**Farman I. MAMEDOV, Shahla Yu. SALMANOVA**

# ON STRONG SOLVABILITY OF THE DIRICHLET PROBLEM FOR SEMILINEAR ELLIPTIC EQUATIONS WITH DISCONTINUOUS COEFFICIENTS

**Abstract**

*In the article the strong solvability of the Dirichlet problem for semilinear elliptic equations with discontinuous coefficients is proved in Sobolev spaces* $W_p^2(\Omega)$, $p>1$.

**1. Introduction.** Let $E_n - n$ dimensional Euclidean space of the points $x=(x_1,x_2,...,x_n)$, $\Omega$ is a bounded domain in $E_n$ with the boundary $\partial\Omega$ from class $C^2$.
Consider $\Omega$ the following Dirichlet problem in

$$\sum_{i,j=1}^{n} a_{ij}(x)\, u_{x_i x_j} + |u|^{q-1}u = f(x), \quad x\in\Omega, \tag{1.1}$$

$$u|_{\partial\Omega}=0. \tag{1.2}$$

Assume,that the coefficients $a_{ij}(x)$, $i,j=1,2,...,n$ of the operator

$$L=\sum_{i,j=1}^{n} a_{ij}(x)\frac{\partial^2}{\partial x_i \partial x_j}$$

are measurable bounded functions satisfying the following conditions:

$$\begin{gathered}\gamma|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(x)\,\xi_i\xi_j \le \gamma^{-1}|\xi|^2, \\ \forall x\in\Omega, \quad \forall\xi\in E_n, \quad \gamma\in(0,1)-const,\end{gathered} \tag{1.3}$$

$$ess\sup_{x\in\Omega}\frac{\sum\limits_{i,j=1}^{n} a_{ij}^2(x)}{\left[\sum\limits_{i=1}^{n} a_{ii}(x)\right]^2} \le \frac{1}{n-1}. \tag{1.4}$$

The condition (1.4) is called Cordes condition.It is fulfilled to within non-singular linear transformation,i.e. we can cover the domain $\Omega$ with finite number $\Omega_i$ of the subdomains $\Omega_i$ so in every there exists coefficients of the equation satisfies condition (1.4) in the image of subdomains $\Omega_i$.

Denote $\dot{W}_p^2(\Omega)$, $p>1$ by the closure of functions $u\in C^\infty\left(\overline{\Omega}\right)\cap C\left(\overline{\Omega}\right)$, $u|_{\partial\Omega}=0$ of class by norm

$$\|u\|_{W_p^2(\Omega)}=\left[\int\limits_{\Omega}\left(|u|^p+\sum_{i=1}^{n}|u_i|^p+\sum_{i,j=1}^{n}|u_{ij}|^p\right)dx\right]^{1/p}.$$

Here for $u_i, u_{ij}$ denote by the derivative $u_{x_i}, u_{x_i x_j}$ respectively $i,j = 1,...n$. Denote $p'$ by a conjugate number ,i.e. $1 < p < \infty$, $\frac{1}{p'} + \frac{1}{p} = 1$.

The notation $C_{i.j}(...)$ means that the positive constant $C_{i.j}$ depends only on the content of brackets,but $C_{i.j}(\Omega)$ means that it depends only on smoothness $\Omega$.

The function $u(x) \in \dot{W}_p^2(\Omega)$ is called a strong solution (a.e.) of the problem (1.1), (1.2), if it satisfies the equation (1.1) a.e. in $\Omega$.

We observe that in the case of linear equations the questions of the strong solvability of elliptic and parabolic equations with discontinuous coefficients satisfying Cordes condition,is studied in [1-3]. Obtaining of the estimation

$$\int_\Omega (\Delta u)^2 dx \le C_{1.1}(n,\gamma,\delta) \int_\Omega Lu \cdot \Delta u dx. \tag{1.5}$$

playes the main role in study of linear equations.

It is clear by example (see [4, p. 48], that if coefficients of the operator $L$ are discontinuous and the Cordes condition is not valid,then equation $Lu = f$ is non-solvable in $\dot{W}_p^2(\Omega)$ for any $p > 1$.

As to the strong solvability for linear elliptic equations with continuous coefficients in $\dot{W}_p^2(\Omega)$ (for arbitrary $p > 1$) we refer to papers [4,5]. For this kind of equations with coefficients from $VMO$ class the questions of the strong solvability in the $\dot{W}_p^2(\Omega)$ spaces for arbitrary $p > 1$ is considered in [6-8].

