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ON ASYMPTOTIC ESTIMATION OF APPROXIMATION OF FUNCTIONS BY GENERAL MELLIN TYPE SINGULAR INTEGRALS

Abstract

In 1972 Kolbe and Nessel used the Mellin transformation method and determined the classes of saturation of a Mellin type singular integral in metric of the space $L^p(0,\infty)$, $(p \geq 1)$. Futher, R. G. Mamedov and his followers studied the classes and orders of saturation of Mellin convolution type m-singular integrals [6]. The present paper is devoted to investigation of asymptotic order of approximation of a family of general integral operators to the functions whose differential properties are characterized by M-derivatives. Furthermore, it is shown construction of new linear aggregates on the basis of the given linear operators of Mellin convolution type that represent a higher order of approximation of function.

Notice that the obtained results contain appropriate results of Kolbe, Nessel and R.G. Mamedov as special cases.

Asymptotic equalities for approximation of functions by linear positive operators were obtained in many papers [1,2,3,4,5] and others.

Let $f(r)$ be a real function determined and measurable on $R^+ = (0,+\infty)$. Assume [2,6]

$$\|f\|_{L^p(R^+)} = \begin{cases} \left(\int\limits_0^\infty |f(r)|^p \frac{dr}{r}\right)^{1/p} & \text{for} \quad 1 \leq p < \infty \\ ess \sup\limits_{0<r<\infty} |f(r)| & \text{for} \quad p = \infty \end{cases}$$

By $L^p(R^+)$ we denote a space of functions $f(r)$ for which $\|f\|_{L^p(R^+)}$ is finite. Let $C(R^+)$ be a space of bounded and continuous on R^+ functions $f(r)$ for which [6]

$$\lim_{p\to 1} \|f(r\rho) - f(r)\|_{C(R^+)} = 0,$$

with the norm

$$\|f\|_{C(R^+)} = \sup_{0<r<\infty} |f(r)|.$$

In the sequel, under $X(R^+)$ we'll understand the space $L^p(R^+)$ $(1 \leq p < \infty)$ or $C(R^+)$ and

$$L^{*_1}(R^+) = \left\{ f \in L(R^+) \left| \int\limits_0^\infty f(r) \frac{dr}{r} = 1 \right. \right\},$$

$$L^{*_2}(R^+) = \left\{ f/f(r) \geq 0 \text{ on } R^+ \text{ and } \int\limits_0^\infty f(r) \frac{dr}{r} = 1 \right\},$$

$$L^{*3}\left(R^{+}\right)=\left\{f\in L^{*2}\left(R^{+}\right)\left|f\left(r\right)=f\left(r^{-1}\right)\right.\right\}.$$

Let $f\in X\left(R^{+}\right)$ and $K_{\lambda}\left(r\right)\in L^{*1}\left(R^{+}\right)$, then the expression

$$A_{\lambda}^{[N]}\left(f;r\right)=\int\limits_{0}^{\infty}\left[\sum_{l=1}^{N}\alpha_{l}\left(\lambda\right)f\left(\frac{r}{\rho^{b_{l}(\lambda)}}\right)\right]K_{\lambda}\left(\rho\right)\frac{d\rho}{\rho} \tag{1}$$

is said to be a general N-singular integral of Mellin type [8], where λ changes on some totally disconnected set D,λ_0 is a limit point of this set and $\sum\limits_{l=1}^{N}\alpha_{l}\left(\lambda\right)=1$ for each $\lambda\in D$, $\sum\limits_{l=1}^{N}\left|\alpha_{l}\left(\lambda\right)\right|\le C<+\infty$, where C is independent of λ, $b_{l}\left(\lambda\right)\ge 0\ \left(l=\overline{1-N};\lambda\in D\right)$ and $\sup\limits_{l,\lambda}\left\{b_{l}\left(\lambda\right)\right\}=b<+\infty$, moreover for natural n

$$\left|\sum_{l=1}^{N}\alpha_{l}\left(\lambda\right)b_{l}^{n}\left(\lambda\right)\right|\ge q>0.$$

