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AXISYMMETRIC LONGITUDINAL WAVE PROPAGATION IN CIRCULAR CYLINDER EMBEDDED WITH COMPRESSIBLE ELASTIC MEDIUM WITH INITIAL FINITE COMPRESSING STRAINS

Abstract

Within the scope of the piecewise homogeneous body model with the use of the three-dimensional linearized theory of elastic wave propagation in the initially stressed bodies the axisimmetric longitudinal wave propagation in the finite pre-strained cylinder surrouded with the finite pre-compressed infinite elastic body is investigated. It is assumed that the materials of the components are compressible and the elasticity relations of those are given through the harmonic potential. The numerical results illustrating the influence of the finite initial compressing strains on the wave propagation velocity are presented and discussed.

1. Introduction

At present, many investigations on wave propagation in initially (residual) stressed bodies are carried out with the use of Three-dimensional Linearized Theory of Wave Propagation in Initially-stressed Bodies (TLTWPISB). Detailed analysis of the results of these investigations are in the monograph [1-3]. In these monographs the principles of construction of TLTWPISB are stated, principal equations and relations of this theory are cited. A brief review of investigations of the last years is in the papers [4-12].

Analysis of the monographs and papers mentioned above show that at present there is no investigations on the wave propagation in fibrous pre-stressed (pre-strained) composites. Considering this state, in the given paper we make attempt in this direction and study propagation (dispersion) of longitudinal, axisymmetric waves in one-directional fibrous composite made of compressible material with initial finite strains. It is assumed that concentration of fibers in the composite is small and the indicated composite may be modelled as an infinite elastic body containing an infinitely-long cylinder of a circular cross section. Concrete numerical results and their analysis are represented for the case when a finite initial strain appears because of action of uniformly distributed normal compressing efforts at infinity. And it is accepted that these efforts act along the cylinder (fiber).

2. Problem Statement. Principal equations and relations

Let's consider an infinite elastic body containing an infinitely long cylinder of a circular cross section. Assume that the materials of an infinite body and cylinder are highelastic and compressible. In natural state, by R we denote a radius of the indicated cylinder. We determine the state of the points of the cylinder and surrounding body with Lagrangian coordinates in a Cartesian system of coordinates $Oy_1y_2y_3$, and also in a cylindric system of coordinates $Or\theta y_3$. Accept that the axis Oy_3 is directed along the central axis of the cylinder and at each component of the considered body it holds a homogeneous axisymmetric (with respect to the axis Oy_3) finite initial strain. Such an initial strain may appear for example, in compressing the body along the axis Oy_3.

At initial state we determine the state of position of the points with Lagrangian coordinates in Cartesian system of coordinates $O'y_1'y_2'y_3'$ and in cylindrical system of coordinates $O'r'\theta'y_3'$. Accept that elastic relations of the material of components are determined through harmonic potential. We distinguish the quantities relating to a cylinder and to an infinite body by the upper indices (2) and (1), respectively. Moreover, the quantities relating to initial state - by an additional upper index O. Thus, considering the above-stated one, the quantities relating to initial state may be determined in the form

$$u_m^{(k),0} = \left(\lambda_m^{(k)} - 1\right) y_m, \quad \lambda_1^{(k)} = \lambda_2^{(k)} \neq \lambda_3^{(k)}, \quad \lambda_m^{(k)} = const_m, \quad m = 1,2,3; k = 1,2, \tag{1}$$

where $u_m^{(k),0}$ are permutations vector components, $\lambda_m^{(k)}$ are elongation parameters in the direction of the axis Oy_m.

According to the above-stated one and relation (1) we have

$$y_i' = \lambda_i^{(k)} y_i, \quad r' = \lambda_1^{(k)} r, \quad R' = \lambda_1^{(2)} R, \tag{2}$$

where $k = 1$ for $r > R$ (or for $\sqrt{y_1^2 + y_2^2} > R$) and $k = 2$ for $r < R$ (or for $\sqrt{y_1^2 + y_2^2} < R$).