Note that,study of semilinear equations (1.1) with small non-linearity $(0 < q < 1)$ doesn't require the restriction on norm of right-hand side [9].

In the case ,when the solution of problem (1.1),(1.2) (for $\Omega = R_n$) can be written as a non-linear integral equation

$$u(x) = \int_\Omega G(x,y)\,|u(x)|^q\, dy + f(x), \tag{1.6}$$

where $G(x,y) = |x-y|^{\alpha - n}$ $(0 < \alpha < n)$ is Riesz kernel ,the most general criterion of solvability of non-linear integral equations (1.6) in terms of non-linear capacity are considered in works [10-12]. If the operator $L$ has not sufficient smooth coefficients,then the cited results cannot be applied to the problem (1.1), (1.2) for non-linear integral equations. Conserning it we note that on the case of semilinear equation with discontinuous coefficients and with linear elliptic operator in the principal part has not been studied.

The aim of the present article is to prove the following :

1) the strong solvability of the Dirichlet problem (1.1),(1.2) in the $\dot{W}_2^2(\Omega)$ spaces, when the coefficients are discontinuous and satisfy the Cordes condition, and certain conditions are imposed on $f(x) \in L_2(\Omega)$ and $q$.

2) the strong solvability of the Dirichlet problem (1.1),(1.2) in the $\dot{W}_p^2(\Omega)$, $1 < p < \infty$ spaces in the case when coefficients are continuous and $f(x) \in L_p(\Omega)$ satisfies certain conditions.

## 2. Equations with discontinuous coefficients

**Theorem 2.1.** *Let $n > 4$, $1 < q < \frac{n}{n-4}$ and relative to the coefficients of the operator $L$ the conditions (1.3), (1.4) be satisfied, $\partial\Omega \in C^2$. Then for any*

$f(x) \in L_2(\Omega)$ *satisfying the condition*

$$\|f\|_{L_2(\Omega)} \leq C_{2.1}(n,\gamma,q,\delta,\Omega)\,(mes_n\Omega)^{\frac{-n+(n-4)q}{2n(q-1)}}.$$

*the Dirichlet problem (1.1), (1.2) has at least one solution from* $\dot{W}_2^2(\Omega)$.

**Proof.** We apply the Shauder method on continuous map of convex and compact set into itself in Banach spaces (see. [5, p. 257]).

As the Banach spaces we take $L_{2q}(\Omega)$. In this space we define the set of functions $V_2 = \{u \in \dot{W}_2^2(\Omega) \big| \ \|u\|_{W_2^2(\Omega)} \leq K\}$, where number $K$ will be choosen later. For each $u(x) \in L_{2q}(\Omega)$, $f(x) \in L_2(\Omega)$ denote $v(x) \in \dot{W}_2^2(\Omega)$ by the solution of the following problem

$$Lv + |u|^{q-1}u = f(x), \quad x \in \Omega, \tag{2.1}$$

$$u|_{\partial\Omega} = 0. \tag{2.2}$$

The problem(2.1),(2.2) is solvable in the $\dot{W}_2^2(\Omega)$ spaces for arbitrary $u(x) \in V_2$, $f(x) \in L_2(\Omega)$, such that at this conditions we come to the Dirichlet problem for equation

$$Lv = F(x), \quad x \in \Omega, \tag{2.3}$$

where $F = f(x) - |u|^{q-1}u \in L_2(\Omega)$ (see[13]).

Indeed

$$\|F\|_{L_2(\Omega)} \leq \|f\|_{L_2(\Omega)} + \left\||u|^{q-1}u\right\|_{L_2(\Omega)} = \|f\|_{L_2(\Omega)} + \|u\|^q_{L_{2q}(\Omega)}.$$

$\|u\|_{L_{2q}(\Omega)}$ is finite by virtue of that the space $\dot{W}_2^2(\Omega)$ continuously embeds to $L_{2q}(\Omega)$ when $1 \leq q < \frac{n}{n-4}$, consequently $F \in L_2(\Omega)$ (see[5,p.154]).

Denote by $A$ on operator which throws $u$ to $v$ :

$$Au = v.$$

Show continuity of the operator $A$ in $L_{2q}(\Omega)$.