Notice that for $K_{\lambda}\left(\rho\right)=\lambda K\left(\rho^{\lambda}\right)$, (1) is said to be a general Mellin type N-singular integral with Fejer type kernel [8] and is denoted by

$$B_{\lambda}^{[N]}\left(f;r\right)=\lambda\int\limits_{0}^{\infty}\left[\sum_{l=1}^{N}\alpha_{l}\left(\lambda\right)f\left(\frac{r}{\rho^{b_{l}(\lambda)}}\right)\right]K\left(\rho^{\lambda}\right)\frac{d\rho}{\rho} \tag{2}$$

In the case when

$$a_{l}\left(\lambda\right)=\begin{cases}\left(-1\right)^{l-1}\begin{pmatrix}N\\ l\end{pmatrix},1\le l\le N,\\ 0,\ l>N\end{cases}$$

$$b_{l}\left(\lambda\right)=\begin{cases}l,\ 1\le l\le N,\\ 0,\ l>N\end{cases}$$

from (1) and (2) we get

$$T_{\lambda}^{[N]}\left(f;r\right)=\int\limits_{0}^{\infty}\left[\sum_{l=1}^{N}\left(-1\right)^{l-1}\begin{pmatrix}N\\ l\end{pmatrix}f\left(\frac{r}{\rho^{l}}\right)\right]K_{\lambda}\left(\rho\right)\frac{d\rho}{\rho}, \tag{3}$$

$$\Phi_{\lambda}^{[N]}\left(f;r\right)=\lambda\int\limits_{0}^{\infty}\left[\sum_{l=1}^{N}\left(-1\right)^{l-1}\begin{pmatrix}N\\ l\end{pmatrix}f\left(\frac{r}{\rho^{l}}\right)\right]K_{\lambda}\left(\rho^{\lambda}\right)\frac{d\rho}{\rho} \tag{4}$$

(3) is called Mellin's N -singular integral, (4) is said to be Mellin's N singular integral with Fejer type kernel [6,10].

Introduce the denotation

$$\left(\Delta_{\rho}^{*m}f\right)_{M}\left(r\right)=\left(\Delta_{\rho}^{m}f\right)_{M}\left(r\right)+\left(\Delta_{\rho-1}^{m}f\right)_{M}\left(r\right) \tag{5}$$

where

$$\left(\Delta_\rho^m f\right)_M(r) = \sum_{l=0}^{m} (-1)^{m-l} \begin{pmatrix} m \\ l \end{pmatrix} f\left(\frac{r}{\rho^l}\right)$$

is Mellin's m-th finite difference of the function f at the point r [6].

The expression $E^{(1)} f(r) = -r\frac{df(r)}{dr}$ is called M-derivative of the function f at the point r [6].

Let

$$E^{(l)} f(r) = \begin{cases} f(r) & \text{for } l = 0, \\ E^{(1)}\left(E^{(l-1)} f\right)(r) & \text{for } l \geq 1. \end{cases}$$

$E^{(l)} f(r)$ is called l-order M derivative of the function f at the point r [6].

Definition. *1) Let the function $f(r)$ be determined in the vicinity of the point r_0. If there exists the limit*

$$\lim_{\rho\to 1} \frac{\left(\Delta_\rho^{*m} f\right)_M(r_0)}{2\ln^m \rho},$$

then it is called Riemann's general M-derivative of order m of the function $f(r)$ at the point r_0 and is denoted by $E^{\{m\}} f(r_0)$.

2) If for the function $f \in X(R^+)$ there exists a function $g \in X(R^+)$ such that it holds the equality

$$\lim_{\rho\to 1} \left\| \frac{\left(\Delta_\rho^{*m} f\right)_M(r_0)}{2\ln^m \rho} - g(r) \right\|_{X(R^+)} = 0,$$

then $g(r)$ is said to be Riemann's strong general M-derivative of order m of the function f in the space $X(R^+)$ and is denoted by $E_{X(R^+)}^{\{m\}} f(r)$.

The present paper studies asymptotic order of approximation of a family of integral operators (1)-(4) to the functions whose differential properties are characterized by M-derivatives.