Below, we'll mark the quantities related with a system of coordinates $O'y_1'y_2'y_3'$ or $O'r'\theta'y_3'$ by an upper prime.

So, let's investigate propagation of axisymmetric longitudinal waves in the direction of the axis $O'y_3'$ in the above-indicated infinite body involving a finite initial strain determined by means of (1). We carry out investigations in the scope of a piece-wise homogeneous body model with the use of TLTWPISB in coordinates connected with initial state. According to [3] in a cylindrical system of coordinates, in the considered case we have the following motion equations

$$\begin{aligned} &\frac{\partial}{\partial r'} Q_{r'r'}^{\prime(k)} + \frac{\partial}{\partial y_3'} Q_{r3}^{\prime(k)} + \tfrac{1}{r'} \left(Q_{r'r'}^{\prime(k)} - Q_{\theta'\theta'}^{\prime(k)}\right) = \rho^{\prime(k)} \frac{\partial^2}{\partial t^2} u_{r'}^{\prime(k)}, \\ &\frac{\partial}{\partial r'} Q_{3r'}^{\prime(k)} + \frac{\partial}{\partial y_3'} Q_{33}^{\prime(k)} + \frac{1}{r'} Q_{3r'}^{\prime(k)} = \rho^{\prime(k)} \frac{\partial^2}{\partial t^2} u_3^{\prime(k)}, \end{aligned} \tag{3}$$

and mechanical relations

$$\begin{aligned}
Q'^{(k)}_{r'r'} &= \omega'^{(k)}_{1111}\frac{\partial u'^{(k)}_{r'}}{\partial r'} + \omega'^{(k)}_{1122}\frac{\partial u'^{(k)}_{r'}}{\partial r'} + \omega'^{(k)}_{1133}\frac{\partial u'^{(k)}_{3}}{\partial y'_3},\\
Q'^{(k)}_{\theta'\theta'} &= \omega'^{(k)}_{2211}\frac{\partial u'^{(k)}_{r'}}{\partial r'} + \omega'^{(k)}_{2222}\frac{\partial u'^{(k)}_{r'}}{\partial r'} + \omega'^{(k)}_{2233}\frac{\partial u'^{(k)}_{3}}{\partial y'_3},\\
Q'^{(k)}_{33} &= \omega'^{(k)}_{3311}\frac{\partial u'^{(k)}_{r'}}{\partial r'} + \omega'^{(k)}_{3322}\frac{\partial u'^{(k)}_{r'}}{\partial r'} + \omega'^{(k)}_{3333}\frac{\partial u'^{(k)}_{3}}{\partial y'_3},\\
Q'^{(k)}_{r'3} &= \omega'^{(k)}_{1313}\frac{\partial u'^{(k)}_{r'}}{\partial y'_3} + \omega'^{(k)}_{1313}\frac{\partial u'^{(k)}_{3}}{\partial r'},\\
Q'^{(k)}_{3r'} &= \omega'^{(k)}_{3113}\frac{\partial u'^{(k)}}{\partial y'_3} + \omega'^{(k)}_{3131}\frac{\partial u'^{(k)}_{3}}{\partial r'}.
\end{aligned} \tag{4}$$

In (3) and (4) the following denotation were accepted:$Q'^{(k)}_{r'r'}$,..., $Q'^{(k)}_{3r'}$ are disturbances of Kirkhoff stress tensor components in a cylindrical system of coordinates; $u'^{(k)}_{r'}$ and $\partial u'^{(k)}_{3}$ are disturbances of permutation vector components; $\omega'^{(k)}_{1111}$,..., $\omega'^{(k)}_{3131}$ are the constants whose values are calculated by mechanical constants and initial strains; $\rho'^{(k)} = \rho^{(k)}/\left(\lambda^{(k)}_1\lambda^{(k)}_2\lambda^{(3)}_3\right)$ and $\rho^{(k)}$ are densities of the k-th material in natural state.

