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THE LOAD CARRYING CAPACITY OF ANNULAR PLATES MADE OF FIBROUS COMPOSITE, FASTENED ON BOTH CONTOURS

Abstract

A problem on load-carrying capacity of a statically indeterminable circular plate made of certain class fibrous composite whose constituents possess plastic properties, is considered in the paper. It is assumed that the plate is fastened on both contours in different ways and is under the action of axially symmetric lateral load linearly alternating in radius. Ultimate load is determined depending on mechanical properties of constituents and some typical geometrical parameters. The results of the numerical calculations for different special cases are given in the form of graphs.

Let a circular annular plate of thickness H, inner and external radii A and B, respectively, be fastened on both contours in different ways and be under the action of down-directed axially-symmetric lateral load $P(r)$ in the form

$$P(r) = (B - A)^{-1} [P_1 (B - r) + P_2 (r - A)] \tag{1}$$

where r is a radial coordinate in a cylindrical system of coordinates $r\varphi z$ (the axis z is down-directed), P_1 and P_2 are the values of lateral loads in inner and external contours, respectively (fig 1a).

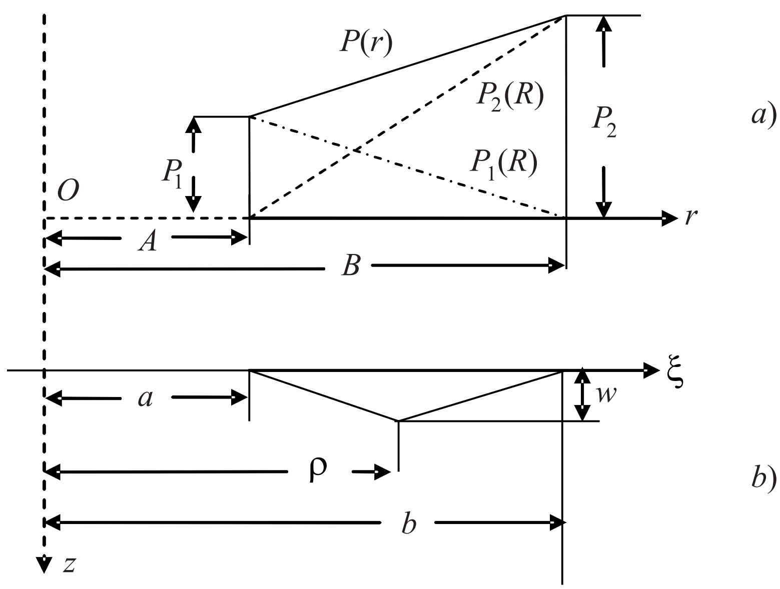

Fig. 1.

If we denote principal bending moments by M_1 and M_2 in radial and peripheral directions, the equilibrium equation will be of the form

$$\frac{dM_1}{dr} + \frac{M_1 - M_2}{r} = \frac{AQ}{r} - \frac{1}{r}\int_A^r P(r)\, r dr \tag{2}$$

where Q is the unknown support reaction in inner contour related to unit of length.

Accept the following denotation for dimensionless quantities

$$m_i = \frac{4M_i}{\sigma_0 H^2} \quad (i = 1, 2), \quad p = 4\frac{P}{\sigma_0}, \quad p_i = 4\frac{P_i}{\sigma_0}, \quad q = 4\frac{Q}{\sigma_0 H}$$
$$\xi = \frac{r}{H}, \quad a = \frac{A}{H}, \quad b = \frac{B}{H} \tag{3}$$

Then equilibrium equation (2) will take the form:

$$(\xi m_1)' - m_2 = aq - \frac{p_1}{b-a}\int\limits_a^\xi [b - na + (n-1)\xi]\,\xi d\xi \tag{4}$$

where $n = \dfrac{p_2}{p_1}$, and the prime means a derivative with respect to ξ.

It is accepted that the composite consists of ideally-plastic matrix having different yield points for compression σ_0 and tension $k\sigma_0$, where $0 \le k \le 1$ and ideally-plastic reinforsing thin fibers. They also possess different ultimate forces for comression and tension different in each direction.

Let S_{0i}^+ and $S_{0i}^- = \mu_i S_{0i}^+$ where $0 \le \mu_i \le 1$ will be ultimate forces for the fibers for tension and compression, respectively. Here $S_{0i}^+ = F_i^+ \sigma_{0i}^+$, $S_{0i}^- = F_i^- \sigma_{0i}^-$ but F_i^+ , F_i^- are cross-section areas of the fibers; σ_{0i}^+ , σ_{0i}^- are yield points for the fibers for tension and contraction, $i = 1, 2$ are orthogonal directions coinciding with principal ones [1].