Let $u_n \to u_0$ in $L_{2q}(\Omega)$ in $n \to \infty$, where $u_n$, $u_0 \in L_{2q}(\Omega)$ and $v_n = Au_n$; $v_0 = Au_0$. Then

$$\begin{aligned} Lv_n &= -|u_n|^{q-1}u_n + f \\ Lv_0 &= -|u_0|^{q-1}u_0 + f \end{aligned} \tag{2.4}$$

Show that $v_n \to v_0$ in norm of the $L_{2q}(\Omega)$ space. We have:

$$L(v_n - v_0) = -\left(|u_n|^{q-1}u_n - |u_0|^{q-1}u_0\right). \tag{2.5}$$

Multiply by $\Delta(v_n - v_0)$ both sides of the equality (2.5), we obtain

$$L(v_n - v_0)\,\Delta(v_n - v_0) = -\left(|u_n|^{q-1}u_n - |u_0|^{q-1}u_0\right)\Delta(v_n - v_0).$$

Hence, by virtue of estimate (1.5) and Hölder inequality,we obtain

$$\int_\Omega [\Delta(v_n - v_0)]^2\,dx \leq$$

$$\leq C_{2.2}(n,\gamma,\delta)\left(\int\limits_{\Omega}[\Delta(\upsilon_n-\upsilon_0)]^2dx\right)^{1/2}\left\||u_n|^{q-1}u_n-|u_0|^{q-1}u_0\right\|_{L_2(\Omega)}. \quad (2.6)$$

Let 's estimate the second multiplier of the right-hand of the inequality (2.6) from above. We have:

$$\left||u_n|^{q-1}u_n-|u_0|^{q-1}u_0\right|\leq q\left(|u_n|^{q-1}-|u_0|^{q-1}\right)|u_n-u_0|.$$

It' s obvious

$$\left\||u_n|^{q-1}u_n-|u_0|^{q-1}u_0\right\|_{L_2(\Omega)}\leq$$

$$\leq q\left\|(u_n-u_0)|u_n|^{q-1}\right\|_{L_2(\Omega)}+q\left\|(u_n-u_0)|u_0|^{q-1}\right\|_{L_2(\Omega)}. \quad (2.7)$$

Applying the Hölders inequality to inequality (2.7), we have

$$\left\||u_n|^{q-1}u_n-|u_0|^{q-1}u_0\right\|_{L_2(\Omega)}\leq q\|u_n-u_0\|_{L_{2q}(\Omega)}\left(\|u_n\|^{q-1}_{L_{2q}(\Omega)}+\|u_0\|^{q-1}_{L_{2q}(\Omega)}\right). \quad (2.8)$$

By virtue of $u_n\to u_0$ in $L_{2q}(\Omega)$ follows $\sup\|u_n\|_{L_{2q}(\Omega)}<\infty$, therefore from (2.8) and (2.6) we obtain:

$$\|\Delta(\upsilon_n-\upsilon_0)\|_{L_2(\Omega)}\to 0 \quad (n\to\infty).$$

Applying the estimate

$$\|\upsilon_n-\upsilon_0\|_{W_2^2(\Omega)}\leq C_{22}\|\Delta(\upsilon_n-\upsilon_0)\|_{L_2(\Omega)},$$

we have

$$\|\upsilon_n-\upsilon_0\|_{W_2^2(\Omega)}\to 0 \quad (n\to\infty). \quad (2.9)$$

By virtue of the embedding theorem $W_2^2(\Omega)\to L_{2q}(\Omega)$, we have

$$\|\upsilon_n-\upsilon_0\|_{L_{2q}(\Omega)}\to 0 \text{ for } n\to\infty.$$

The continuity of the operator $A$ is proved.

Show that the set $V_2$ is convex and compact in $L_{2q}(\Omega)$ and the operator $A$ throws it into itself.

For $u_1$, $u_2\in V_2$ and $\upsilon=tu_1+(1-t)u_2$, $t\in[0,1]$ we have

$$\|\upsilon\|_{W_2^2(\Omega)}=\|tu_1+(1-t)u_2\|_{W_2^2(\Omega)}\leq$$

$$\leq t\|u_1\|_{W_2^2(\Omega)}+(1-t)\|u_2\|_{W_2^2(\Omega)}\leq tK+(1-t)K=K, \quad (2.10)$$

that means convexity $V_2$.

By virtue of the compact embedding theorem

$$W_2^2\hookrightarrow L_{2q} \text{ for } 1<q<\frac{n}{n-4} \text{ the set } V_2\subset L_{2q}(\Omega) \quad (2.11)$$

is compact.