Accept that mechanical relations of materials and infinite surrounding body of the cylinder are given through harmonic potential that have the following expression:

$$\Phi^{(k)} = \frac{1}{2}\lambda^{(k)}s^{(k)^2}_1 + \mu^{(k)}s^{(k)}_2 \tag{5}$$

where

$$\begin{aligned}
s^{(k)}_1 &= \sqrt{1+2\varepsilon^{(k)}_1} + \sqrt{1+2\varepsilon^{(k)}_2} + \sqrt{1+2\varepsilon^{(k)}_3} - 3,\\
s^{(k)}_2 &= \left(\sqrt{1+2\varepsilon^{(k)}_1} - 1\right)^2 + \left(\sqrt{1+2\varepsilon^{(k)}_2} - 1\right)^2 + \left(\sqrt{1+2\varepsilon^{(k)}_3} - 1\right)^2
\end{aligned} \tag{6}$$

In (5), (6) $\lambda^{(k)}$ and $\mu^{(k)}$ are the constants of the material, $\varepsilon^{(k)}_i$ $(i = 1, 2, 3)$ are the principal values of the Green strain tensor.

Conducting appropriate mathematical calculations stated for example in [3] we get the following expression for $\lambda^{(k)}_1 = \lambda^{(k)}_2$, $\omega'^{(k)}_{1111}$,..., $\omega'^{(k)}_{3131}$.

$$\begin{aligned}
&\lambda^{(k)}_2 = \lambda^{(k)}_1 = \left[2 - \frac{\lambda^{(k)}}{\mu^{(k)}}\left(\lambda^{(k)}_3 - 3\right)\right]\left[2\left(\frac{\lambda^{(k)}}{\mu^{(k)}} + 1\right)\right]^{-1},\\
&\omega'^{(k)}_{1111} = \left(\lambda^{(k)}_3\right)^{-1}\left(\lambda^{(k)} + 2\mu^{(k)}\right),\ \ \omega'^{(k)}_{1122} = \left(\lambda^{(k)}_3\right)^{-1}\lambda^{(k)},\\
&\omega'^{(k)}_{1133} = \left(\lambda^{(k)}_1\right)^{-1}\lambda^{(k)},\ \ \omega'^{(k)}_{1221} = \left(\lambda^{(k)}_3\right)^{-1}\mu^{(k)},\\
&\omega'^{(k)}_{1313} = 2\mu^{(k)}\left(\lambda^{(k)}_1 + \lambda^{(k)}_3\right)^{-1},\\
&\omega'^{(k)}_{3113} = 2\mu^{(k)}\left(\lambda^{(k)}_1\right)^{-2}\left(\lambda^{(k)}_3\right)^{2}\left(\lambda^{(k)}_1 + \lambda^{(k)}_3\right)^{-1}.
\end{aligned} \tag{7}$$

We write boundary and contact conditions in the scope in which we'll study wave propagation

$$Q'^{(1)}_{r'r'}\Big|_{r'=R'} = Q'^{(2)}_{r'r'}\Big|_{r'=R'}, \quad Q'^{(1)}_{r'3}\Big|_{r'=R'} = Q'^{(2)}_{r'3}\Big|_{r'=R'},$$

$$u'^{(1)}_{r'}\Big|_{r'=R'} = u'^{(2)}_{r'}\Big|_{r'=R'}, \quad u'^{(1)}_{3}\Big|_{r'=R'} = u'^{(2)}_{3}\Big|_{r'=R'},$$

$$\left|Q'^{(1)}_{r'r'}\right|; \left|Q'^{(1)}_{\theta'\theta'}\right|; \left|Q'^{(1)}_{33}\right|; \left|Q'^{(1)}_{3r'}\right|; \left|u'^{(1)}_{3}\right|; \left|u'^{(1)}_{r'}\right| \to 0 \text{ as } r' \to \infty. \tag{8}$$

Thus, the problem statement is exhausted by the above-mentioned ones. Notice that when $\lambda_1^{(k)} = \lambda_2^{(k)} = \lambda_3^{(k)} = 1.0$ $(k = 1, 2)$ the stated statement passes to the statement of appropriate problems of classic (linear) theory of elastodynamics for the considered infinite body containing an infinite cylinder of a circular cross section.