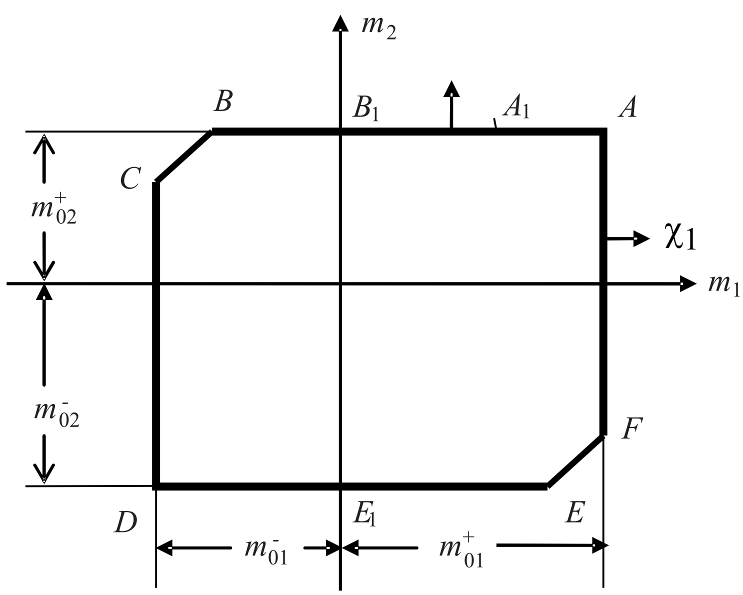

Fig. 2.

We'll assume that the plate is subjected to the yield condition that in the plane $m_1 m_2$ is an improper hexagon $ABCDEF$ (fig.2) [1], [2].

For the sides AB and AF of the hexagon we have

$$m_i = m_{0i}^+ = c_0 + c_{1i}^+ s_{0i}^+ - c_{2i}\left(s_{0i}^+\right)^2, \tag{5}$$

for the sides CD and DE

$$m_i = -m_{0i}^- = -\left[c_0 + c_{1i}^- s_{0i}^+ - c_{2i}\left(s_{0i}^+\right)^2\right]. \tag{6}$$

Here m_{0i}^+ and m_{0i}^- are limiting values of positive and negative bending moments.

For the sides EF and BC we have

$$am_1 - m_2 = aa_1 - a_2 \ , \tag{7}$$

and

$$am_1 - m_2 = aa_3 - a_4 \ . \tag{8}$$

respectively.

Here we accept the following denotation:

$$\begin{array}{l} c_0 = \dfrac{2k}{1+k}; \qquad c_{2i} = \dfrac{2\left(1-\mu_i\right)^2}{1+k} \ ; \\ c_{1i}^+ = 4\left[d_i'' - \mu_i d_i' + \dfrac{(1-k)(1-\mu_i)}{2(1+k)}\right] \ ; \\ c_{1i}^- = 4\left[d_i' - \mu_i d_i'' - \dfrac{(1-k)(1-\mu_i)}{2(1+k)}\right] \ ; \\ a = \dfrac{(1-k)(1-\mu_1)s_{01}^+ + k}{(1-k)(1-\mu_2)s_{02}^+ + k} \ ; \\ a_1 = \left(1-k^2\right)^{-1}\left[(1-k)k + \left(1+k^2\right)(1-\mu_1)s_{01}^+ - \right. \\ \left. -2k(1-\mu_2)s_{02}^+\right] + 4\left(d_1'' - \mu_1 d_1'\right)s_{01}^+ \ ; \\ a_2 = \left(1-k^2\right)^{-1}\left[(k-1)k - \left(1+k^2\right)(1-\mu_2)s_{02}^+ + \right. \\ \left. +2k(1-\mu_1)s_{01}^+\right] + 4\left(d_2' - \mu_2 d_2''\right)s_{02}^+ \ ; \\ a_3 = -a_1 + 4(1-\mu_1)\left(d_1' + d_1''\right)s_{01}^+ \ ; \\ a_4 = -a_2 + 4(1-\mu_2)\left(d_2' + d_2''\right)s_{02}^+ \ . \end{array} \tag{9}$$

where $s_{0i}^+ = S_{0i}^+/\sigma_0 H^2$ and d_1' and d_i'' are dimensionless distances (related to the thickness H) from the mean plane to the layers laid above and below from it, reinforced by fibers, m_{0i}^+ and m_{0i}^- are limiting values of positive and negative bending moments.

