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ERGODIC DISTRIBUTIONS FOR CERTAIN MODIFICATIONS OF THE $M/M/n/m$ QUEUEING SYSTEM

Abstract

Modifications of the M/M/n/m and M/M/n/∞ queueing systems are examined. These modifications consist in an opportunity of transition in a state of inaccessibility and blocking of an input flow. Ergodic distributions of number of customers in queueing systems are found.

1. Introduction. We study queue with n servers for which the number of places in queue cannot exceed number m. Let customers arrive to system on one. We assume also, that time intervals between the moments of arrival of customers and service times are independent random variables exponentially distributed with parameter λ and μ respectively.

The single-server queueing systems of type M/G/1/m with limited queue and regenerating level of input flow are examined in works [1–3]. If the length of queue reaches number m then arrival of customers in system is blocked and renews only when the length of queue decreases up to some threshold level $l \in [0,\, m-1]$. Algorithms for definition of ergodic distribution of length of queue in such systems which realization demands enough bulky calculations are represented in works [2, 3].

Let's assume, that through time intervals exponentially distributed with parameter ν the system can pass in a state of inaccessibility when any server has no an opportunity to serve the customers, new customers cannot arrive to system, but all the customers who are being in system (on service and in queue), remain and wait for end of a state of inaccessibility. Duration of a state of inaccessibility is exponentially distributed with parameter γ.

Below for queue M/M/n/m with possible states of inaccessibility and blocking of an input flow, for queue M/M/n/m with blocking of an input flow and for queue M/M/n/∞ with the opportunity of transition in a state of inaccessibility we deduce universal formulas for ergodic distribution of number of customers in system. Universality of the received formulas allows us to receive directly such ergodic distribution for any number of servers n, set maximal length of queue m and a threshold level of blocking $l \in [0,\, m-1]$.

2. Queueing system with possible states of inaccessibility and blocking of an input flow. We introduce standard numbering of states of queueing system: state s_k $\left(k=\overline{0,\ n+m}\right)$ means presence in system of k customers (blocking of an input flow is not carried out, the system is accessible). By s_{k1} $\left(k=\overline{0,\ n+m}\right)$ we shall separately designate the states corresponding to the period of inaccessibility of system which comes during stay of system in a state s_k. We shall denote the states corresponding to the period of blocking of an input flow till the moment of reduction of length of queue up to number l as s_{k0} $\left(k=\overline{n+l+1,\ n+m-1}\right)$, and as s_{k01} $\left(k=\overline{n+l+1,\ n+m-1}\right)$ we shall designate states of inaccessibility to which the system passes directly from a corresponding state s_{k0}.

Let us denote by $p_k(t)$, $p_{k1}(t)$, $p_{k0}(t)$, $p_{k01}(t)$ probability of stay of system at time t in a state s_k, s_{k1}, s_{k0} and s_{k01} respectively. Evidently (see, for example, [4, p. 69; 5, p. 61]), that following limits exist

$$p_k=\lim_{t\to\infty}p_k(t),\quad p_{k1}=\lim_{t\to\infty}p_{k1}(t)\qquad \left(k=\overline{0,\ n+m}\ \right);$$

$$p_{k0}=\lim_{t\to\infty}p_{k0}(t),\quad p_{k01}=\lim_{t\to\infty}p_{k01}(t)\qquad \left(k=\overline{n+l+1,\ n+m-1}\ \right).$$

Using graph of states of system, we shall write the equations for finding of steady-state probabilities p_k, p_{k1}, p_{k0}, p_{k01}:

