ka >> 1,$$

$$\sigma_{2D}^{mono}\frac{2}{\pi}\sin^2\theta_0\left\{\frac{\sin\varepsilon(2\cos\theta_0)}{2\cos\theta_0}\right\}^2, \ \varepsilon = ka >> 1. \quad (11)$$

Calculation of these quantities under different values of impedance η, incidence angle θ_0 and frequency parameter $\varepsilon = ka$ are plotted in figures 1-4. The values of these parameters are also in these figures. The cited results on these figures show that the suggested method allows to calculate necessary quantities with good accuracy.

Fig 1.

Fig 2.

Fig 3.

Fig 4.

References

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