APPLIED PROBLEMS OF MATHEMATICS AND MECHANICS

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MATHEMATICAL MODELING OF MOTION OF AVALANCHE - LIKE FLOWS AND CALCULATION OF DETERMINING PARAMETERS

Abstract

In the paper, mathematical model of motion of different type multi -phase media on inclined surface describing the current avalanche-like flows is constructed on the basis of averaging method. We consider turbulence models of different complexity with regard to rheology of multi-phase media by means of which the parameters of turbulent transfer applaring in averagied flow equations may be given.

1. Introduction. The structure of avalanche-like flow may be represented in the form of granulated liquid that is a group of discrete solid particles dispersed in liquid in such a way that the particles of solid component are in contact with neighboring particles.

Landslips, mudflows, snow avalanches, floadings are dangerous natural processes and they create obstacles in constructing civil and industrial buildings, roads, developing lands and extracting mineral products. They are dangerous for human life. At present everywhere realization of measures in engineering defence of terretories and objects is not always sensible and economically warranted. Therefore, study of motion of landslips, mudflows, snow avalanches and prediction of development of these processes are very urgent and have practical value.

The requirement to mathematical models of motion of lavanche-like like flows is that the model must be constructed on the base of mechanics of composite media in the frames of macroscopic - phenomenological representations and simultaneously possess the properties of granulated medium (hard component), liquid and gas (water, air).

2. Mathematical model. In describing motion of avalanche-like flow we use a model of multi-phase medium that ”pours” down (motion arises from rest state). Variable steepness slope- that is long and wide in the form of variable length and depth channel, i.e. the effects related with interactions on lateral boundaries of the flow, are taken into account. The flow motion is subjected to the action of gravity force and friction force. This model may be constructed by various methods. In some cases for each mixture component we can write a sysyem of equations and phase interactions take into account with the help of additional terms. In other cases we can approximately represent multi-phase flow as homogeneous one with averaged properties. The advantage of the last method is comparative simplicity of statement and solution of practical problems. Approximation degree of the obtained results that in some cases are far from truth, is the defect of this method.

In models considered up to now where turbulence of the flow is taken into account [1, p. 57], the multi-phase property was ignored, if multi-phase property was taken into account, the turbulence was ignored [2]. It was assumed that for describing turbulence at a point it suffices to know only the scale of velocity, but stress components may be expressed by this scale by means of Kolmogorov-Prandtl relation for turbulent viscosity. However, in transferring separate components, the stresses

are inadequately reflected by these relations, even if the transfer of velocity scale is correctly described. In complicated turbulent flows of multi-phase avalanche-like media, difference in evolution of different Reynolds stress components should be taken into consideration and transfer of these components must be taken properly into account. To this end the models using transfer equations for separate components of turbulent stresses $\overline{U_i \cdot U_j}$ [3] must be worked out. These equations may be derived exactly, but for obtaining a close system, modal relations must be introduced as well. The advantage of exact equations is that there automatically appear the terms taking into account buoyancy forces, rotational motions and other factors and therefore they are very complicated, especially if they satisfy invariance and realizability requirements. Therefore, inspite of their great potential abilities they are not frequently used in practice up to now. In motion of various character avalanche-like flows, consideration of temperature change of medium is inevitable. We can assume that exact equation for thermoelastic Reynolds stresses (for Reynolds great numbers), small scale dissipative turbulent motions are isotropic (local isotropy). Then we can express the dissipative term as follows:

$$\varepsilon_{ij} = \frac{2}{3}\varepsilon\sigma_{ij}$$

Here ε is velocity of dissipation of kinetic energy $K = \frac{1}{2}\overline{U_i \cdot U_j}$, (scalar products of the same name quantities are denoted by dash).

Exact equation for a general case is not cited here because of its awkwardness.

In many cases mean value of flow characteristics change slowly in vertical direction, so, change of these quantities in horizontal direction may be determined by solving two-dimensional equations of mean flow for values averaged in depth.

Integrating three-dimensional equations in depth and assuming that pressure distribution is hydrostatic, we can derive mean flow equations for quantities averaged in depth. As a result we get:

Continuity equation

$$\frac{\partial h}{\partial t} + \frac{\partial}{\partial x}(h\overline{u}) + \frac{\partial}{\partial y}(h\overline{v}) = 0 \tag{1}$$

Equation of motion amount (in direction X)

$$\frac{\partial \overline{u}}{\partial t} + \overline{u}\frac{\partial \overline{u}}{\partial x} + v\frac{\partial \overline{v}}{\partial y} = -g\frac{\partial}{\partial x}(h + z_b) +$$

$$+\frac{1}{\rho h}\left(\frac{\partial(h\overline{\tau}_{xx})}{\partial x} + \frac{\partial(h\overline{\tau}_{xx})}{\partial y}\right) + \frac{\tau_{sx} - \tau_{bx}}{\rho h} + \frac{1}{\rho h}\left(\frac{\partial \overline{F}}{\partial x} + \frac{\partial \overline{G}}{\partial y}\right) \tag{2}$$

Equation of motion amount (in direction Y)

$$\frac{\partial \overline{v}}{\partial t} + \overline{u}\frac{\partial \overline{v}}{\partial x} + \overline{v}\frac{\partial \overline{v}}{\partial y} = -g\frac{\partial}{\partial y}(h + z_b) +$$

$$+\frac{1}{\rho h}\left(\frac{\partial(h\overline{\tau}_{xy})}{\partial x} + \frac{\partial(h\overline{\tau}_{yy})}{\partial y} + \tau_{sy} - \tau_{by} + \frac{\partial \overline{F}}{\partial x} + \frac{\partial \overline{G}}{\partial y}\right) \tag{3}$$

Equation of medium temperature

$$\frac{\partial \overline{\Phi}}{\partial t} + \overline{u}\frac{\partial \overline{\Phi}}{\partial x} + \overline{v}\frac{\partial \overline{\Phi}}{\partial y} =$$

$$= \frac{1}{\rho h} \left(\frac{\partial \left(h \overline{J_x} \right)}{\partial x} + \frac{\partial \left(h \overline{J_y} \right)}{\partial y} + q_s + \frac{\partial \overline{P}}{\partial x} + \frac{\partial \overline{Q}}{\partial y} \right) \qquad (4)$$

The last two terms in equations (2)-(4) are dispersing.