The curvature speed in radial and peripheral directions χ_1 and χ_2 are expressed by the flexure speed w in the following way

$$H\chi_1 = -w'', \quad \chi_2 = -w'\xi^{-1} \tag{10}$$

If an annular plate is simply supported on both contours, then under the action of the indicated kind lateral load for bending in plastic state it will take the form schematically shown in fig.1 b.

Then, as is seen from the figure, $w' > 0$ for $a \leq \xi \leq \rho$ and $w' < 0$ for $\rho \leq \xi \leq b$ where ρ is an unknown radius dividing two zones with different plastic states, that should be determined in the course of solution of the problem. Since radial bending moment is positive for the indicated kind of bending (tension of the lower, compression of the upper fibers) then for using yield level it suffices to consider only the right hand part of the yield hexagon. Proceeding from the associated flow condition

$$\chi_i = \lambda \frac{\partial f}{\partial m_i} \tag{11}$$

where λ is a positive coefficient, and f is a yield function, we conclude that in the domain $a \le \xi \le \rho$ plastic operation $m_2 = -m_{02}^{-}\,(0 \le m_1 \le m_{01}^{*})$ and for $\rho \le \xi \le b$, the operation $m_2 = m_{02}^{+}\,(m_{01}^{*} \ge m_1 \ge 0)$ is realized.

Thus, allowing for the what has been said equilibrium equation (4) takes the form

$$(\xi m_1)' = -m_{02}^{-} + aq - \frac{p_1}{b-a}\int\limits_a^{\xi}\left[b - na + (n-1)\,\xi\right]\xi d\xi, \quad a \le \xi \le \rho$$

$$(\xi m_1)' = m_{02}^{+} + aq - \frac{p_1}{b-a}\int\limits_a^{\xi}\left[b - na + (n-1)\,\xi\right]\xi d\xi, \quad \rho \le \xi \le b$$

Hence we find

$$m_1 = \begin{cases} -m_{02}^{-} + aq - \left[12\,(b-a)\right]^{-1} p_1 \left[4a^3 - 6a^2 b + 2b\xi^2 - \xi^3 + \right. \\ \left. +n\left(2a^3 - 2a\xi^2 - \xi^3\right)\right] + \dfrac{C_1}{\xi}, \qquad a \le \xi \le \rho \\ \\ m_{02}^{+} + aq - \left[12\,(b-a)\right]^{-1} p_1 \left[4a^3 - 6a^2 b + 2b\xi^2 - \xi^3 + \right. \\ \left. +n\left(2a^3 - 2a\xi^2 - \xi^3\right)\right] + \dfrac{C_2}{\xi}, \qquad \rho \le \xi \le b \end{cases} \tag{12}$$

Using the boundary conditions $m_1\,(a) = 0$ and $m_1\,(b) = 0$ we define C_1 and C_2 and (12) takes the form

$$\begin{gathered} m_1 = \Big\{ aq - m_{02}^{-} - \left[12\,(b-a)\right]^{-1} \times \\ \times p_1 \left[a^2\,(3a - 4b) - \xi\left(\xi^2 + a\xi + a^2\right) + \right. \\ \left. +2b\,(a+\xi)\,\xi + (a-\xi)^2\,(a+\xi)\,n\right]\Big\}\,(\xi - a)\,\xi^{-1}, \quad a \le \xi \le \rho \\ m_1 = \Big\{ aq + m_{02}^{+} - \left[12\,(b-a)\right]^{-1} \times \\ \times p_1 \left[2a^2\,(2a - 3b) - \xi^3 + b\left(\xi^2 + b\xi + b^2\right)\right] + \left[2a\left(a^2 - \xi^2 - \right.\right. \\ \left.\left. -b\xi - b^2\right) + \left(\xi^2 + b^2\right)(\xi + b)\right] n\Big\}\,(\xi - b)\,\xi^{-1}, \qquad \rho \le \xi \le b \end{gathered} \tag{13}$$

