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INTERACTION OF PREFRACTURE STRIPS IN FRICTION LINING

Abstract

Mathematical description of crack initiation in friction lining under braking is given. It is assumed that crack initiation and fracture of lining's material occurs under multiple braking. The problem on equilibrium of friction lining with nuclear crack is reduced to the solution of the system of nonlinear singular integrodifferential equation with Cauchy type kernel. The condition of appearance of a crack is formulated taking account of the criterion of the limit traction of the bonds in the material.

Problem Statement. Friction couple "drum-lining" of drum block brake mechanisms functions under complicated stres state conditions. Elaboration of mathematical model within of which one could effectively prognose development of crack type defects in friction lining under braking is of great importance.

In the course of operation of friction pair "drum-lining" there will arise prefracture zones (interlayers of overstressed material) in friction linings and they will be modelled as areas of weakened interparticle bonds of material.

Interaction of lips of these areas is modelled by introducing between the prefracture strip lips the bonds of the given strain diagram of material's bonds.

Interaction of prefracture zones lips is modelled by introducing adhesion forces (bonds) between the prefracture zone lips. Physical nature of such bonds and sizes of prefracture areas wherein interaction of interparticle bonds areas lips is realized depends on the form of the material $[1-4]$. Thus embryonic cracks are modelled by prefracture zones with bonds between the lips that are considered as the areas of weakened interpartide bonds of the material. Since the prefracture zones of lining are small in comparison with the remaining part of friction lining, we can remove them mentally and replace by sections whose surfaces interact between themselves by some law that corresponds to the action of the removed material. It is accepted that the law on deformation of bonds is given. In the investigated case, arise of crack type defect represents a process of transition of prefracture zone to the area of broken bonds between lining's material surfaces. We model friction lining by a curved (annular) beam with cross section close to narrow rectangle. Assume that external contour of the lining is close to annular one. As is known, real processed surface is never absolutely smooth and always has micro or macroscopic irregularities of technological character making roughness and they have essential influence on different operational properties of triboconjunction $[5]$.

The friction lining will bend in the curvature plane under the action of contact pressure on external boundary. Refer the lining to polar system of coordinates $r\theta$ having chosen the origin at the centre of concentric circles L_θ, L with radii R_0 and R, respectively. Consider some arbitrary realization of a rough external surface of the lining. In domain occupied by friction lining, the stress tensor components $\sigma_r, \sigma_\theta, \tau_{r\theta}$ should satisfy the equations of plane theory of elasticity $[6]$. Represent the boundary of external contour of lining L' in the form

$$r = \rho(\theta), \ \rho(\theta) = R + \varepsilon H(\theta),$$

where $\varepsilon = R_{\max}/R$ is a small parameter; $R_{\max}$ is the greatest height of bulg (valley) of unevenness of external surface of the lining; $H(\theta)$ is a function independent on small parameter.

Internal concave surface of the lining is fixed, i.e. is rigidly fastened with rigid block

$$v_r = 0; \quad v_\theta = 0 \quad \text{for } r = R_0, \tag{1}$$

where v_r and v_θ are radial and tangential displacement of lining points, respectively.

Convex external surface of friction lining in the braking process is under the action of contact pressure and tangential stresses connected with contact pressure by Coulomb law. For an external surface on the contour L' we have for

$$r = \rho(\theta) \quad \sigma_n = -p(\theta); \quad \tau_{nt} = -fp(\theta), \tag{2}$$

where $p(\theta)$ is contact pressure; f is friction coefficient of friction couple.

For linear ends of lining we accept that forces acting on these ends are statically equivalent to the principal vector and principal moment equal zero. On linear ends of lining, the boundary conditions are accepted in the form

$$\theta = \pm\theta_0 \quad \int\limits_{R_0}^{R} \sigma_\theta dr = 0, \quad \int\limits_{R_0}^{R} \tau_{r\theta} dr = 0, \quad \int\limits_{R_0}^{R} \sigma_\theta \cdot r dr = 0 \tag{3}$$

Assume that prefracture zones are oriented in the direction of maximal stretching stresses arising in friction lining.