3. Solution method

Substitute expression (4) into equation (3) and derive motion equations in permutations. For solving these equations we use the following representation for desired permutations [3].

$$u'^{(k)}_{r'} = -\frac{\partial^2}{\partial r' \partial y'_3} X^{(k)},$$
$$u'^{(k)}_{3} = \frac{1}{\omega'^{(k)}_{1133} + \omega'^{(k)}_{1313}} \left(\omega'^{(k)}_{1111} \Delta'_1 + \omega'^{(k)}_{3113} \frac{\partial^2}{\partial y'^2_3} - \rho'^{(k)} \frac{\partial^2}{\partial t^2} \right) X^{(k)}, \tag{9}$$

where the function $X^{(k)}$ satisfies the following equation:

$$\left[\left(\Delta'_1 + \left(\xi'^{(k)}_2\right)^2 \frac{\partial^2}{\partial y'^2_3} \right) \left(\Delta'_1 + \left(\xi'^{(k)}_3\right)^2 \frac{\partial^2}{\partial y'^2_3} \right) - \rho'^{(k)} \left(\frac{\omega'^{(k)}_{1111} + \omega'^{(k)}_{1331}}{\omega'^{(k)}_{1111}\omega'^{(k)}_{1331}} \Delta'_1 + \right.\right.$$
$$\left.\left. + \frac{\omega'^{(k)}_{3333} + \omega'^{(k)}_{3113}}{\omega'^{(k)}_{1111}\omega'^{(k)}_{1331}} \frac{\partial^2}{\partial y'^2_3} \right) \frac{\partial^2}{\partial t^2} + \frac{\rho'^{(k)}}{\omega'^{(k)}_{1111}\omega'^{(k)}_{1331}} \frac{\partial^4}{\partial t^4} \right] X^{(k)} = 0. \tag{10}$$

In (9) and (10) we used the following denotation

$$\Delta'_1 = \frac{d^2}{dr'^2} + \frac{1}{r'}\frac{d}{dr'},$$
$$\left(\xi'^{(k)}_{2,3}\right)^2 = d^{(k)} \pm \left[\left(d^{(k)}\right)^2 - \omega'^{(k)}_{3333}\omega'^{(k)}_{3113} \left(\omega'^{(k)}_{1111}\omega'^{(k)}_{1331} \right)^{-1} \right]^{\frac{1}{2}}, \tag{11}$$
$$d^{(k)} = \left(2\omega'^{(k)}_{1111}\omega'^{(k)}_{1331} \right)^{-1} \left[\omega'^{(k)}_{1111}\omega'^{(k)}_{3333} + \omega'^{(k)}_{1331}\omega'^{(k)}_{3113} - \left(\omega'^{(k)}_{1133} + \omega'^{(k)}_{1313} \right) \right].$$

According to the problem statement we represent the function $X^{(n)} = X^{(n)}(r', y'_3, t)$ $(n = 1, 2)$ in the form $X^{(n)} = X_1^{(n)}(r') \cos(ky'_3 - \omega t)$ and substituting this representation into equation (10) we get the following equation for the function $X_1^{(n)}(r')$:

$$\left(\Delta'_1 + \left(\zeta'^{(n)}_2\right)^2 \right) \left(\Delta'_1 + \left(\left(\zeta'^{(n)}_3\right)^2 \right) \right) X_1^{(n)}(r') = 0, \tag{12}$$

where the constants $\zeta_{2,3}^{\prime(n)}$ are determined from the following equation:

$$\omega_{1111}^{\prime(n)}\omega_{1331}^{\prime(n)}\left(\zeta^{\prime(n)}\right)^4-k^2\left(\zeta^{\prime(n)}\right)^2\left[\omega_{1111}^{\prime(n)}\left(\rho^{(n)}c^2-\omega_{3333}^{\prime(n)}\right)+\omega_{1331}^{\prime(n)}\left(\rho^{(n)}c^2-\right.\right.$$
$$\left.\left.-\omega_{3113}^{\prime(n)}+\left(\omega_{1111}^{\prime(n)}+\omega_{1313}^{\prime(n)}\right)\right]+k^4\left(\rho^{(n)}c^2-\omega_{3333}^{\prime(n)}\right)\left(\rho^{(n)}c^2-\omega_{1313}^{\prime(n)}\right) \tag{13}$$

In (13) phase velocity of wave propagation is denoted by $c=\omega/k$. So, by equation (13), (12) and damping condition in (8) we determine the function $X_1^{(n)}(r')$ in the following way

$$\begin{aligned} X_1^{(1)}(r') &= B_2^{(1)}E_0^{(1)}\left(kr'\zeta_2^{\prime(1)}\right)+B_3^{(1)}E_0^{(1)}\left(kr'\zeta_3^{\prime(1)}\right), \\ X_1^{(2)}(r') &= B_2^{(2)}E_0^{(2)}\left(kr'\zeta_2^{\prime(2)}\right)+B_3^{(2)}E_0^{(2)}\left(kr'\zeta_3^{\prime(2)}\right), \end{aligned} \tag{14}$$

where

$$E_0^{(2)}\left(kr'\zeta_n^{\prime(2)}\right)=\begin{cases} J_0\left(kr'\zeta_n^{\prime(2)}\right) & \text{if } \left(\zeta_n^{\prime(2)}\right)^2>0, \\ I_0\left(kr'\left|\zeta_n^{\prime(2)}\right|\right) & \text{if } \left(\zeta_n^{\prime(2)}\right)^2<0, \end{cases} \tag{15}$$

$$E_0^{(1)}\left(kr'\zeta_n^{\prime(1)}\right)=\begin{cases} Y_0\left(kr'\zeta_n^{\prime(1)}\right) & \text{if } \left(\zeta_n^{\prime(1)}\right)^2>0, \\ K_0\left(kr'\left|\zeta_n^{\prime(1)}\right|\right) & \text{if } \left(\zeta_n^{\prime(1)}\right)^2<0, \end{cases} \tag{16}$$

In (15) and (16) by $J_0(x)$, $Y_0(x)$ we denote the first and second kind Bessel function, respectively, of zero order, and by $I_0(x)$ and $K_0(x)$ we denote a Bessel function of purely imaginary argument and Mc.Donald function, respectively, of zero order.

So, substituting (14)-(16) into equation (9), from (4) and (8) we derive the variance equation

$$\det\|\alpha_{ij}\|=0, \qquad i,j=1,2,3,4, \tag{17}$$

where α_{ij} are the coefficients of unknowns in algebraic equations obtained from contact conditions (8) for the unknowns $B_2^{(1)}$, $B_3^{(1)}$, $B_2^{(2)}$ and $B_3^{(2)}$ that enter into expression (14). Therewith, in general form we can represent these coefficients in the following form:

$$\alpha_{ij}=\alpha_{ij}\left(c/c_2^{(2)},kR,\mu^{(2)}/\mu^1,\lambda^{(2)}/\mu^{(2)},\lambda^{(1)}/\mu^{(1)},\lambda_3^{(2)},\lambda_3^{(1)}\right), \tag{18}$$

where $c_2^{(2)}\sqrt{\mu^{(2)}/\rho^{(2)}}$. For shortening the volume of the paper we don't cite explicit form of expression (18). Thus, solving equations (17) and (18) we construct dispersive curves $c=c(kR)$ and study the influence of problem parameters on these curves.