$$\begin{gathered}
\mu p_1+\gamma p_{01}-(\lambda+\nu)p_0=0;\quad \lambda p_{k-1}+(k+1)\mu p_{k+1}+\\
+\gamma p_{k1}-(\lambda+k\mu+\nu)p_k=0\quad \left(k=\overline{1,\ n-1}\ \right);\\
\lambda p_{k-1}+n\mu p_{k+1}+\gamma p_{k1}-(\lambda+n\mu+\nu)p_k=0\quad \left(k=\overline{n,\ n+l-1}\ \right);\\
\lambda p_{n+l-1}+n\mu(p_{n+l+1}+p_{n+l+1,0})+\gamma p_{n+l,1}-(\lambda+n\mu+\nu)p_{n+l}=0;\\
\lambda p_{k-1}+n\mu p_{k+1}+\gamma p_{k1}-(\lambda+n\mu+\nu)p_k=0\quad \left(k=\overline{n+l+1,\ n+m-2}\ \right);\\
\lambda p_{n+m-2}+\gamma p_{n+m-1,1}-(\lambda+n\mu+\nu)p_{n+m-1}=0;\\
\lambda p_{n+m-1}+\gamma p_{n+m,1}-(n\mu+\nu)p_{n+m}=0;\\
\nu p_k-\gamma p_{k1}=0\quad \left(k=\overline{0,\ n+m}\ \right);\\
\nu p_{k0}-\gamma p_{k01}=0\quad \left(k=\overline{n+l+1,\ n+m-1}\ \right);\\
n\mu p_{k-1,0}+\gamma p_{k01}-(n\mu+\nu)p_{k0}=0\quad \left(k=\overline{n+l+1,\ n+m-2}\ \right);\\
\gamma p_{n+m-1,0,1}+n\mu p_{n+m}-(n\mu+\nu)p_{n+m-1,0}=0.
\end{gathered}\tag{1}$$

By means of the equations (1) we can express at once probabilities p_{k0}, p_{k1} and p_{k01} by p_k:

$$\begin{gathered}
p_{k0}=p_{n+m}\quad \left(k=\overline{n+l+1,\ n+m-1}\ \right);\\
p_{k1}=\beta p_k\quad \left(k=\overline{0,\ n+m}\ \right);\\
p_{k01}=\beta p_{k0}=\beta p_{n+m}\quad \left(k=\overline{n+l+1,\ n+m-1}\ \right),
\end{gathered}\tag{2}$$

where $\beta=\nu/\gamma$. Then a normalization condition together with which we should solve system (1), we can write in the form of

$$(1+\beta)\sum_{k=0}^{n+m-1}p_k+(\beta+m-l)p_{n+m}=1.\tag{3}$$

After introduction of designations

$$u_k=k\mu p_k-\lambda p_{k-1}\ \left(k=\overline{1,\ n}\ \right);\quad u_k=n\mu p_k-\lambda p_{k-1}\ \left(k=\overline{n+1,\ n+m}\ \right)$$

the system (1) is reduced to a kind convenient for the further solving

$$u_1 = 0; \quad u_{k+1} - u_k = 0 \quad \left(k = \overline{1,\ n+l-1}\,;\ k = \overline{n+l+1,\ n+m-2}\right);$$
$$u_{n+l+1} - u_{n+l} + n\mu p_{n+m} = 0; \quad u_{n+m-1} + \lambda p_{n+m-1} = 0; \quad u_{n+m} = 0.$$

So,

$$u_k = 0 \qquad \left(k = \overline{1,\ n+l}\right);$$
$$u_k = -n\mu p_{n+m} \quad \left(k = \overline{n+l+1,\ n+m-1}\right); \qquad u_{n+m} = 0.$$

From here we consistently find

$$p_k = \frac{\alpha^k}{k!}\,p_0 \quad \left(k = \overline{1,\ n}\right); \quad p_{n+k} = \frac{\alpha^n}{n!}\,\eta^k p_0 \quad \left(k = \overline{1,\ l}\right); \tag{4}$$

$$p_{n+k} = \frac{\alpha^n}{n!}\,\eta^k p_0 - \sum_{j=0}^{k-l-1} \eta^j p_{n+m} \quad \left(k = \overline{l+1,\ m-1}\right); \tag{5}$$

$$p_{n+m} = \frac{\alpha^n}{n!}\,\eta^m p_0 - \sum_{j=1}^{m-l-1} \eta^j p_{n+m}, \tag{6}$$

where $\alpha = \lambda/\mu; \quad \eta = \alpha/n.$

By means of the equation (6) we can express p_{n+m} by p_0. Then in case of $\eta \neq 1$ we receive

$$p_{n+m} = \frac{\alpha^n}{n!}\,\frac{\eta^m}{\sum\limits_{j=0}^{m-l-1} \eta^j}\,p_0 = \frac{\alpha^n}{n!}\,\frac{(1-\eta)\eta^m}{1-\eta^{m-l}}\,p_0. \tag{7}$$

