MECHANICS

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RESEARCH OF MAIN CHARACTERISTICS CHANGE OF SEISMIC WAVES IN SATURATED EARTH SOLIDS

Abstract

Energy dissipation stipulated by viscoelastic properties of material should be taken into account while investigating wave processes in multilayer media. We reserch properties of viscous wave propagation in laminated earth solids on the example of plane waves in unbounded laminated medium. The fact that even ignoring the dissipation effects in soft layers some forms are attenuated for certain frequencies, is typical for laminated media. These are so-called non-transmission zones. Taking into account viscous properties of material in soft layers the amplitude of majority of possible wave forms decrease during their propagation. Attenuation rate of the waves that were not attenuated for perfect ideal laminated medium, is many times less than the attenuation rate of waves in non-transmission zones.

1. Introduction. Inhomogeneity of structure of earth solid simulated as a viscoelastic medium, reduces to change of configuration of propagating waves of attenuation and dispersion, and also cumulation-amplification of waves [1, p.152]. Besides, by propagation of periodic waves in periodic inhomogeneous medium there appear opacity (choking), when propagating waves either don't exist in general, or exponentially decrease due to growth of length.

In this paper we study these quality effects on the example of propagation of harmonic waves in an inhomogeneous viscoelastic earth solid.

2. Problem Statement. Let's consider a periodically inhomogeneous plane-laminated medium and derive an equation describing the wave propagation in the direction Oz, perpendicular to the plane of layers. In such a statement deflected mode will be monoaxial with an axis Oz, and we'll wtire the motion equation in the form [1, p.35]

$$\frac{\partial \sigma}{\partial z} = \rho(z) \frac{\partial^2 \vartheta}{\partial t^2}, \tag{2.1}$$

where

$$\sigma(z,t) = E(z) \cdot \frac{\partial \vartheta(z,t)}{\partial z}, \tag{2.2}$$

$\sigma(z,t)$ is stress, $\vartheta = \vartheta(\vartheta, t)$ is displacement, $\rho(z)$ is density of the earth solid material, $E(z)$ is Young modulus.

Proceeding from physical characteristics of the earth solid we'll assume

$$\begin{array}{ll} E(z) = E_0 + v_0 \cos(T_0 z), & E_0 = const \\ \rho(z) = \rho_0 + v_1 \cos(T_0 z), & \rho_0 = const \end{array} \tag{2.3}$$

where v_0, v_1 are small (in comparison with E_0, ρ_0), $T_0 = \dfrac{2\pi}{k}$, k is homogeneity period along the axis Oz. Introducing the denotation

$$c_0^2 = E_0/\rho_0; \qquad \varepsilon = \frac{v_1}{\rho_0} - \frac{v_0}{E_0} \tag{2.4}$$

we'll have $\varepsilon << 1$. After substitution of (2.2) and (2.3) into (2.1), assumiong that the quantity $\frac{\partial E}{\partial z}\frac{\partial \vartheta}{\partial z}$ is slightigly small in comparison with the quantities $E_0\frac{\partial^2 \vartheta}{\partial z^2}$, $v_0\frac{\partial^2 \vartheta}{\partial z^2}$, $\rho\frac{\partial^2 \vartheta}{\partial t^2}$, $v_1\frac{\partial^2 \vartheta}{\partial t^2}$, ignoring the quantities of order ε^2 in comparison with the quantities or order ε, we get the equation

$$\frac{\partial^2 \vartheta}{\partial z^2} = \frac{1}{c_0^2}\left(1+\varepsilon\cos\left(T_0 z\right)\right)\cdot\frac{\partial^2 \vartheta}{\partial z^2} \tag{2.5}$$

In the case of harmonic seismic wave

$$\vartheta\left(z,t\right) = A\left(z\right)\cdot e^{i\omega t} \tag{2.6}$$

where ω is frequency of seismic wave. Allowing for (2.6) in (2.5) we get:

$$A''\left(z\right) + \frac{\omega^2}{c_0^2}\left(1+\varepsilon\cos\left(T_0 z\right)\right)A\left(z\right) = 0 \tag{2.7}$$

3. Problem solution. In principle, the small parameter method may be applied to the solution of this equation, however, the use of the theory of Mathieu-Hill equations is more useful.