4. Numerical results and their discussion

In this section we'll mark the quantities relating to the cylinder by the upper index (f), the quantities relating to surrounding infinite body - by an upper index (m). Accept that $\rho^{(f)}/\rho^{(m)}=1.0$, $\lambda^{(f)}/\mu^{(m)}=1.5$ and consider a dispersive curve

$c = c(kR)$ and analyse influence of initial contractions, i.e. influence of parameters $\lambda_3^{(f)}$ and $\lambda_3^{(m)}$ when $\lambda_3^{(f)} < 1.0$; $\lambda_3^{(m)} < 1.0$ to these curves. Assume $\lambda_3^{(f)} = \lambda_3^{(m)}$ and introduce denotation $\lambda_3 = \lambda_3^{(f)} = \lambda_3^{(m)}$. Let's consider the following two cases. In the first case accept $\mu^{(f)}/\mu^{(m)} = 5.0$; in the second case accept $\mu^{(f)}/\mu^{(m)} = 0.2$.

Notice that construction of above-mentioned dispersive curves is conducted in the following form. At first, for each fixed value kR we find definite number of sequential root of equation (17) (denote these roots by $\left(c/c_2^{(f)(0)}\right)_1 < ... < \left(c/c_2^{(f)(0)}\right)_2$, where $c_2^{(f)(0)} = \sqrt{\mu^{(f)}/\rho^{(f)}}$. Here the values λ_3 are also fixed. Notice, that the first four roots of equation (17) as $kR \to 0$ have the following values:

$$\left(\frac{c}{c_2^{(f)0}}\right)_1 = \sqrt{\frac{\rho^{(f)}\omega'^{(m)}_{3113}}{\rho^{(m)}\mu^{(f)}}} < \left(\frac{c}{c_2^{(f)0}}\right)_2 = \sqrt{\frac{\rho^{(f)}\omega'^{(m)}_{3333}}{\rho^{(m)}\mu^{(f)}}} <$$
$$< \left(\frac{c}{c_2^{(f)0}}\right)_3 = \sqrt{\frac{\omega'^{(f)}_{3113}}{\mu^{(f)}}} < \left(\frac{c}{c_2^{(f)0}}\right)_4 = \sqrt{\frac{\omega'^{(f)}_{3333}}{\mu^{(f)}}}. \tag{19}$$

These roots follow from equation (13) and by the expression (7) the values of these roots depend on λ_3. In obtaining numerical results the values $\lambda_3 = 1.0$; 0.9 and 0.8 are considered. For these values λ_3 the following denotation

$$\frac{c_n^{(m)0}}{c_2^{(f)0}} = \left.\left(\frac{c_n^{(m)}}{c_2^{(f)0}}\right)\right|_{\lambda_3=1.0}, \quad \frac{c_n^{(f)0}}{c_2^{(f)0}} = \left.\left(\frac{c_n^{(f)}}{c_2^{(f)0}}\right)\right|_{\lambda_3=1.0},$$
$$\frac{c_n^{(m)\alpha}}{c_2^{(f)0}} = \left.\left(\frac{c_n^{(m)}}{c_2^{(f)0}}\right)\right|_{\lambda_3=\alpha\neq 1.0}, \quad \frac{c_n^{(f)\alpha}}{c_2^{(f)0}} = \left.\left(\frac{c_n^{(f)}}{c_2^{(f)0}}\right)\right|_{\lambda_3=\alpha\neq 1.0}, n = 1, 2, \tag{20}$$

was introduced.

In the above mentioned first (second) case, i.e. in the case when $\mu^{(f)}/\mu^{(m)} = 5.0$ (when $\mu^{(f)}/\mu^{(m)} = 0.2$) for each chosen value of λ_3 the inequalities $c_2^{(m)} < c_1^{(m)} < c_2^{(f)} < c_1^{(f)}$ ($c_2^{(f)} < c_1^{(f)} < c_2^{(m)} < c_1^{(m)}$) are fulfilled. And by the expression (7) the values $c_2^{(m)}$, $c_1^{(m)}$, $c_2^{(f)}$ and $c_1^{(f)}$ monotonically decrease due to decrease of λ_3.

