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NONSTATIONARY WAVES PROPAGATION IN VISCOELASTIC RECTANGULAR BARS

Abstract

The nonstationary wave propagation process in viscoelastic rectangular bars subjected to the action of suddenly applied axial forces, is studied.

Based on the results of [4], a method for construction of a solution, taking into account of properties of viscoelastic relation between strains and stress, is worked out. The solution is obtained for the case of Kelvin-Foight material and it was reduced to numerical calculations and illustrations.

In the present paper the nonstationary waves propagation in semi-infinite rectangular bars, whose material possesses the viscoelastic properties, is investigated. At that we accept that stress deviator linearly depends both on strain deviator, and on its time derivatives. Problems on waves propagation in viscoelastic and viscoplastic bars occur in the papers of I.I. Zverev [2], E.G.Li and I.Kanter [3] considered elastoviscous bars of finite length.

To the problem of dynamics of elastic rectangular bars were devoted the papers [4,5], where it was obtained analytical solutions for some types of boundary conditions. The present paper is viscoelastic analogy of the problem considered in [4], for Kelvin-Foight material.

Determinative relations describing this material are accepted in the form:

$$\begin{aligned} s_{ij} &= 2\mu_0\left(e_{ij} + \eta\frac{\partial e_{ij}}{\partial t}\right) \\ \sigma_{kk} &= 3K_0\varepsilon_{ii} \end{aligned} \tag{1}$$

here η is coefficient of viscosity, μ_0, K_0 are elactic constants, s_{ij} and e_{ij} are components of stress and strain deviators:

$$\begin{aligned} e_{ij} &= \varepsilon_{ij} - \frac{1}{3}\delta_{ij}\varepsilon_{kk} \\ s_{ij} &= \sigma_{ij} - \frac{1}{3}\delta_{ij}\varepsilon_{kk} \end{aligned}$$

where σ_{ij}, ε_{ij} are the real components of corresponding tensors.

Impart to (1) the classic form:

$$\sigma_{ij} - K_0\delta_{ij}\varepsilon_{kk} = 2\mu_0\left(\varepsilon_{ij} - \frac{1}{3}\delta_{ij}\varepsilon_{kk} + \eta\frac{\partial\varepsilon_{ij}}{\partial t} - \frac{1}{3}\eta\delta_{ij}\frac{\partial\varepsilon_{kk}}{\partial t}\right)$$

Hence

$$\sigma_{ij} = 2\mu_0\left(\varepsilon_{ij} + \eta\frac{\partial\varepsilon_{ij}}{\partial t}\right) + \frac{1}{3}\delta_{ij}\left[\left(3K_0 - 2\mu_0\right)\varepsilon_{kk} - 2\eta\mu_0\frac{\partial\varepsilon_{kk}}{\partial t}\right]$$

that after Laplace transformation has the following form:

$$\overline{\sigma}_{ij} = \frac{1}{3}\delta_{ij}\left(3K_0 - 2\mu_0 - 2\eta p\right)\overline{\varepsilon}_{kk} + 2\mu\left(1+\eta p\right)\overline{\varepsilon}_{ij}$$

reminding "elastic" relations:

$$\overline{\sigma}_{ij} = \delta_{ij}\lambda\left(p\right)\varepsilon_{kk} + 2\mu\left(p\right)\varepsilon_{ij} \tag{2}$$

here

$$\lambda\left(p\right) = \frac{3K_0 - 2\mu\left(p\right)}{3}; \qquad \mu\left(p\right) = \mu_0\left(1+\eta p\right) \tag{3}$$

where p is parameter of Laplace transformation.