Now we can write the equation (5) in the form of

$$\begin{aligned} p_{n+k} &= \frac{\alpha^n}{n!}\,\eta^k p_0 - \frac{\alpha^n}{n!}\,\frac{(1-\eta)\eta^m}{1-\eta^{m-l}} \sum_{j=0}^{k-l-1} \eta^j p_0 = \\ &= \frac{\alpha^n}{n!}\,\frac{\eta^k - \eta^m}{1-\eta^{m-l}}\,p_0, \qquad \left(k = \overline{l+1,\ m-1}\right); \qquad \eta \neq 1. \end{aligned} \tag{8}$$

So, in view of relations (2) all probabilities are expressed by p_0. We can find p_0 by means of the normalization condition (3):

$$\begin{aligned} \frac{1}{p_0} &= (1+\beta)\left(\sum_{k=0}^{n} \frac{\alpha^k}{k!} + \frac{\alpha^n}{n!}\left(\sum_{k=1}^{l} \eta^k + \sum_{k=l+1}^{m-1} \frac{\eta^k - \eta^m}{1-\eta^{m-l}}\right)\right) + \\ &+ \frac{\alpha^n}{n!}\,\frac{(\beta+m-l)(1-\eta)\eta^m}{1-\eta^{m-l}} = (1+\beta)\sum_{k=0}^{n} \frac{\alpha^k}{k!} + \\ &+ (1+\beta)\frac{\alpha^n}{n!}\left(\frac{\eta(1-\eta^l)}{1-\eta} + \frac{\eta^{l+1}(1-\eta^{m-l-1})}{(1-\eta)(1-\eta^{m-l})}\right) + \\ &+ \frac{\alpha^n}{n!}\,\frac{(\beta+m-l)(1-\eta)\eta^m - (1+\beta)(m-l-1)\eta^m}{1-\eta^{m-l}}, \qquad \eta \neq 1. \end{aligned}$$

From here

$$p_0 = \frac{1-\eta^{m-l}}{A_n(\alpha,\ \eta)+\beta B_n(\alpha,\ \eta)}, \qquad \eta \neq 1, \tag{9}$$

$$A_n(\alpha,\ \eta) = (1-\eta^{m-l})\sum_{k=0}^{n}\frac{\alpha^k}{k!} + \frac{\alpha^n}{n!}\left(\frac{\eta-\eta^{m-l+1}}{1-\eta} - (m-l)\eta^{m+1}\right);$$

$$B_n(\alpha,\ \eta) = (1-\eta^{m-l})\sum_{k=0}^{n}\frac{\alpha^k}{k!} + \frac{\alpha^n}{n!}\left(\frac{\eta-\eta^{m-l+1}}{1-\eta} - (m-l-1+\eta)\eta^{m}\right).$$

The relations (2), (4), (7), (8) and (9) completely define ergodic distribution of number of customers in system in case of $\eta \neq 1$:

$$\begin{gathered}
p_k = \frac{\alpha^k}{k!}\,p_0 = \frac{\alpha^k}{k!}\frac{1-\eta^{m-l}}{A_n(\alpha,\ \eta)+\beta B_n(\alpha,\ \eta)} \qquad \left(k=\overline{1,\ n}\right);\\
p_{n+k} = \frac{\alpha^n}{n!}\,\eta^k p_0 = \frac{\alpha^n}{n!}\,\frac{\eta^k(1-\eta^{m-l})}{A_n(\alpha,\ \eta)+\beta B_n(\alpha,\ \eta)} \qquad \left(k=\overline{1,\ l}\right);\\
p_{n+k} = \frac{\alpha^n}{n!}\,\frac{\eta^k-\eta^m}{1-\eta^{m-l}}\,p_0 =\\
= \frac{\alpha^n}{n!}\frac{\eta^k-\eta^m}{A_n(\alpha,\ \eta)+\beta B_n(\alpha,\ \eta)} \qquad \left(k=\overline{l+1,\, m-1}\right);\\
p_{n+m} = p_{k0} = \frac{\alpha^n}{n!}\,\frac{(1-\eta)\eta^m}{1-\eta^{m-l}}\,p_0 = \frac{\alpha^n}{n!}\,\frac{(1-\eta)\eta^m}{A_n(\alpha,\ \eta)+\beta B_n(\alpha,\ \eta)},\\
p_{k01} = \beta p_{n+m} \qquad \left(k=\overline{n+l+1,\ n+m-1}\ \right);\\
p_{k1} = \beta p_k \qquad \left(k=\overline{0,\ n+m}\ \right).
\end{gathered} \tag{10}$$