Now, we use the continuity condition $[m_1]_{\xi=\rho} = 0$ that gives two conditions in the form of $m_1\,(\rho) = m_{01}^{*}$ where m_{01}^{*} is the value of radial bending moment at the points E and A_1 we find two equations for defining three unknowns q, p, ρ

$$\begin{gathered} m_{01}^{*} = \Big\{ aq - m_{02}^{-} - \left[12\,(b-a)\right]^{-1} \times \\ \times p_1 \left[a^2\,(3a - 4b) - \rho\left(\rho^2 a\rho + a^2\right) + \right. \\ \left. +2b\,(a+\rho)\,\rho + (a-\rho)^2\,(a+\rho)\,n\right]\Big\}\,(\rho - a)\,\rho^{-1}, \\ m_{01}^{*} = \Big\{ aq + m_{02}^{+} - \left[12\,(b-a)\right]^{-1} \times \\ \times p_1 \left[2a^2\,(2a - 3b) - \rho^3 + b\left(\rho^2 - b\rho + b^2\right)\right] + \\ + \left[2a\left(a^2 - \rho^2 - b\rho - b^2\right) + \left(\rho^2 + b^2\right)(\rho + b)\right] n\Big\}\,(\rho - b)\,\rho^{-1}, \end{gathered} \tag{14}$$

We determine kinematically possible field of flexure velocities and find the third missing equation. Using the flow low and yield hexagon we conclude that $\chi_1 =$

$-w'' = 0$ both in the zone $a \le \xi \le \rho$ $(\chi_2 < 0, w' > 0)$ and in the zone $\rho \le \xi \le b$ $(\chi_2 > 0, w' < 0)$. Thus, we'll have

$$w = \begin{cases} C_3\xi + C_4 & a \le \xi \le \rho \\ C_5\xi + C_6 & \rho \le \xi \le b \end{cases}$$

Now, using boundary conditions $w(a) = 0,\ w(b) = 0$, continuity condition w for $\xi = \rho$ (that gives two conditions), we find

$$\begin{aligned} w_1 &= w_0 \frac{\xi - a}{\rho - a}, \qquad a \le \xi \le \rho \\ w_2 &= w_0 \frac{b - \xi}{b - \rho}, \qquad \rho \le \xi \le b \end{aligned} \tag{15}$$

where w_0 is maximal flexure velocity for $\xi = \rho$, the condition $w_1' = -w_2'$ for $\xi = \rho$ allows to find the unknown radius ρ in the form

$$\rho = \frac{a+b}{2} \tag{16}$$

Substituting this value of ρ in (14) we define q and p_1, depending on mechanical properties of composite constituents and geometrical parameters of the plate in the form

$$\begin{gathered} p_1 a^2 = \frac{12\left[2(1+l) m_{01}^* + (l-1)\left(m_{02}^- + m_{02}^+\right)\right]}{\left[3 + l + (5 + 7l) n\right](l-1)^2} \\ qa = \frac{l+1}{l-1} m_{01}^* + m_{02}^- + \\ + \frac{\left[2(1+l) m_{01}^* + (l-1)\left(m_{02}^- + m_{02}^+\right)\right]}{8(l-1)^3} \times \\ \times \frac{\left[14 - 31l + 11l^2 + 3l^3 + \left(3 + 7l + 5l^2 + l^3\right) n\right]}{\left[3 + l + (5 + 7l) n\right]} \end{gathered} \tag{17}$$

where $l = \dfrac{b}{a}$

Let's consider some special cases

1) Let $p_1 = p_2 = p_0$, $(n = 1)$ i.e the plate is under the action of everywhere uniformly distributed lateral load. Then from (17) we have

$$\begin{gathered} p_0 a^2 = \frac{3\left[2(1+l) m_{01}^* + (l-1)\left(m_{02}^- + m_{02}^+\right)\right]}{2(1+l)(l-1)^2} \\ qa = (l-1)^{-1}(l+1) m_{01}^* + m_{02}^- + \\ + \frac{\left[2(1+l) m_{01}^* + (l-1)\left(m_{02}^- + m_{02}^+\right)\right]\left(17 - 24l + 16l^2 + 4l^3\right)}{64(1+l)(l-1)^3} \end{gathered} \tag{18}$$

2) Assume $p_2 = 0,\ p_1 \neq 0$ $(n = 0)$. Then

$$\begin{gathered} p_1 a^2 = \frac{12\left[2(1+l) m_{01}^* + (l-1)\left(m_{02}^- + m_{02}^+\right)\right]}{(3+l)(l-1)^2} \\ qa = (l-1)^{-1}(l+1) m_{01}^* + m_{02}^- + \\ + \frac{\left[2(1+l) m_{01}^* + (l-1)\left(m_{02}^- + m_{02}^+\right)\right]\left[\left(14 - 31l + 11l^2 + 3l^3\right)\right]}{8(3+l)(l-1)^3} \end{gathered} \tag{19}$$