Show that for certain choose $K$ the operator $A$ throws $V_2$ into itself.

For solution of the Dirichlet problem of equation (2.3) we have

$$\|v\|_{W_2^2(\Omega)} \le C_{2.3}(\delta,\gamma,n)\,\|F\|_{L_2(\Omega)} \le C_{2.3}\left[\left\||u|^{q-1}u\right\|_{L_2(\Omega)} + \|f\|_{L_2(\Omega)}\right]. \qquad (2.12)$$

Further

$$\left\||u|^{q-1}u\right\|_{L_2(\Omega)} = \left(\int_\Omega |u|^{2q}\,dx\right)^{1/2} \le (mes_n\Omega)^{1/2-\frac{n-4}{2n}q}\left(\int_\Omega u^{\frac{2n}{n-4}}dx\right)^{\frac{n-4}{2n}q} \le$$

$$\le (mes_n\Omega)^{1/2\left(1-\frac{n-4}{n}q\right)} C_{2.4}(n,\Omega,q)\,\|u\|^q_{W_2^2(\Omega)} \qquad (2.13)$$

Here we use the above mentioned embedding theorem (2.11).

Allowing the estimate (2.13) in (2.12), we obtain:

$$\|v\|_{W_2^2(\Omega)} \le C_{2.5}\left[(mes_n\Omega)^{1/2\left(1-\frac{n-4}{n}q\right)}\|u\|^q_{W_2^2(\Omega)} + \|f\|_{L_2(\Omega)}\right] \le$$

$$\le C_{2.5}\left[K^q(mes_n\Omega)^{1/2\left(1-\frac{n-4}{n}q\right)} + \|f\|_{L_2(\Omega)}\right], \qquad (2.14)$$

where $C_{2.5} = C_{2.5}(n,\delta,\gamma,q,\Omega)$.

Let 's require that $K$ satisfies the following estimation:

$$C_{2.5}\left[K^q(mes_n\Omega)^{1/2\left(1-\frac{n-4}{n}q\right)} + \|f\|_{L_2(\Omega)}\right] \le K. \qquad (2.15)$$

For existence this number $K$, $\|f\|_{L_2(\Omega)} \le C_{2.6}(mes_n\Omega)^{-\left(\frac{n-(n-4)q}{2n(q-1)}\right)}$ is sufficient, where $C_{2.6} = C_{2.6}(n,\delta,\gamma,q,\Omega)$.

Indeed,let' s introduced the following notation:

$$a = (mes_n\Omega)^{1/2\left(1-\frac{n-4}{n}q\right)}, \quad b = \|f\|_{L_2(\Omega)}.$$

Then the inequality (2.15) has a view:

$$aK^q + b \le K, \quad \text{i.e.} \quad aK^q - K + b \le 0, \quad K > 0. \qquad (2.16)$$

The function $F(K) = aK^q - K$, $K \ge 0$, attain the minimally value when $K_0 = \left(\frac{1}{qa}\right)^{\frac{1}{q-1}}$. Indeed, $f'(K) = aqK^{q-1} - 1$ , then for $K_0^{q-1} = \frac{1}{qa}$ we have $f'(K_0) = 0$, $f''(K_0) > 0$. Consequently, for $b \le f(K_0)$ the inequality (2.16) is solvable relative to the $K$.

The theorem 2.1 is proved.

In the case $1 \le n \le 4$ the following is valid:

**Theorem 2.2.** *Let relative to the coefficients of the operator $L$ the conditions (1.3) and (1.4) be satisfied and $1 \le n < 4$ $(n = 4)$, $1 < q < \infty$, $\partial\Omega \in C^2$. Then for any $f(x) \in L_2(\Omega)$ satisfying condition*

$$\|f\|_{L_2(\Omega)} \le C_{2.7}(n,\gamma,\delta,q,\Omega)(mes_n\Omega)^{-\frac{n+q(n-4)}{2n(q-1)}}$$

$$\left(\|f\|_{L_2(\Omega)} \le C_{2.8}(n,\gamma,\delta,q,\Omega)(mes_n\Omega)^{-\frac{1}{2(q-1)}}\right).$$

*the Dirichlet problem (1.1),(1.2) has at least one solution from* $\dot{W}_2^2(\Omega)$.