Theorem 1. *Let $f \in L^\infty(R^+)$ have $2n$-th a order finite M-derivative $E^{(2n)} f(r)$ at the point $r = r_0$, $K_\lambda(\rho) \in L^{*3}(R^+)$ and $\tau_\lambda^{[2n+a]} = \int\limits_1^\infty \ln^{2n+a} \rho K_\lambda(\rho) \frac{d\rho}{\rho} < +\infty$. Then if the condition*

$$\lim_{\lambda\to\lambda_0} \left[\tau_\lambda^{[2n+a]} / \tau_\lambda^{[2n]}\right] = 0 \tag{6}$$

is fulfilled for one value of $\alpha > 0$, then it holds the relation

$$\lim_{\lambda\to\lambda_0} \frac{1}{\tau_\lambda^{[2n]} \left\{\sum\limits_{l=1}^{N} a_l(\lambda) b_l^{2n}(\lambda)\right\}} \left\{ A_\lambda^{[N]}(f; r_0) - f(r_0) - 2\sum_{l=1}^{N} a_l(\lambda) \times \right.$$

$$\left. \times \sum_{k=1}^{n-1} \frac{b_l^{2k}(\lambda)}{(2k)!} \tau_\lambda^{[2k]} E^{(2k)} f(r_0) \right\} = \frac{2}{(2n)!} E^{(2n)} f(r_0) \tag{7}$$

Proof. Since

$$f\left(\frac{r_0}{\rho^{b_l(\lambda)}}\right)=f(r_0)+\sum_{k=1}^{2n-2}\frac{E^{(k)}f(r_0)}{k!}b_l^k(\lambda)\ln^k\rho+$$

$$+\frac{E^{(k)}f(r_0)+\alpha_1(b_l(\lambda)\ln\rho)}{(2n)!}b_l^{2n}(\lambda)\ln^{2n}\rho$$

and

$$f\left(r_0\rho^{b_1(\lambda)}\right)=f(r_0)+\sum_{k=1}^{2n-2}\frac{(-1)^k E^{(k)}f(r_0)}{k!}b_l^k(\lambda)\ln^k\rho+$$

$$+\frac{E^{(2n)}f(r_0)+\alpha_2\left(b_l(\lambda)\ln\rho^{-1}\right)}{(2n)!}b_l^{2n}(\lambda)\ln^{2n}\rho \tag{8}$$

then we have

$$f\left(\frac{r_0}{\rho^{b_l(\lambda)}}\right)+f\left(r_0\rho^{b_1(\lambda)}\right)=2f(r_0)+2\sum_{k=1}^{n}\frac{E^{(2k)}f(r_0)}{(2k)!}b_l^{2k}(\lambda)\ln^{2k}\rho+$$

$$+\frac{\alpha_1(b_l(\lambda)\ln\rho)+\alpha_2\left(b_l(\lambda)\ln\rho^{-1}\right)}{(2n)!}b_l^{2n}(\lambda)\ln^{2n}\rho\quad\left(l=\overline{1,N}\right)$$

$$A_\lambda^{[N]}(f;r)-f(r_0)-2\left\{\sum_{k=1}^{n}a_l(\lambda)f\sum_{k=1}^{n-1}\frac{E^{(2k)}f(r_0)}{(2k)!}\right\}b_l^{2k}(\lambda)\int_1^\infty\ln^{2k}\rho+$$

$$+\frac{\alpha_1(b_l(\lambda)\ln\rho)+\alpha_2\left(b_l(\lambda)\ln\rho^{-1}\right)}{(2n)!}b_l^{2n}(\lambda)\ln^{2n}\rho\quad\left(l=\overline{1,N}\right),$$

where $\alpha_1(b_l(\lambda)\ln\rho)$; $\alpha_2\left(b_l(\lambda)\ln\rho^{-1}\right)$ tends to zero as $\rho\to 1+$ uniformly with respect to l and λ for some constant M_1; $\left|\alpha_1(b_l(\lambda)\ln\rho)+\alpha_2\left(b_l(\lambda)\ln\rho^{-1}\right)\right|\le M_1\,(p\in R)$