Let in friction lining there be N prefracture zones of length $2l_k$ $(k = 1, 2, ..., N)$. A the center of prefracture zone we arrange an origin of local system of coordinates $x_k O_k y_k$ whose axes x_k coincide with prefracture zones and make angle α_k, with the axis Ox $(\theta = 0)$. The sizes of prefracture zones are not known beforehand and should be determined in the course of the problem solution. Under braking, in bonds connecting the prefracture zone lips, there will arise normal $q_{y_k}(x_k)$ and tangential $q_{x_k y_k}(x_k)$ tractions under the a ction of external loads on friction lining. The values of these stresses are unknown beforehand and are to be determined.

The principal equations of the stated problem should be completed with relations connecting the opening of prefracture zones lips and the bond tractions. Without loss of generality, we can write these relations [7] in the form

$$\left(v_k^+(x_k, 0) - v_k^-(x_k, 0)\right) - i\left(u_k^+(x_k, 0) - u_k^-(x_k, 0)\right) =$$
$$= C(x_k, \sigma_k)\left[q_{y_k}(x_k) - iq_{x_k y_k}(x_1)\right] \quad (k = 1, 2, ..., N), \tag{4}$$

where the function $C(x_k, \sigma_k)$ may be considered as effective compliance of bonds depending on tension of bonds; $\sigma_k = \sqrt{q_{y_k}^2 + q_{x_k y_k}^2}$ is the modulus of the vector of bond tractions.

For predicting ultimate state of friction lining, when the crack may appear in the material, we use [7] the criterium of critical opening of prefracture zone lips

$$\left|\left(u_k^+(x_k, 0) - u_k^-(x_k, 0)\right) - i\left(v_k^+(x_k, 0) - v_k^-(x_k, 0)\right)\right| = \delta_c \quad (k = 1, 2, ..., N) \tag{5}$$

where δ_c is characteristics of resistance of friction lining material to cracking and it is defined experimentally.

In order to determine stress-strain state of friction lining with a system of prefracture zone under braking it is necessary to solve jointly the equations of plane theory of elasticity [6] under lining loading boundary conditions (1) – (3). To these boundary conditions we add conditions in bonds under braking

$$\text{for } y_k = 0, \quad |x_k| \le l_k \quad \sigma_{y_k} = q_{y_k}(x_k), \ \tau_{x_k y_k} = q_{x_k y_k}(x_k) \quad (k = 1, 2, ..., N) \tag{6}$$

and also additional complex relations (4).

The enumerated differential equations and conditions compose a closed system for determining stress and strains in friction lining in braking process.

Solution method of the problem. We look for stresses and displacements in the lining in the form of expansion in small parameter of ε. Expanding the expressions for stresses in the circle of $r = R$ in series, we find the values of stress tensor components for $r = \rho(\theta)$. Using the perturbations method, we get boundary conditions of the problem at each approximation. For the zero approximation we have:

$$\text{for } r = R \ \sigma_r^{(0)} = -p^{(0)}(\theta), \ \tau_{r\theta}^{(\theta)} = -fp^{(\theta)}(\theta) \ \text{ for } r = R_0 \ v_r^{(0)} = 0, \ v_\theta^{(0)} = 0 \quad (7)$$

$$\text{for } \theta = \pm\theta_0 \quad \int_{R_0}^{R} \sigma_\theta^{(0)} dr = 0, \quad \int_{R_0}^{R} \tau_{r\theta}^{(0)} dr = 0, \quad \int_{R_0}^{R} \sigma_\theta^{(0)} r dr = 0 \quad (8)$$

$$\text{for } |x_k| \le l_k^{(0)}, \quad \sigma_{y_k}^{(0)} = q_{y_k}^{(0)}(x_k), \ \tau_{x_k y_k}^{(0)} = q_{x_k y_k}^{(0)}(x_k) \quad (k = 1, 2, ..., N) \quad (9)$$