The equation (2.7) acquires standard form of the Mathieu equation [2, p.231]. If we introduce the denotation

$$\frac{T_0}{z} \equiv \xi; \quad \frac{\omega^2}{c_0^2} = \eta; \quad \eta\varepsilon = \gamma \tag{3.1}$$

instead of (2.7) we'll wave:

$$\frac{d^2 A}{d\xi^2} + \left(\eta + \gamma\cos\left(2\xi\right)\right)A\left(\xi\right) = 0 \tag{3.2}$$

By Floquet's theorem [2, p.234], general solution of equation (3.2) will take the form:

$$A = c_1 F_1\left(\xi\right)e^{n\xi} + c_2 F_2\left(\xi\right)e^{-n\xi} \tag{3.3}$$

where c_1, c_2 and c are arbitrary constants, F_1, F_2 are periodic functions of variable ξ with period $\pi; n = const$ is determined by η and γ.

The solutions of the Mathieu equation are called Mathieu functions [3, p.320], and the tables were made up for them. Not going into the theory of these functions, we notice that all the plane of variables (η, γ) is divided into three subdomains (fig.1). In the domain I, where $\eta < -\gamma$ the propagating waves can't exist, in domains II and III in the zones shaded by waves n is pure imaginary quantity, consequently, the solution (3.3) in this case will be a superposition of two sustained waves propagating in opposite directions, in shaded zones of subdomains II and III the quantity n is complex, consequently, here propagating waves are exponentially attenuated, and these zones are called opacity zones.

Thus, a periodic inhomogeneity medium functions as a filter, leaks the waves with the same frequencies and dampens the waves with frequencies corresponding to opacity zones. In general case, the dependence of the number n on ω is nonlinear, consequently, the propagating waves disperse.

If periodicity of medium's properties is not trigonometric, then repeating the reasonings by means of which equation (2.7) was obtained, we arrive at the equation

$$\frac{d^2A}{dz^2}+\frac{\omega^2}{c^2}\left(1+\varepsilon f\left(z\right)\right)A\left(z\right)=0 \tag{3.4}$$

where $f(z)$ is an arbitrary periodic function, equation (3.4) is said to be Hill's equation.

When $f(z)$ has the form represented in figure 3, for a piece-wice homogeneous medium we get

$$A\left(z\right)=\begin{cases} c_1e^{ik_1z}+D_1e^{-ik_1z}, & -l_1<x<0 \\ c_1e^{ik_1z}+D_1e^{-ik_1z}, & -0<x<l_2 \end{cases} \tag{3.5}$$

where $k_1^2=k^2\left(1+\varepsilon f_1\right),\quad k_2^2=k^2\left(1+\varepsilon f_2\right),\quad k=\omega^2/c_0^2$ is a wave number.

By Floquet's theorem for Hill's equation, for a wave propagating in positive direction of the axis Oz, there should be

$$A\left(z\right)=F\left(z\right)e^{i\overline{k}z} \tag{3.6}$$

where $\overline{k}$ is a desired wave number, $F(z)$ is periodic with period $d=l_1+l_2$. We compare (3.5) and (3.6) on the interval $l_2<z<d$, use the equality $F\left(z-d\right)=F\left(z\right)$ and arrive at the expression

$$A\left(z\right)=c_1e^{i\overline{k}d}\cdot e^{ik_1(z-d)}+D_1e^{i\overline{k}d}\cdot e^{-ik_1(z-d)}. \tag{3.7}$$

Having required continuity of the "amplitude" $A(z)$ and its first derivative on the interface of layers with different properties, we arrive at the homogeneous linear system with respect to the coefficients C_1D_1, C_2D_2 from whose condition of existence of non-trivial solution (equality of adeterminant to zero), we get the equation

$$2e^{i\overline{k}d}\left(\cos\left(k_1l_1\right)\cdot\cos\left(k_2l_2\right)-\right.$$
$$\left.-\frac{1}{2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)\cdot\sin\left(k_1l_1\right)\cdot\sin\left(k_2l_2\right)\right)=e^{2i\overline{k}d}-1. \tag{3.8}$$

The roots $z_1=e^{i\overline{k}_1d}$ and $z_2=e^{i\overline{k}_2d}$ are conjugated, consequently $z_1\cdot z_2=1$, whence allowing for the properties of the roots of quadratic equation we find

$$z_1+z_2=2\left(\cos\overline{k}d\right)=$$
$$=2\left(\cos\left(k_1l_1\right)\cdot\cos\left(k_2l_2\right)-\frac{1}{2}\left(\frac{k_1}{k_2}+\frac{k_2}{k_1}\right)\cdot\sin\left(k_1l_1\right)\cdot\sin\left(k_2l_2\right)\right). \tag{3.9}$$

(the index $\overline{k}$ is omitted).