Consequently, by (1) we find

$$A_\lambda^{[N]}(f;r)-f(r_0)-2\left\{\sum_{l=1}^{N}a_l(\lambda)\cdot\sum_{k=1}^{n-1}\frac{E^{(2k)}f(r_0)}{(2k)!}\right\}b_l^{2k}(\lambda)\int_1^\infty K_\lambda(\rho)\frac{d\rho}{\rho}=$$

$$=2\left\{\sum_{l=1}^{N}a_l(\lambda)b_l^{2n}(\lambda)\right\}\frac{E^{(2n)}f(r_0)}{(2k)!}\int_1^\infty\ln^{2n}\rho\cdot K_\lambda(\rho)\frac{d\rho}{\rho}+R_\lambda, \tag{9}$$

where

$$R_\lambda=\sum_{l=1}^{N}a_l(\lambda)\int_1^\infty\left[f\left(\frac{r_0}{\rho^{b_l(\lambda)}}\right)+f\left(r_0\rho^{b_l(\lambda)}\right)-\right.$$

$$\left.-2f(r_0)-\sum_{k=1}^{n-1}\frac{E^{(2k)}f(r_0)}{(2k)!}b_l^{2k}(\lambda)\ln^{2k}\rho\right]K_\lambda(\rho)\frac{d\rho}{\rho}.$$

Hence we have

$$\frac{1}{\tau_\lambda^{[2n]}\left\{\sum_{l=1}^{N}a_l(\lambda)b_l^{2n}(\lambda)\right\}}\times$$

$$\times\left[A_{\lambda}^{[N]}(f;r)-f(r_0)-2\sum_{l=1}^{N}a_l(\lambda)\cdot\sum_{k=1}^{n-1}\frac{E^{(2k)}f(r_0)}{(2k)!}b_l^{2k}(\lambda)\tau_{\lambda}^{[2n]}\right]=$$

$$=\frac{2}{(2n)!}E^{(2n)}f(r_0)+\frac{R_{\lambda}}{\left[\sum\limits_{l=1}^{N}a_l(\lambda)b_l^{2n}(\lambda)\right]\tau_{\lambda}^{[2n]}} \qquad (10)$$

Considering (6), passing to limit as $\lambda\to\lambda_0$, by

$$R_{\lambda}=0\left\{\left[\sum_{l=1}^{N}a_l(\lambda)b_l^{2k}(\lambda)\right]\tau_{\lambda}^{[2n]}\right\},\qquad \lambda\to\lambda_0$$

we complete the proof of the theorem.

Corollary. *Let $f\in L^{\infty}(R^{+})$ have a finite M- derivative of $2n$-th order $E^{(2n)}f(r)$ at the point $r=r_0$, $K_{\lambda}(\rho)\in L^{*_3}(R^{+})$. Then it holds the relation*

$$\lim_{\lambda\to\infty}\frac{\lambda^{2n}}{\tau^{[2n]}\sum\limits_{l=1}^{N}a_l(\lambda)b_l^{2n}(\lambda)}\left[B_{\lambda}^{[N]}(f;r_0)-f(r_0)-\sum_{l=1}^{N}a_l(\lambda)\times\right.$$

$$\left.\times\sum_{k=1}^{n-1}\frac{b_l^{2k}(\lambda)}{(2k)!}\frac{\tau^{[2k]}}{\lambda^{2k}}E^{(2k)}f(r_0)\right]=\frac{2}{(2n)!}E^{(2n)}f(r_0), \qquad (11)$$

in particular,

$$\lim_{\lambda\to\infty}\frac{\lambda^{2n}}{\tau^{[2n]}\sum\limits_{l=1}^{N}(-1)^{l-1}\begin{pmatrix}N\\ l\end{pmatrix}l^{2n}}\left[\Phi_{\lambda}^{[N]}(f;r_0)-f(r_0)-2\sum_{k=N}^{n-1}a_l(\lambda)\times\right.$$

$$\left.\times\left(\sum_{l=1}^{2N}(-1)^{l-1}\begin{pmatrix}2N\\ l\end{pmatrix}l^{2k}\right)\cdot\frac{\tau^{[2k]}}{\lambda^{2k}}\frac{E^{(2k)}f(r_0)}{(2k)!}\right]=\frac{2}{(2n)!}E^{(2n)}f(r_0), \qquad (12)$$

where $\tau^{[2n]}<+\infty$ and $2N\le n$.