With moving off the roots of the values of kR from zero, i.e. with growth of kR, the roots of the equation (17) appear between the roots of (19) and using these roots we construct dispersive curves $c = c(R)$ that for the cases $\mu^{(f)}/\mu^{(m)} = 5.0$ ($\mu^{(f)}/\mu^{(m)} = 0.2$) are given in figures 1 and 2 (fig.3 and 4).

In order to illustrate more clearly the influence of initial strain on the velocity (in quality and quantity sense) of wave propagation, in these figures the indicated curves are constructed only for two sequential values of λ_3. Moreover, in these figures the curves drawn with dotted (solid) lines are obtained for $\lambda_3 = 1.0$, 0.9 ($\lambda_3 = 0.9$, 0.8), respectively. In these figures, the velocities determined by the expression (19) are shown with straight lines. It follows from these figures that the above-mentioned N number of roots found for each value of kR from the solution of dispersive equation (17) form the first $N-1$, $N-2$, $N-3$ and $N-4$ modes for the velocity c for which the following inequalities are fulfilled: $c_2^{(m)} < c < c_1^{(m)}$ (i), $c_1^{(m)} < c < c_2^{(f)}$ (ii), $c_2^{(f)} < c < c_1^{(f)}$ (iii) and $c > c_1^{(f)}$ (iiii) ($c_2^{(f)} < c < c_1^{(f)}$ (i), $c_1^{(f)} < c < c_2^{(m)}$

(ii), $c_2^{(m)} < c < c_1^{(m)}$ (iii) and $c > c_1^{(m)}$ (iiii)), for the above-mentioned first (second) case, i.e. for the case when $\mu^{(f)}/\mu^{(m)} = 5.0$ ($\mu^{(f)}/\mu^{(m)} = 0.2$).

Fig. 1.

Fig. 2.

Notice that in the figures 3 and 4 for the explicitness of illustartion, the parts of dispersive curves corresponding to the velocity $c > c_1^{(m)}$ are not shown. Therewith for $\mu^{(f)}/\mu^{(m)} = 5.0$ ($\mu^{(f)}/\mu^{(m)} = 0.2$) it was accepted that $N = 7$ ($N = 5$).

Fig. 3.

Fig. 4.

So, by the above-stated one, each dispersive curve is divided into I, II, III and IV parts for which the inequalities (i), (ii), (iii) and (iiii) are fulfilled, respectively. These parts are separated from each other with straight lines indicating the values of $c_1^{(m)}$, $c_2^{(f)}$ and $c_1^{(f)}$ ($c_1^{(f)}$, $c_2^{(m)}$ and $c_1^{(m)}$) for the case when $\mu^{(f)}/\mu^{(m)} = 5.0$ ($\mu^{(f)}/\mu^{(m)} = 0.2$) obtained for the chosen λ_3. Interior to each of these parts the dispersive curves

$c = c(kR)$ and their derivatives, i.e. $dc/d(kR)$ are continuous. This continuity holds at transition points between the IV and III and also III and II parts. However, in transition points between the parts I and II only the function itself is continuous, its derivative is discontinuous.

The obtained results certify that the propagation velocity of the considered wave has a lower bound. And this bound equals $c_2^{(m)}$ for the case $\mu^{(f)}/\mu^{(m)} = 5$, and $c_2^{(f)}$ for the case when $\mu^{(f)}/\mu^{(m)} = 0.2$. Since the value of $c_2^{(m)}$ and also of $c_2^{(f)}$ diminishes due to decrease of λ_3, so the lower bound of the propagation velocity of symmetric wave in the considered body decreases according to increase of initial compressing load acting along the wave propagation. The results indicated in the figures 1-4 certify that the indicated parts I, II, III, and IV downward shift with growth of initial compressing load. Therewith, by the expressions (7) and (19) "length" of these parts also decrease. Thus, initial compressing load acting along the axisymmetric wave propagation in the considered case calls in principal, decrease of propagation velocity of this wave.

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