If $\eta = 1$ that is $\alpha = n$, then from (4)−(7) and (2) we have

$$\begin{gathered}
p_k = \frac{n^k}{k!}\,p_0 \quad \left(k=\overline{1,\ n}\right); \qquad p_{n+k} = \frac{n^n}{k!}\,p_0 \quad \left(k=\overline{1,\ l}\right);\\
p_{n+m} = \frac{n^n}{n!}\,p_0 - (m-l-1)p_{n+m} \quad \Rightarrow\\
\Rightarrow \quad p_{n+m} = \frac{n^n}{n!(m-l)}\,p_0 = p_{k0} \quad \left(k=\overline{n+l+1,\ n+m-1}\right);\\
p_{n+k} = \frac{n^n}{n!}\,p_0 - (k-l)p_{n+m} = \frac{n^n}{n!}\,\frac{m-k}{m-l}\,p_0 \quad \left(k=\overline{l+1,\ m-1}\right);\\
p_{k1} = \beta p_k \qquad \left(k=\overline{0,\ n+m}\ \right);\\
p_{k01} = \beta p_{n+m} \qquad \left(k=\overline{n+l+1,\ n+m-1}\ \right).
\end{gathered} \tag{11}$$

By means of the normalization condition (3) we find

$$p_0 = \frac{1}{C_n},\ C_n = \frac{n^n}{n!}\,\frac{\beta+m-l}{m-l} + (1+\beta)\left(\sum_{k=0}^{n}\frac{n^k}{k!} + \sum_{k=1}^{l}\frac{n^n}{k!} + \frac{n^n}{n!}\,\frac{m-l-1}{2}\right). \tag{12}$$

For the single-server queueing system ($n = 1$) formulas (9), (10) get a kind (case of $\alpha \neq 1$):

$$p_k = \frac{(1-\alpha)(1-\alpha^{m-l})\alpha^k}{A_1(\alpha)+\beta B_1(\alpha)} \quad \left(k=\overline{0,\ l+1}\right);$$
$$p_k = \frac{(1-\alpha)(\alpha^k-\alpha^{m+1})}{A_1(\alpha)+\beta B_1(\alpha)} \quad \left(k=\overline{l+2,\ m}\right);$$
$$p_{m+1} = \frac{(1-\alpha)^2\alpha^{m+1}}{A_1(\alpha)+\beta B_1(\alpha)} = p_{k0} \quad \left(k=\overline{l+2,\ m}\right); \tag{13}$$
$$p_{k1} = \beta p_k \quad \left(k=\overline{0,\ m+1}\right); \quad p_{k01} = \beta p_{m+1} \quad \left(k=\overline{l+2,\ m}\right);$$
$$A_1(\alpha) = 1-\alpha^{m-l}-(m-l)(1-\alpha)\alpha^{m+2};$$
$$B_1(\alpha) = 1-\alpha^{m-l}-(m-l-1+\alpha)(1-\alpha)\alpha^{m+1}.$$

If $\alpha = 1$, that is $\lambda = \mu$, then from (11) and (12) we have

$$p_0 = \frac{2(m-l)}{(1+\beta)(m-l)\left(m-l+3+2\sum\limits_{k=1}^{l}\frac{1}{k!}\right)+2(\beta+m-l)};$$
$$p_k = \frac{p_0}{(k-1)!} \quad \left(k=\overline{1,\ l+1}\right); \quad p_k = \frac{m-k+1}{m-l}p_0 \quad \left(k=\overline{l+2,\ m}\right);$$
$$p_{m+1} = \frac{p_0}{m-l} = p_{k0} \quad \left(k=\overline{l+2,\ m}\right);$$
$$p_{k1} = \beta p_k \quad \left(k=\overline{0,\ m+1}\right); \quad p_{k01} = \beta p_{m+1} \quad \left(k=\overline{l+2,\ m}\right).$$