Theorem 2. *Let $f(r)\in L^{\infty}(R^{+})$ and $K_{\lambda}(\rho)\in L^{*_3}(R^{+})$ satisfy the condition*

$$\lim_{\lambda\to\lambda_0}\left[\tau_{\lambda}^{[N+\alpha]}/\tau_{\lambda}^{[N]}\right]=0 \qquad (13)$$

even if for one value of $\alpha>0$, where $\tau_{\lambda}^{[N+\alpha]}=\int\limits_{1}^{\infty}\ln^{N+\alpha}\rho K_{\lambda}(\rho)\frac{d\rho}{\rho}<+\infty$. It there exists Riemann's general finite M-derivative $E^{\{N\}}f(r)$ at the point $r=r_0$, then the asymptotic equality

$$T_{\lambda}^{[N]}(f;r_0)-f(r_0)=(-1)^{N+1}E^{\{N\}}f(r_0)\tau_{\lambda}^{[N]}+0\left(\tau_{\lambda}^{[N]}\right) \qquad (14)$$

is true as $\lambda\to\lambda_0$

Proof. Since

$$\frac{1}{\tau_{\lambda}^{[N]}}\left[T_{\lambda}^{[N]}(f;r_0)-f(r_0)\right]-(-1)^{N+1}E^{\{N\}}f(r_0)=$$

$$=\frac{2\cdot(-1)^{N+1}}{\tau_\lambda^{[N]}}\left(\int\limits_1^{1+\delta}+\int\limits_{1+\delta}^{\infty}\right)\left[\frac{\left(\Delta_\rho^{*N}f\right)_M(r_0)}{2\ln^N\rho}-E^{\{N\}}f(r_0)\right]\times$$

$$\times\ln\rho K_\lambda(\rho)\frac{d\rho}{\rho}=L_1+L_2 \tag{15}$$

for any, then from $\delta>0$,

$$\left(\Delta_\rho^{*N}f\right)_M(r_0)=2\ln^N\rho E^{\{N\}}f(r_0)+0\left(\ln^N\rho\right)$$

as $\rho\to1+0$ it follows

$$L_1=0\,(1) \tag{16}$$

for $\lambda\to\lambda_0$.

Furthermore, for all $\delta>0$

$$|L_2|\le2\left\{\left(2^N-1\right)\|f\|_{L^\infty(R^+)}+|f(r_0)|\ln^{-N}(1+\delta)+\left|E^{\{N\}}f(r_0)\right|\right\}\times$$

$$\times\frac{1}{\tau_\lambda^{[N]}}\int\limits_{1+\delta}^{\infty}\ln^N\rho\cdot K_\lambda(\rho)\frac{d\rho}{\rho}$$

Therefore, by (13) we find

$$L_2=0\,(1) \tag{17}$$

as $\lambda\to\lambda_0$.

(14) follows from (15), (16) and (17) . The theorem is proved.

Corollary. *Let $f(r)\in L^\infty(R^+)$ and $\tau_\lambda^{[N]}=\int\limits_1^\infty\ln^N\rho K(\rho)\frac{d\rho}{\rho}<+\infty\quad(N>0)$. If Riemann's general finite M-derivative $E^{\{N\}}f(r_0)$ is at the point $r=r_0$, then the asymptotic equality*

$$\lim_{\lambda\to\infty}\frac{\lambda^N}{\tau^{[N]}}\left[\Phi_\lambda^{[N]}(f;r_0)-f(r_0)\right]=(-1)^{N+1}E^{\{N\}}f(r_0). \tag{18}$$

is true.