In proof of this theorem for the Banach space is taken the space $C\left(\overline{\Omega}\right)$ $(L_{2q}(\Omega))$ and applies the compact embedding theorem

$$W_2^2(\Omega) \to C\left(\overline{\Omega}\right)\ \left(W_2^2(\Omega) \to L_{2q}(\Omega)\right).$$

**3. Equations with continuous coefficients**

In this punct we consider the Dirichlet problem for semilinear elliptic equations (1.1) with continuous coefficients.

Otherwise,we consider the Dirichlet problem in $\Omega$

$$\sum_{i,j=1}^{n} a_{ij}(x)\, u_{ij} + |u|^{q-1} u = f(x)\,, \quad x \in \Omega, \tag{3.1}$$

$$u|_{\partial\Omega} = 0. \tag{3.2}$$

when the coefficients $a_{ij}(x)$, $i,j = 1,2,...,n$ of operator $L = \sum\limits_{i,j=1}^{n} a_{ij}(x) \dfrac{\partial^2}{\partial x_i \partial x_j}$ is a bounded measurable functions satisfying the conditions (1.3) and

$$a_{ij}(x) \in C\left(\overline{\Omega}\right), \quad i,j = 1,2,...,n. \tag{3.3}$$

Note that in the previous punct we are satisfied the case $1 \le q < \dfrac{n}{n-4}$, $n > 4$ in equation (1.1). It is connected with application of exictence aprior estimation (1.5) to equation with discontinuous coefficients.In the case equation with continuous coefficients we apply aprior estimation in $W_p^2(\Omega)$ (5, lemma 9.17). This estimation allows to consider an arbitrary exponent of nonlinearity of the equation (1.1).

The following is valid:

**Theorem 3.1.** *Let relative to the coefficients of the operator* $L$ *the conditions (1.3),(3.3) be satisfied and* $1 < q < \infty$, $p > \dfrac{n}{2p'}$, $\partial\Omega \in C^2$. *Then for any* $f(x) \in L_p(\Omega)$ *satisfying condition*

$$\|f\|_{L_p(\Omega)} \le C_{3.1}(\gamma, n, q, \delta, \Omega)\,(mes_n\Omega)^{-\frac{n+(n-2p)q}{pn(q-1)}} \tag{3.4}$$

*the Dirichlet problem (3.1), (3.2) has at least one solution from* $\dot{W}_p^2(\Omega)$.

**Proof.** Let $V_p = \{u \in \dot{W}_p^2(\Omega) \Big|\, \|u\|_{W_p^2(\Omega)} \le K\}$.

Denote by the $\upsilon(x) \in \dot{W}_p^2(\Omega)$ the solution of problem

$$L\upsilon + |u|^{q-1} u = f(x)\,, \quad x \in \Omega, \tag{3.5}$$

$$\upsilon|_{\partial\Omega} = 0. \tag{3.6}$$

where $u(x) \in L_{pq}(\Omega)$ is arbitrary function.

The problem (3.5)-(3.6) is solvable in the $\dot{W}_p^2(\Omega)$ space for arbitrary $u(x) \in V_p$, $f \in L_p(\Omega)$, such that at this conditions we have a deal with solvability of the Dirichlet problem (2.3), where $F = f(x) - |u|^{q-1} u \in L_p(\Omega)$ (on solvability of the Dirichlet problem see. [5, theorem 9.15]).

By virtue of embedding $W_2^2(\Omega) \to L_{2q}(\Omega)$ we have

$$\|F\|_{L_p(\Omega)} \le \|f\|_{L_p(\Omega)} + \left\| |u|^{q-1} u \right\|_{L_p(\Omega)} = \|f\|_{L_p(\Omega)} + \|u\|^q_{L_{pq}(\Omega)} \ ,$$

consequently $F \in L_p(\Omega)$.