If we assume, that $\eta < 1$, and we pass to a limit as $m \to \infty$ in relations (9), (10), then ergodic distribution for queueing system with n servers and possible states of inaccessibility and without restrictions on length of queue can be written

$$p_k = \frac{\frac{\alpha^k}{k!}}{(1+\beta)\left(\sum\limits_{s=0}^{n}\frac{\alpha^s}{s!}+\frac{\eta}{1-\eta}\frac{\alpha^n}{n!}\right)} \quad \left(k=\overline{0,\ n}\right);$$
$$p_{n+k} = \frac{\alpha^n}{n!}\eta^k p_0 \quad (k=1,\ 2,\ \ldots); \quad p_{k1} = \beta p_k \quad (k=0,\ 1,\ 2,\ \ldots).$$

For single-server system we receive corresponding formulas from relations (13) as $\alpha < 1$, $m \to \infty$,

$$p_k = \frac{\alpha^k(1-\alpha)}{1+\beta}, \quad p_{k1} = \frac{\beta\alpha^k(1-\alpha)}{1+\beta} \quad (k=0,\ 1,\ 2,\ \ldots).$$

3. Queueing system with blocking of an input flow. If transition to state of inaccessibility is impossible, we should exclude states s_{k1} $\left(k=\overline{0,\ n+m}\right)$ and s_{k01} $\left(k=\overline{n+l+1,\ n+m-1}\right)$ from the list of states of system. Corresponding system of the equations for steady-state probabilities p_k, p_{k0} we receive from relations

(1) as $\nu=\gamma=0$

$$
\begin{gathered}
\mu p_1-\lambda p_0=0; \quad \lambda p_{k-1}+(k+1)\mu p_{k+1}-(\lambda+k\mu)p_k=0 \quad \left(k=\overline{1,\ n-1}\ \right);\\
\lambda p_{k-1}+n\mu p_{k+1}-(\lambda+n\mu)p_k=0 \quad \left(k=\overline{n,\ n+l-1}\ \right);\\
\lambda p_{n+l-1}+n\mu(p_{n+l+1}+p_{n+l+1,0})-(\lambda+n\mu)p_{n+l}=0;\\
\lambda p_{k-1}+n\mu p_{k+1}-(\lambda+n\mu)p_k=0 \quad \left(k=\overline{n+l+1,\ n+m-2}\ \right);\\
\lambda p_{n+m-2}-(\lambda+n\mu)p_{n+m-1}=0; \qquad \lambda p_{n+m-1}-n\mu p_{n+m}=0;\\
p_{n+m}=p_{k,0} \quad \left(k=\overline{n+l+1,\ n+m-1}\ \right);\\
\sum_{k=0}^{n+m-1} p_k+(m-l)p_{n+m}=1.
\end{gathered} \tag{14}
$$

If we put $\beta=0$ in relations (9), (10) for $\eta\neq 1$ and in (11), (12) for $\eta=1$, then solutions of system of the equations (14) can be written :

$$
\begin{gathered}
p_0=\frac{1-\eta^{m-l}}{A_n(\alpha,\eta)}; \qquad p_k=\frac{\alpha^k}{k!}p_0 \qquad \left(k=\overline{1,n}\right);\\
p_{n+k}=\frac{\alpha^n}{n!}\eta^k p_0 \quad \left(k=\overline{1,l}\right); \qquad p_{n+k}=\frac{\alpha^n}{n!}\frac{\eta^k-\eta^m}{A_n(\alpha,\eta)} \quad \left(k=\overline{l+1,m-1}\right);\\
p_{n+m}=p_{k0}=\frac{\alpha^n}{n!}\frac{(1-\eta)\eta^m}{A_n(\alpha,\eta)} \quad \left(k=\overline{n+l+1,n+m-1}\right); \qquad \eta\neq 1;
\end{gathered} \tag{15}
$$

$$
\begin{gathered}
p_0=\frac{1}{\sum\limits_{k=0}^{n}\frac{n^k}{k!}+\sum\limits_{k=1}^{l}\frac{n^n}{k!}+\frac{n^n}{n!}\,\frac{m-l+1}{2}}; \quad p_k=\frac{n^k}{k!}p_0 \quad \left(k=\overline{1,\ n}\right);\\
p_{n+k}=\frac{n^n}{k!}p_0 \quad \left(k=\overline{1,\ l}\right); \qquad p_{n+k}=\frac{n^n}{n!}\,\frac{m-k}{m-l}p_0 \quad \left(k=\overline{l+1,\ m-1}\right);\\
p_{n+m}=\frac{n^n}{n!(m-l)}p_0=p_{k0} \qquad \left(k=\overline{n+l+1,\ n+m-1}\right); \qquad \eta=1.
\end{gathered} \tag{16}
$$