The motion equations are

$$\begin{array}{l} div\ \overrightarrow{T}_k = \rho\dfrac{\partial^2 U_k}{\partial t^2} \\ \overrightarrow{U} = \overrightarrow{U}\left(u,\vartheta,w\right) = \overrightarrow{i}_k U_k \\ \overrightarrow{T}_k = \overrightarrow{i}_r \sigma_{kr} \end{array} \tag{4}$$

where $\overrightarrow{U}$ is displacement vector, and i_k are unit orts of coordinate axis, with the boundary and initial conditions:

$$\left.\begin{array}{l} u = \vartheta = w = 0 \\ \dfrac{\partial u}{\partial t} = \dfrac{\partial \vartheta}{\partial t} = \dfrac{\partial w}{\partial t} = 0 \end{array}\right\} \text{ for } \ t = 0 \tag{5}$$

$$\left.\begin{array}{l} \sigma_{33} = \sigma_0\left(x,y\right)f\left(t\right) \\ u = \vartheta = 0 \end{array}\right\} \text{ for } \ z = 0 \tag{6}$$

$$\left.\begin{array}{l} \sigma_{11} = 0 \\ \vartheta = w = 0 \end{array}\right\} \text{ for } \ x = \pm a \tag{7}$$

$$\left.\begin{array}{l} \sigma_{22} = 0 \\ u = w = 0 \end{array}\right\} \text{ for } \ y = \pm b \tag{8}$$

with taking into account (1) form closed system in the domain $-a \leq x \leq a\,;\ -b \leq y \leq b\,;\ z \geq 0$. As in [4], in view of statement multidimensionality, for two variables Laplace and Fourier transformations is applied. Relations (2) keep its form after Fourier integrals effect as well, and under these conditions the solutions, obtained in [4], in transformed planes completely determine here solution of problem (2)-(8) as well, therewith difference that in the present case λ and μ are the functions with respect to p of (3) type:

$$\begin{array}{l} \overline{u}_s = \dfrac{\partial \varphi}{\partial x} + \dfrac{\partial \psi_1}{\partial y} - q\dfrac{\partial \psi_2}{\partial x} \\ \overline{\vartheta}_s = \dfrac{\partial \psi}{\partial y} - \dfrac{\partial \psi_1}{\partial x} - q\dfrac{\partial \psi_2}{\partial y} \\ \overline{w}_c = q\varphi - \dfrac{\partial^2 \psi_2}{\partial x^2} - \dfrac{\partial^2 \psi_2}{\partial y^2} \end{array}$$

here $\overline{u}_s$, $\overline{\vartheta}_s$, $\overline{w}_c$ are twofold transformations of components of displacement vector (by Laplace and Fourier), and indices s and c indicate on sines and cosines of Fourier transformation, respectively, q is its parameter.

$$\varphi=\sum_k\sum_m A_{km}\cos\alpha_k x\cos\beta_m y$$
$$\psi_2=\sum_k\sum_m B_{km}\cos\alpha_k x\cos\beta_m y$$
$$\psi_1=0$$

$$\alpha_k=\left(\frac{1}{2}+k\right)\frac{\pi}{a};\ \ \beta_k=\left(\frac{1}{2}+m\right)\frac{\pi}{b}$$

$$\overline{A}_{km}(p,q)=\frac{f(p)\,q}{\lambda(p)+2\mu(p)}\cdot\frac{\sigma_{km}}{\left(\alpha_m^2+\beta_m^2+\frac{p^2}{c_1^2(p)}+q^2\right)\left(\alpha_m^2+\beta_m^2+q^2\right)}$$

$$\overline{B}_{km}(p,q)=\frac{f(p)\,q}{\mu(p)}\cdot\frac{\sigma_{km}}{\left(\alpha_m^2+\beta_m^2+\frac{p^2}{c_2^2(p)}+q^2\right)\left(\alpha_m^2+\beta_m^2+q^2\right)}$$

$$\sigma_{km}=\frac{1}{ab}\int_{-a}^{b}\int_{-b}^{b}\sigma_0(x,y)\cos\alpha_m x\cdot\cos\beta_m y\,dxdy$$

$$c_1^2(p)=\frac{\left(K_0+\frac{4}{3}\mu_0\right)+\frac{4}{3}\eta\mu_0 p}{\rho}=e_1+e_2 p$$
$$c_2^2(p)=\frac{\mu_0(1+\eta p)}{\rho}=s_1+s_2 p$$