From the last relation we can find the opacity zones (non-transmission or choking), determining a frequency range for which the expression in the paranthesis in (3.9) will be greater than a unit in modulus. Thus, piecewise-homogeneous media can filter waves in a definite frequency bands as well. For real earth solids opacity zones appear in frequencies of order $10^4\div10^8Gc$.

If we consider the earth medium as an homoheneous medium, i.e. $\rho\left(z\right)=\rho_0=const$ after some transformations we get a motion equation in displacements [1, p.36]

$$\frac{\partial^2\vartheta}{\partial t^2}-c_0^2\frac{\partial^2u}{\partial t^2}=-c_0^2\int\limits_{-\infty}^{t}R\left(t-\tau\right)\frac{\partial^2}{\partial z^2}\vartheta\left(z,\tau\right)d\tau \tag{3.10}$$

whose solution is found in the form:

$$\vartheta(z,\tau)=\beta e^{i(\omega t-kz)} \tag{3.11}$$

Substitution of (3.11) and (3.10) gives

$$e^{i(\omega t-kz)}\left(-\omega^2+c_0^2k^2\right)=c_0^2k^2\int\limits_{-\infty}^{t}R(t-\tau)\,e^{-i\omega t-ikz}d\tau. \tag{3.12}$$

Let's consider a special case, when the relaxation function

$$R(t-\tau)=A\cdot e^{-\beta(t-\tau)},\quad A,\beta\sim const.$$

then from equation (3.12) we get

$$-\omega^2+c_0^2k^2=\frac{Ac_0^2k^2}{\beta ei\omega}\qquad\text{or}\qquad k^2=\frac{\omega^2(\beta+i\omega)}{(\beta+i\omega-A)\,c_0^2}.$$

Fig.1.

Change of contractive (normal) stresses in earth solid with regard to earth's rheology $\alpha=0$ an elastic model, $\alpha=[0;0,2]$ a viscoelastic model in different relaxation functions.

A real part of k corresponds to dependence of phase velocity on frequency (obviously nonlinear), imaginary part to dependence of attenuation coefficient of wave amplitude on frequency. Thus, viscoelasticity of the material reduces simultaneously to dispersion and attenuation of waves.

Numerical realization of the obtained theoretical result is given for the case of abyssal rocks of earth arranged in depth of $10\div 30$ km, where lamination of solid specially differs [4, p.116]. The obtained number values of physical characteristics of waves for elastic and viscoelastic media distinctly shows difference between the results of corresponding problems.

The cited calculation experiments have for an object of quality analysis of stress changes in earth during passage of longitudinal and transverse seismic waves.

The patterm of change of contractive normal stresses arising under the action of longitudinal waves is given in figure 1. These waves propagate with the velocity of elastic waves and there is no essential difference between these models.

Fig.2.
Change of stretching (destroying) stresses depending on the account of earth's rheology. $\alpha = 0$ an clastic model, $\alpha = [0; 0, 2]$ a viscoelastic model for different creeping functions.

Fig.3.
Dispersive curves for longitudinal waves in the area with wells for different rheological characteristics of earth (solid lines for an elastic model of earth, a dotted line for a viscoelastic model of earth).

The coefficient $\alpha = 0 \div 0, 2$ determines the level of calculation of solid's rheology. $\alpha = 0$ corresponds to the solution of elastic problem.

The graph of change of stretching stresses arising under the action of shear and reflected longitudial waves is in figure 2. It is known that the earth is structurally granular medium that doesn't resist to tension. This king of loading destroys earth rock, account of earth's viscosity remarkably decreases the dectruction probability at initial (peak) stages of earthquakes.

The results of calculation of a problem on dispersion of longitudinal seismic waves in the area with wells are given in figure 3. The values of frequencies of vibrations of underground pipelines for elastic and viscoelastic models of earth, are reduced.

Account of viscosity properties of an earth medium refines the solution of the problem, changung the values of stresses about $5-35\%$ for different values of period of seicmic actions. This is explained by dampening properties of enveronment and perceptible values of the adjoint earth solid.

4. Conclusions. It is seen from the obtained solution that ignorance of rheological properties of medium while studying occuring wave prcesses reduces to qualitative and quantitative improper results. Account of real physical properties of earth solid refines the solution of the problem for $5 \div 7\%$, depending on the choice of the model of periodicity of layers.

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