Here $\overline{u}$ and $\overline{v}$ are horizontal velocities; $h(t, x, y)$ is flow depth, τ_s, τ_b are tangential stresses of surface and bottom, respectively, q_s is distributed heat flow through surface (heat flow from bottom may be assumed to be equal to zero in majority of cases), are denoted:

$$\overline{F} = \int_{z_b}^{z_b+h} \rho (u - \overline{u})(v - \overline{v}) \, dz; \qquad\qquad \overline{G} = \int_{z_b}^{z_b+h} \rho (u - \overline{u})^2 \, dz;$$

$$\overline{P} = \int_{z_b}^{z_b+h} P (u - \overline{u})(v - \overline{v}) \, dz; \qquad\qquad \overline{Q} = \int_{z_b}^{z_b+h} \rho (v - \overline{v})^2 \, dz;$$

$$\overline{\Phi} = \frac{1}{h} \int_{z_b}^{z_b+h} \Phi \, dz; \qquad \overline{U} = \frac{1}{h} \int_{z_b}^{z_b+h} U \, dz; \qquad \overline{v} = \frac{1}{h} \int_{z_b}^{z_b+h} v \, dz;$$

z_b are coordinates of inchined surface streamlined by multi-phase, multi-component flow.

Now, turbulence models are necessary in order to determine depth averaged values of turbulent stresses $\overline{\tau}_{ij}$ and depth averaged flows $\overline{J}_i$ and also bottom tangential stresses (friction stresses) τ_b.

Flow shift at the surface (wind shift) and heat influx from the heat surface is determined by turbulent average flow of mixed mass. The quantities τ_b and q_s are usually determined by means of simple empiric laws that contain friction coefficients and heat transfer. In the equations mentioned above, in many practice situations we can neglect the terms containing turbulent stresses $\overline{\tau}_{ij}$ averaged in depth in comparison with other terms, and effect of turbulence tells on tangential bottom stresses τ_b.

In these cases only relation of bottom tangential stresses depth averaged velocities should be determined in model. To this end we use the relation derived from the friction law

$$\tau_{bx} = \rho \cdot c_f \overline{u} \sqrt{\overline{u}^2 + \overline{v}^2} \; ; \qquad \tau_{by} = \rho \cdot c_f \overline{v} \sqrt{\overline{u}^2 + \overline{v}^2} \qquad (5)$$

where c_f is empirical friction coefficient dependent on the state of the given surface the values for smooth and rough bottoms may be found in technical references).

Unlike turbulent stresses, turbulent mass and heat flows $\overline{J}_i$ always play an important role and require model assumptions.

In equations (1)-(4) there are also so-called dispersive terms that appear because of vertical inhomogeneity of quantities related with mean flow. These terms have no connection with turbulence, they arise only because of depth averaging process and modelling of these terms is not turbulence modelling. All the turbulence models considered here, are based on turbulent viscosity (diffusion) concept. A great majority of investigations of such content assumptions such as steadyness of turbulent viscosity (diffusion) coefficients whose acceptance essentially simlifies the mathematical problem.

When turbulence is generated mainly in bottom and lateral areas (e.g. flow in chanel and old riversbeds) depth averaged coefficient of turbulent diffusion is well described by the expression that contains dynamic velocity u_τ and flow depth h :

$$\overline{R}_t = c \cdot \overline{u}_\tau \cdot h \qquad (6)$$

Here c is an empiric constant depending on flow geometry ($c \approx 0,135 \div 0.275$) and it changes depending on the ratio of flow width to its depth.

Notice that in calculations of depth averaged quantities the depth average value of the turbulent diffusion coefficient $\overline{R}_t$ belong to transfer on the whole not only to turbulent but also to dispersive one that arises because of heregeneity of vertical distribution of velocities, temperatures and concentrations.

Potentially universal model of motion of avalanche like flows should allow consideration of backround outbursts to environment. Limit values of quantities typical for free flow are given on free boundaries. On rigid boundaries (lateral surface and botton) immediately after viscous underlayer it is given resultunt velocity (u_{pez}) allowing for boundary layer (of variable thickness) because of adhesion of heteregeneous media:

$$(u_{pez}) = \frac{u_\tau}{\chi} \ln \left(y_0^+ \cdot E_0 \right) \qquad (7)$$

where $y_0^+ = y_0 \cdot u_\tau / v$; χ is the Carman constant ($\sim 0, 4$), E_0 is roughness coefficient ($\sim 9 \div 17$), the point y_0 is chosen so that y_0^+ the dimensionless distance from the wall y_0^+ is in the interval $30 \div 100$; v is kinematic molecular viscosity coefficient.

Conclusions. In the paper we considered different degree complexity turbulence models with regard to rheology of multi-phase madia by whose means we can give turbulent transfer parameters appearing in averaged flow equations. These models were worked out to investigate natural phenomena such as motion of avalanche-like flows and airdynamics of environment in nature catastrophes as floodings and etc.

Elaboration of effective methods of solution of practical problems on the base of the suggested model will be represented in future papers of the author.

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