For single-server system ($n=1$) from formulas (15), (16) we obtain

$$
\begin{gathered}
p_k=\frac{(1-\alpha)(1-\alpha^{m-l})\alpha^k}{A_1(\alpha)} \quad \left(k=\overline{0,\ l+1}\right);\\
p_k=\frac{(1-\alpha)(\alpha^k-\alpha^{m+1})}{A_1(\alpha)} \quad \left(k=\overline{l+2,\ m}\right);\\
p_{m+1}=\frac{(1-\alpha)^2\alpha^{m+1}}{A_1(\alpha)}=p_{k,0} \quad \left(k=\overline{l+2,\ m}\right); \qquad \eta\neq 1;
\end{gathered} \tag{17}
$$

$$
\begin{gathered}
p_0=\frac{2}{m-l+5+2\sum\limits_{k=1}^{l}\frac{1}{k!}}; \qquad p_k=\frac{p_0}{(k-1)!} \quad \left(k=\overline{1,\ l+1}\right);\\
p_k=\frac{m-k+1}{m-l}p_0, \quad p_{m+1}=\frac{p_0}{m-l}=p_{k,0} \quad \left(k=\overline{l+2,m}\right); \quad \eta=1.
\end{gathered} \tag{18}
$$

If we put $l = 0$ in relations (17) and (18), then ergodic distribution of numbers of customers for single-server system, in which blocking of an input flow is removed under condition of absence of queue, can be written

$$p_k = \frac{(1-\alpha)(\alpha^k - \alpha^{m+1})}{1-\alpha^m - m(1-\alpha)\alpha^{m+2}} \qquad \left(k = \overline{0,\ m}\right);$$

$$p_{m+1} = \frac{\alpha^{m+1}(1-\alpha)^2}{1-\alpha^m - m(1-\alpha)\alpha^{m+2}} = p_{k,0} \quad \left(k = \overline{2,\ m}\right), \qquad \alpha \neq 1;$$

$$p_0 = \frac{2}{m+5}; \qquad p_k = \frac{2(m-k+1)}{m(m+5)} \qquad \left(k = \overline{1,\ m}\right);$$

$$p_{m+1} = \frac{2}{m(m+5)} = p_{k,0} \quad \left(k = \overline{2,\ m}\right), \qquad \alpha = 1.$$

Let us denote by π_k $\left(k = \overline{0,\ m}\right)$ steady-state probability that the length of queue is equal k. Using relations (15) and (16), we can write formulas for ergodic distribution of length of queue in system with n servers and blocking an input flow

$$\pi_0 = \sum_{j=0}^{n} p_j = \frac{1-\eta^{m-l}}{A_n(\alpha,\ \eta)} \sum_{j=0}^{n} \frac{\alpha^j}{j!}; \qquad \pi_k = p_{n+k} = \frac{\alpha^n}{n!} \frac{\eta^k(1-\eta^{m-l})}{A_n(\alpha,\ \eta)} \quad \left(k = \overline{1,\ l}\right);$$

$$\pi_k = p_{n+k} + p_{n+m} = \frac{\alpha^n}{n!} \frac{\eta^k - \eta^{m+1}}{A_n(\alpha,\ \eta)} \qquad \left(k = \overline{l+1,\ m-1}\right);$$

$$\pi_m = p_{n+m} = \frac{\alpha^n}{n!} \frac{(1-\eta)\eta^m}{A_n(\alpha,\ \eta)}; \qquad \eta \neq 1;$$

$$\pi_0 = \frac{1}{C_n} \sum_{j=0}^{n} \frac{n^j}{j!}; \ \pi_k = \frac{n^n}{k! C_n} \ \left(k = \overline{1, l}\right); \ \pi_k = \frac{n^n}{n!} \frac{m-k+1}{(m-l)C_n} \ \left(k = \overline{l+1, m}\right);$$

$$C_n = \sum_{k=0}^{n} \frac{n^k}{k!} + \sum_{k=1}^{l} \frac{n^n}{k!} + \frac{n^n}{n!} \frac{m-l+1}{2}; \quad \eta = 1.$$

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