In view of formulae identity, it suffices to define longitudinal component expressed by means of the coefficients $\overline{A}_{km}$. Judging by the expression $\overline{A}_{km}$ the inverse transformation by Laplace we can determine by immediate application of the second expansions theorem [1]:

$$L\overline{A}_{km}(p,q)=\frac{f(t)\,\sigma_{km}}{\rho}\frac{2e^{-\frac{\left(q^2+\gamma_{km}^2\right)e_2}{2}t}\sinh\frac{\sqrt{e_2^2\left(\gamma_{km}^2+q^2\right)}}{2}t}{\left(\gamma_{km}^2+q^2\right)^{\frac{3}{2}}\sqrt{e_2^2\left(\gamma_{km}^2+q^2\right)-4e_1}}\frac{q^2dq}{\sqrt{e_1}}$$

Passing to nondimensional quantities:

$$\tau=\frac{e_1}{e_2}t;\quad \widetilde{q}=\frac{e_2}{\sqrt{e_1}}q;\quad \widetilde{z}=\frac{\sqrt{e_1}}{e_2}z;\ \tilde{\gamma}_{km}=\frac{e_2}{\sqrt{e_1}}\gamma_{km};\quad \frac{\pi e_2}{\sqrt{e_1}a}=\xi$$

the longitudinal nondimensional velocity $W_z=\dfrac{\dot{w}}{\sigma_0/\rho\sqrt{e_1}}$ of central axis particle for $a=b$; $\sigma_0(x,y)=const=\sigma_0$ has been calculated.

Graphs in fig.1 illustrate the velocities of end section of central axis for the different values of the parameter $\xi=\dfrac{\pi e_2}{\sqrt{e_1}a}$; as it shown on the graphs, the maximum of strain rate will be for $\tau = 0$. As expected, the viscous component prevents the instantaneous strain initiation; strain or velocity of end section rapidly (but no instantly) go up from zero to some value. Oscillations, as in [4], can be related to cross-section inertia. As the waves deviate from lateral surface, the velocity of bombarding body becomes slower, and then it is quickly reestablished and this process will be repeated in the sequel as well (fig.2).

The width of pit increases as the viscosity grows, that is completely natural (fig.1). If we compare the graphs in fig.3 with corresponding results obtained in [4], the material viscosity strongly influences on velocity distribution along the axis as well.

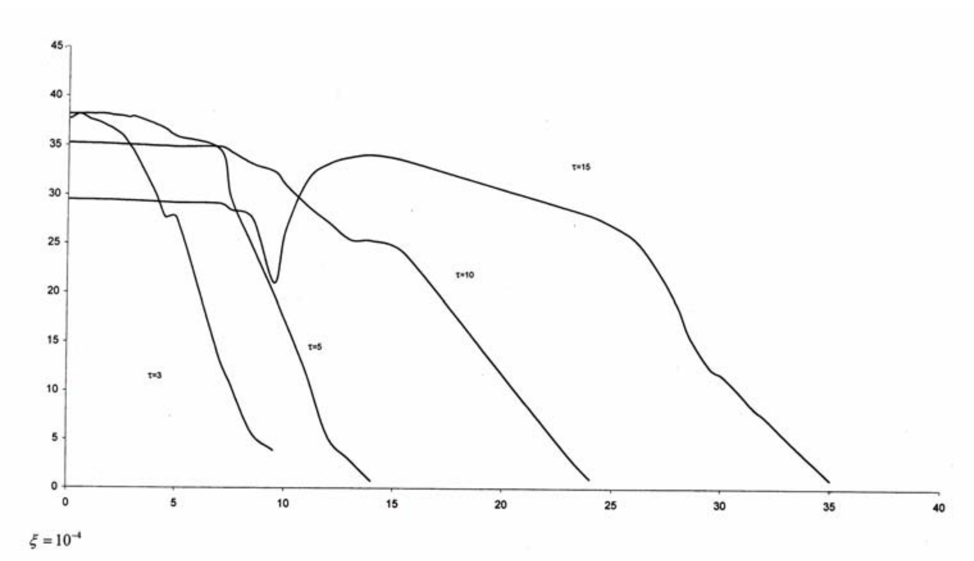

Fig. 1.