Denote by $A$ on operator which throws $u$ to $\upsilon$. Show continuity of the operator $A$. Let $u_n \to u_0$ in $L_{pq}(\Omega)$ at $n \to \infty$, $u_n,\ u_0 \in L_{pq}(\Omega)$ and $\upsilon_n,\ \upsilon_0$ is the solution of problem (2.4) from $\dot{W}{}_p^2(\Omega)$ space. Then for $\upsilon_n - \upsilon_0$ we have equality (2.5). By virtue of aprior estimation

$$\|\upsilon\|_{W_p^2(\Omega)} \le C_{3.2}(n,\gamma,\Omega,p)\, \|L\upsilon\|_{L_p(\Omega)}\,, \quad \upsilon \in \dot{W}{}_p^2(\Omega)\,, \tag{3.7}$$

we have

$$\|\upsilon_n - \upsilon_0\|_{W_p^2(\Omega)} \le C_{3.2} \left\| |u_n|^{q-1} u_n - |u_0|^{q-1} u_0 \right\|_{L_p(\Omega)} \le q \left\| (u_n - u_0)\, |u_n|^{q-1} \right\|_{L_p(\Omega)} +$$

$$+q \left\| (u_n - u_0)\, |u_0|^{q-1} \right\|_{L_p(\Omega)} \le q\, \|u_n - u_0\|_{L_{pq}(\Omega)} \left( \|u_n\|^{q-1}_{L_{pq}(\Omega)} + \|u_0\|^{q-1}_{L_{pq}(\Omega)} \right). \tag{3.8}$$

Such that from $u_n \to u_0$ in $L_{pq}(\Omega)$ follows $\sup \|u_n\|_{L_{pq}(\Omega)} < \infty$, so we obtain

$$\|\upsilon_n - \upsilon_0\|_{W_p^2(\Omega)} \to 0 \quad (n \to \infty)\,.$$

Hence

$$\|\upsilon_n - \upsilon_0\|_{L_{pq}(\Omega)} \to 0 \quad for \quad n \to \infty.$$

The compactness $V_p$ follows from compact embedding theorem $W_p^2(\Omega)$ in $L_{pq}(\Omega)$, but convexity is obvious.

Show that for certain choose $K$ the operator $A$ throws $V_p$ into itself.

For solution of the Dirichlet problem of the equation (3.1) we obtain

$$\|\upsilon\|_{W_p^2(\Omega)} \le C_{3.3}(\gamma,n)\, \|L\upsilon\|_{L_p(\Omega)} \le C_{3.3} \left[ \left\| |u|^{q-1} u \right\|_{L_p(\Omega)} + \|f\|_{L_p(\Omega)} \right] \le$$

$$\le C_{3.4} \left[ (mes_n\Omega)^{1/p\left(1 - \frac{n-2p}{n} q\right)} \|u_n\|^q_{W_p^2(\Omega)} + \|f\|_{Lp(\Omega)} \right] \le$$

$$\le C_{3.4} \left[ (mes_n\Omega)^{1/p\left(1 - \frac{n-2p}{n} q\right)} K^q + \|f\|_{L_p(\Omega)} \right],$$

where $C_{3.4} = C_{3.4}(\gamma, n, \Omega, p, q)$.

If $f \in L_p(\Omega)$ to choose from condition

$$\|f\|_{L_p(\Omega)} \le C_{3.5}\, (mes_n\Omega)^{-\frac{n+(n-2p)q}{pn(q-1)}} \ ,$$

where $C_{3.5} = C_{3.5}(\gamma, n, \Omega, p, q)$, then inequality

$$C_{3.4} \left[ K^q\, (mes_n\Omega)^{1/2\left(1 - \frac{n-2p}{n} q\right)} + \|f\|_{L_p(\Omega)} \right] \le K$$

is solvable relative to the $K > 0$, i.e. the operator $A$ throws $V_p$ into itself.

The theorem 3.1 is proved.

**Remark 3.1.** All theorems of present article are valid also for equation

$$Lu - |u|^{q-1} u = f(x), \quad x \in \Omega.$$

**Remark 3.2.** The affirmation of theorem 3.1 also is valid for nonlinear equation

$$\sum_{i,j=1}^{n} a_{ij}(x, u, u_x) u_{x_i x_j} + |u|^{q-1} u = f(x)$$

with condition of Cordes type

$$\operatorname*{ess\,sup}_{x\in\Omega} \sum_{i,j=1}^{n} \frac{a_{ij}^2(x,\xi,\eta)}{\left(\sum\limits_{i=1}^{n} a_{ii}(x,\xi,\eta)\right)^2} \le \frac{1}{n-1}; \ \xi \in E_1, \ \eta \in E_n.$$

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**Farman I. Mamedov, Shahla Yu. Salmanova**
Institute of Mathematics and Mechanics of NAS of Azerbaijan.
9, F. Agayev str., AZ1141, Baku, Azerbaijan.
Tel.: (99412) 439 47 20 (off.)

Received July 10, 2007; Revised October 22, 2007.