Theorem 3. *a) Let $f(r)\in L^P(R^+)$ $(1\le p<\infty)$ and $K_\lambda(\rho)\in L(R^+)$, then*

$$\left\|A_\lambda^{[N]}(f;r)\right\|_{L^p(R^+)}\le C\|f\|_{L^p(R^+)}\|K_\lambda\|_{L(R^+)} \tag{19}$$

where C is some positive constant.

b) Let $f(r)\in X(R^+)$, $K_\lambda(\rho)\in L^{*_1}(R^+)$ and

$$\lim_{\lambda\to\lambda_0}\left(\int\limits_0^{1+\delta}+\int\limits_{1+\delta}^{\infty}\right)|K_\lambda(\rho)|\frac{d\rho}{\rho}=0$$

for any $\delta\in(0,1)$, then

$$\lim_{\lambda\to\lambda_0}\left\|A_\lambda^{[N]}(f;r)-f(r)\right\|_{X(R^+)}=0. \tag{20}$$

Theorem 4. *Let $f(r) \in X(R^+)$, $K_\lambda(\rho) \in L^{*_3}(R^+)$ and satisfy condition (13) of theorem 2. If the function $f(r)$ has Riemann's strong general M-derivative of order N in the space $X(R^+)$, then*

$$\lim_{\lambda\to\lambda_0}\left\|\frac{1}{\tau_\lambda^{[N]}}\left[T_\lambda^{[N]}(f;r)-f(r)\right]-(-1)^{N+1}E_{X(R^+)}^{\{N\}}f(r)\right\|_{X(R^+)}=0. \qquad (21)$$

Corollary. *Let $f(r) \in X(R^+)$ and $K_\lambda(\rho) \in L^*(R)$. If the function has Riemann's strong general M-derivative of order N in the space $X(R^+)$, then*

$$\lim_{\lambda\to\infty}\left\|\frac{\lambda^N}{\tau_\lambda^{[N]}}\left[\Phi_\lambda^{[N]}(f;r)-f(r)\right]-(-1)^{N+1}E_{X(R^+)}^{\{N\}}\right\|_{X(R^+)}=0. \qquad (22)$$

Notice that some conditions in the formulated theorems may be weakened.

Let $K_\lambda(\rho)$ satisfy the conditions:

1^0. $K_\lambda(\rho)=K_\lambda(\rho^{-1})$ on R^+

2^0. $\int\limits_0^\infty K_\lambda(\rho)\frac{d\rho}{\rho}=1$

3^0. For $K_\lambda(\rho)$ there exists a non-negative majorant $K_\lambda(\rho)$, $|K_\lambda(\rho)|\le K_\lambda^*(\rho)$ and $K_\lambda^*(\rho)\in L(R^+)$.

Assume

$$\tau_\lambda^{*[m]}=\int\limits_1^\infty \ln^m\rho K_\lambda^*(\rho)\frac{d\rho}{\rho}.$$

Theorem 5. *Let $f \in L^\infty(R^+)$ have a finite M-derivative of $2n$-th order $E^{(2n)}f(r)$ at the point $r=r_0$ and the function $K_\lambda(\rho)$ satisfy the conditions 1^0-3^0.*

If

$$\lim_{\lambda\to\lambda_0}\left[\tau_\lambda^{*[2n+\alpha]}/\rho_\lambda^{[2n]}\right]=0 \qquad (23)$$

even if for one $\alpha>0$, relations (6) hold, where $\tau_\lambda^{[2n]}\neq 0$ and $\tau_\lambda^{*[2n+\alpha]}<+\infty$.

Theorem 6. *Let $f \in L^\infty(R^+)$ have Riemann's general finite M-derivative of N-th order $E^{\{N\}}f(r)$ at the point $r=r_0$. If*

$$\lim_{\lambda\to\lambda_0}\left[\tau_\lambda^{*[N+\alpha]}/\tau_\lambda^{[N]}\right]=0 \qquad (24)$$

even if for one value of $\alpha>0$, it holds relation (14), where $\tau_\lambda^{[N]}\neq 0$ and $\tau_\lambda^{*[N+\alpha]}<+\infty$.

Theorem 7. *Let $f(r)$ have Riemann's strong general M-derivative of order N in the space $X(R^+)$ and the function $K_\lambda(\rho)$ satisfy conditions 1^0-3^0. Then condition (24) holds and relation (21) is true.*

Finally notice that the obtained theorems are applicable to operators (1) ((2)-(4)) generated by Mellin type Picard kernels, Mellin type Gauss-Weierstrass kernels, Mellin type Jacksoin's kernels and others.

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