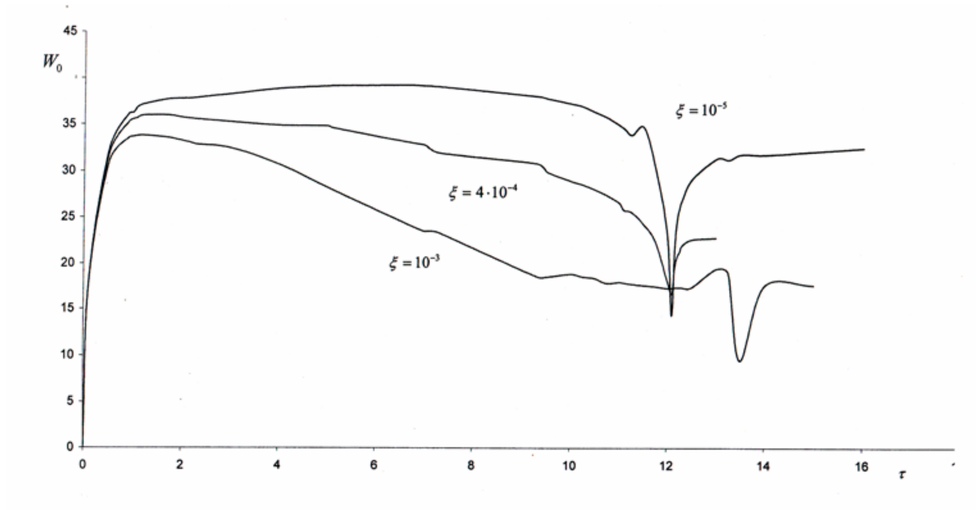

Fig. 2.

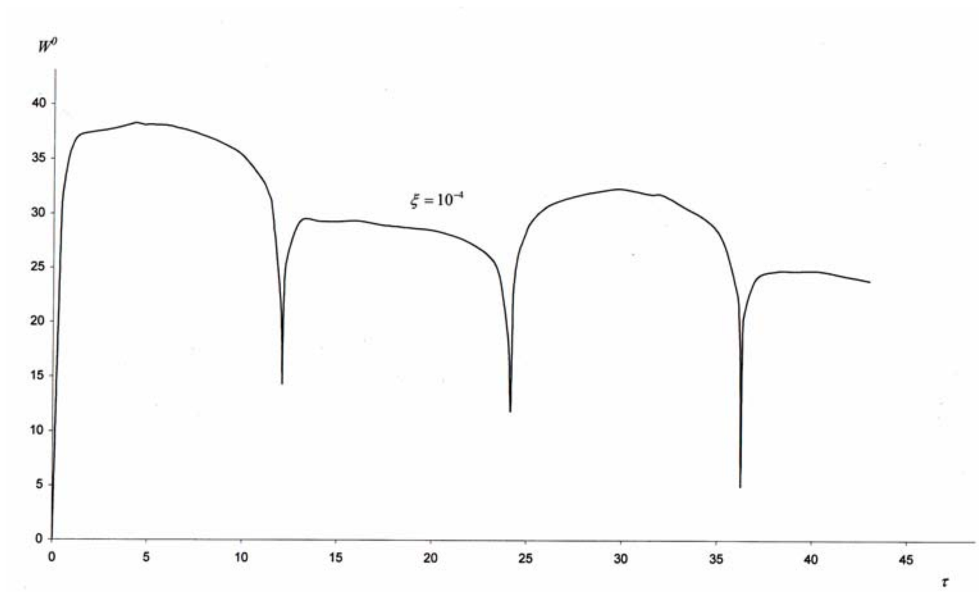

Fig. 3.

References

[1]. Ditkin V.A., Prudnikov A.P. *Integral transformations and operational calculuses.* M. Physmathgiz, 1961, 524 p. (Russian)

[2]. Zverev I.N. *Disturbance propagation in viscoelastic and viscoplastic bar.* PMM, v.XIV, 1950, pp.295-302. (Russian)

[3]. Lee E.H. Kanter I. *Wave propagation in finite rods of viscoelastic material.* J.Appl.Phys., 24, No99, 1953, pp.1115-1122.

[4]. Rassoulova N.B. *Wave propagation in prismatic bar under the action of axial forces.* Izv. AN RF, jur. "MTT", 1997, No6, pp.176-179. (Russian)

[5]. Rassoulova N.B. *On Dynamics of Bar of Rectangular Cross Section.* Trans. ASME, Journal of Applied Mechanics, vol. 68, July 2001, pp.662-666.

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