For the first approximation we have

$$\text{for } r = R \ \ \sigma_r^{(1)} = N - p^{(1)}(\theta), \ \ \tau_{r\theta}^{(1)} = T - fp^{(1)}(\theta) \quad (10)$$

$$\text{for } r = R_0 \ \ v_r^{(1)} = 0, \ \ v_\theta^{(1)} = 0$$

$$\text{for } \theta = \pm\theta_0 \quad \int_{R_0}^{R} \sigma_\theta^{(1)} dr = 0, \quad \int_{R_0}^{R} \tau_{r\theta}^{(1)} dr = 0, \quad \int_{R_0}^{R} \sigma_\theta^{(1)} r dr = 0 \quad (11)$$

$$\text{for } |x_k| \le l_k^{(0)}, \ \sigma_{y_k}^{(1)} = q_{y_k}^{(1)}(x_k), \quad \tau_{x_k y_k}^{(1)} = q_{x_k y_k}^{(1)} \quad (12)$$

Here

$$\begin{gathered} N = -H(\theta)\frac{\partial \sigma_r^{(0)}}{\partial r} + 2\tau_{r\theta}^{(0)}\frac{1}{R}\frac{dH(\theta)}{d\theta}, \quad \text{for } \ r = R \\ T = \left(\sigma_\theta^{(0)} - \sigma_r^{(0)}\right)\frac{1}{R}\frac{dH(\theta)}{d\theta} - H(\theta)\frac{\partial \tau_{r\theta}^{(0)}}{\partial r} \end{gathered} \quad (13)$$

Using Kolosov-Muskheleshvili formula $[6]$, at each approximation we reduce the problem to determination of two analytic functions. Complex potentials describing stress-strain state in friction lining in zero approximation is found in the form

$$\begin{gathered} \Phi^{(0)}(z) = \Phi_0^{(0)}(z) + \Phi_1^{(0)}(z) + \Phi_2^{(0)}(z), \\ \Psi^{(0)}(z) = \Psi_0^{(0)}(z) + \Psi_1^{(0)}(z) + \Psi_2^{(0)}(z) \end{gathered} \quad (14)$$

Here

$$\begin{gathered} \Phi_0^{(0)}(z) = A \ln z + d_0 + d_1 z + \frac{d_{-1}}{z} + \sum_{k=-\infty}^{\infty} a_k^0 z^k, \\ \Psi_0^{(0)}(z) = \frac{d'_{-1}}{z} + \frac{d'_{-2}}{z^2} + \frac{d'_{-3}}{z^3} + \sum_{k=-\infty}^{\infty} b_k^0 z^k, \end{gathered} \quad (15)$$

$$\Phi_1^{(0)} z = \frac{1}{2\pi}\sum_{k=1}^{N}\int_{-l_k^0}^{l_k^0}\frac{g_k^0(t)\,dt}{t-z_k},$$
$$\Psi_1^{(0)} z = \frac{1}{2\pi}\sum_{k=1}^{N} e^{-2i\alpha_k}\int_{-l_k^0}^{l_k^0}\left[\frac{\overline{g_k^0(t)}}{t-z_k} - \frac{\overline{T_k}e^{i\alpha_k}}{(t-z_k)^2} g_k^0(t)\right]dt, \tag{16}$$

$T_1 = te^{i\alpha_k} + z_k^0$; $z_k = e^{-i\alpha_k}\left(z - z_k^0\right)$; $g_k^0(x_k)$ are the desired functions characterizing the opening of prefracture zone lips in zero approximation

$$g_k^0(x_k) = \frac{2\mu}{i(1+k_0)}\frac{\partial}{\partial x_k}\left[\left(u_k^+(x_k,0) - u_k^-(x_k,0)\right) + i\left(v_k^+(x_k,0) - v_k^-(x_k,0)\right)\right], \tag{17}$$

μ is shear modulus of lining's material, $k_0 = 3 - 4\nu$; ν is Poisson's ratio.

The unknown desired functions $g_k^0(x_k)$ and complex potentials $\Phi_2^{(0)}(z)$ and $\Psi_2^{(0)}(z)$ should be determined from boundary conditions on the contour $L\,(r = R)$ and on the lips of end area cracks. Satisfying the boundary condition on external contour of the lining, after some transformations and calculation of appropriate integrals, we find

$$\Phi_2^{(0)} z = \frac{1}{2\pi}\sum_{k=1}^{N}\int_{-l_k^0}^{l_k^0}\left\{\left(\frac{1}{z\overline{T}_k - 1} + \frac{1}{2}\right)\overline{T}_k e^{i\alpha_k} g_k^0(t) + \right.$$
$$\left. + \left[\frac{T_k}{2} - \frac{z^2\overline{T}_k - 2z + T_k}{\left(z\overline{T}_k - 1\right)^2}\right] e^{-i\alpha_k}\overline{g_k^0(t)}\right\} dt, \tag{18}$$
$$\Psi_2^{(0)} z = \frac{1}{2\pi}\sum_{k=1}^{N}\int_{-l_k^0}^{l_k^0}\left\{\frac{e^{i\alpha_k}\overline{T}_k^3}{\left(z\overline{T}_k - 1\right)^2} g_k^0(t) + \right.$$
$$\left. + \left(z^2\overline{T}_k^2 + 4 - 3z\overline{T}_k + zT_k\overline{T}_k^2 - 3T_k\overline{T}_k\right)\frac{\overline{T}_k e^{-i\alpha_k}}{\left(z\overline{T}_k - 1\right)^3}\overline{g_k^0(t)}\right] dt,$$

Satisfying boundary conditions on annular boundaries of the lining by complex potentials (14), and also considering the expansion of the function $p^{(0)}(\theta)$ that is assumed to be known, we get the sysetem of linear algebraic equations with respect to a_k^0 and b_k^0:

$$2A\ln R - A + a_0^0 + \overline{a_0^0} - b_{-2}^0 R^{-2} + d_0 + \overline{d_0} - \frac{d'_{-2}}{R^2} = -(1 - if)A_0,$$
$$A(1-k_0)\ln R_0 - A + a_0^0 + k_0\overline{a_0^0} - b_{-2}^0 R_0^{-2} + d_0 - k_0\overline{d_0} = -A_0^0, \tag{19}$$
$$(1-k_0)\,a_k^0 R^k + \overline{a_{-k}^0} R^{-k} - b_{k-2}^0 R^{k-2} = A'_k$$
$$(1-k)\,a_k^0 R_0^k + k_0\overline{a_{-k}^0} R_0^{-k} - b_{k-2}^0 R_0^{k-2} = A''_k$$

Following [6] we can express the constants $A, d_0, d_1, d_{-1}, d'_{-1}, d'_{-2}, d'_{-3}$ by the three quantities $\varepsilon_*^0, \alpha^0, \beta^0$ that represent (according to terminology accepted in §45 [6]) distortion characteristics. The solution of the obtained system (19) is not difficult and is described in [6].

According to condition (8) for $\theta = \theta_0$ after integration we get three equations for determining quantities $\varepsilon_*^0, \alpha^0$ and β^0.

Satisfying boundary conditions (9) on the prefracture zone lips by functions (14), we get a system of N singular integral equation with respect to the unknown functions $g_k^0(x_k)$ $(k = 1, 2, ..., N)$:

$$\sum_{k=1}^{N} \int_{-l_k^0}^{l_k^0} \left[K_{nk}(t, x)\, g_k^0(t) + L_{nk}\overline{g_1^0(t)} \right] dt = \pi f_k^0(x), \quad |x_k| \le l_k^0. \tag{20}$$

To the singular integral equation (20) we should add an additional equalities expressing the condition of uniquenes of displacements in tracing the contour of external prefracture zone

$$\int_{-l_k^0}^{l_k^0} g_k^0(t)\, dt = 0 \ \ (k = 1, 2, ..., N). \tag{21}$$

Under additional conditions (21) , singular integral equation (20) , by means of algebraization procedure (see [8], application), is reduced to the system of $N \times M$ algebraic equations for determining $N \times M$ unknowns $g_k(t_m)$ $(k = 1, 2, ..., N;\ m = 1, 2, ..., M)$:

$$\begin{gathered} \frac{1}{M} \sum_{m=1}^{m} \sum_{k=1}^{N} l_k = \left[g_k^0(t_m) K_{nk}(l_k t_m, l_1 x_r) + \overline{g_k(t_m)} L(l_k t_m, l_k x_r) \right] = f_n^0(x_r) \\ \sum_{m=1}^{m} g_n(t_m) = 0 \quad n = 1, 2, ..., N;\ r = 1, 2, ..., M-1, \end{gathered} \tag{22}$$

where $t_m = \cos \dfrac{2m-1}{2M}\pi \quad m = 1, 2, ..., M,\ x_r = \cos \dfrac{\pi r}{M} \quad r = 1, 2, ..., M-1.$

The right hand sides of algebraic system (22) contain the unknown values of normal $q_{y_k}^{(0)}$ and tangetial $q_{x_k y_k}^{(0)}$ tractions at nodal points of appropriate prefracture zones. Using the obtained solution, we get

$$g_k^0(x_k) = \frac{2\mu}{i(1+k_0)} \frac{d}{dx_k} \left[C\left(x_k, \sigma_k^0\right) \left(q_{y_k}^0(x_k) - i q_{x_k y_k}^0(x_k) \right) \right] \quad (k = 1, 2, ..., N) \tag{23}$$

where x_k is an affix of points of the lips of the $k-$th prefracture zone. These complex equations help to determine tractions $q_{y_k}^{(0)}$ and $q_{x_1 y_k}^{(0)}$ in the bonds between the lips of appropricte prefracture zones.

For constructing missing algebraic equations for finding approximate values of tractions $q_{y_k}^0(t_m)$ and $q_{x_k y_k}^0(t_m)$ $(m = 1, 2, ..., M)$ we require fulfilment of conditions (23) at the nodal points t_m, in prefracture zone. Moreover, the finite differences method is used. As a result, we get a complex algebraic system of $N \times M$ equations for determining the values $q_{y_k}^0(t_m)$, $q_{x_k y_k}^0(t_m)$ $(k = 1, 2, ..., N,\ m = 1, 2, ..., M)$ at the nodal points of prefracture zones.

For closeness of the obtained algebraic equations, we need $2 \times N$ complex equations determining the sizes of prefracture zones in zero approximation. Since the solution of the system of singular integral equations (20) is sought in the class of everywhere bounded functions (stresses) then to system (22) we should add the condition of stress boundedness at the ends of prefracture areas $x_1 = \pm l_k^0$ $(k = 1, 2, ..., N)$:

$$\sum_{m=1}^{M} (-1)^{M+m} q_k^0 (t_m) \, tg \frac{2m-1}{4M} \pi = 0, \; \sum_{m=1}^{M} (-1)^{M} q_k^0 (t_m) \, ctg \frac{2m-1}{4M} \pi = 0. \quad (24)$$

Because of unknown sizes of prefracture zones, even for linear elastic bonds, the obtained algebraic system (22)-(24) is non-linear. For solving the combined algebraic system we used the successive approximations method.

In the case of nonlinear law of deformation of bonds for determining tractions at prefracture zones an algorithm similar to the method of elastic solutions of A.A. Ilyeshin [9] is also used.

After soving the combined algebraic system we pass to construction of the solution in the first approximation. For $r = R$ we find the functions N and T.

By means of Kolosov Muskheleshvili formula [6] , we can write the boundary conditions of the problem in the first approximation in the form of a boundary value problem for determining complex potentials $\Phi^{(1)}(z)$ and $\Psi^{(1)}(z)$. The functions $\Phi^{(1)}(z)$ and $\Psi^{(1)}(z)$ are sought in the form similar to $(14) - (16)$ with obvious changes. The further course of solution of the problem is the same as in zero approximation. After determining desired quantities for predicting critical value of contact pressure in braking mechanism under which crack's growth happens, criterium (5) is used.

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