3) When $p_1 = 0,\ p_2 \neq 0$, we have

$$p_2 a^2 = \frac{12\left[2\left(1+l\right)m_{01}^{*} + \left(l-1\right)\left(m_{02}^{-} + m_{02}^{+}\right)\right]}{\left(5+7l\right)\left(l-1\right)^2}$$

$$qa = \left(l-1\right)^{-1}\left(l+1\right)m_{01}^{*} + m_{02}^{-} + \tag{20}$$

$$+\frac{\left[2\left(1+l\right)m_{01}^{*} + \left(l-1\right)\left(m_{01}^{-} + m_{02}^{+}\right)\right]\left(3+7l+5l^2+l^3\right)}{8\left(5+7l\right)\left(l-1\right)^3}$$

For numerical calculation we consider the case when reinforcing fibers are placed in two symmetric layers with respect to mean plane of the plate with the same fibers in both directions, i.e the case when $\mu_1 = \mu_2 = \mu$ and $d_i' = -d_i''$.

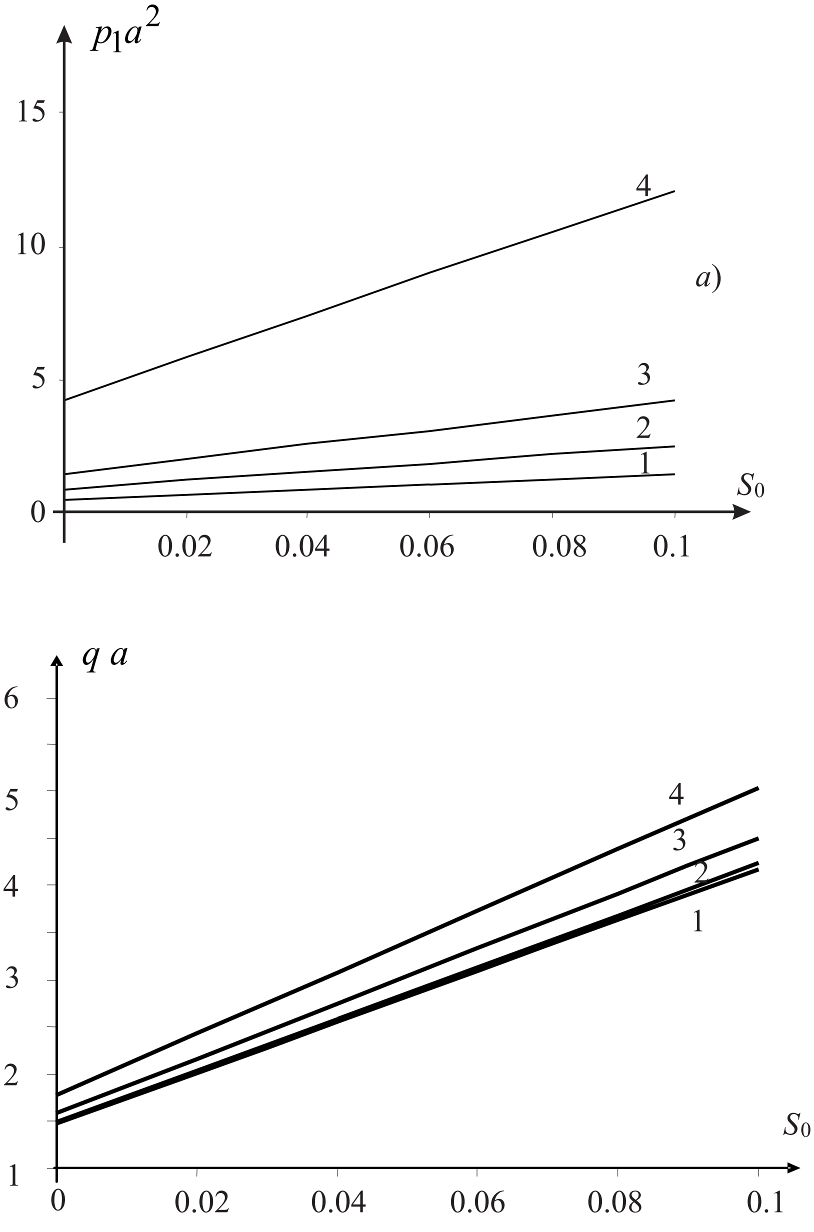

Fig. 3.

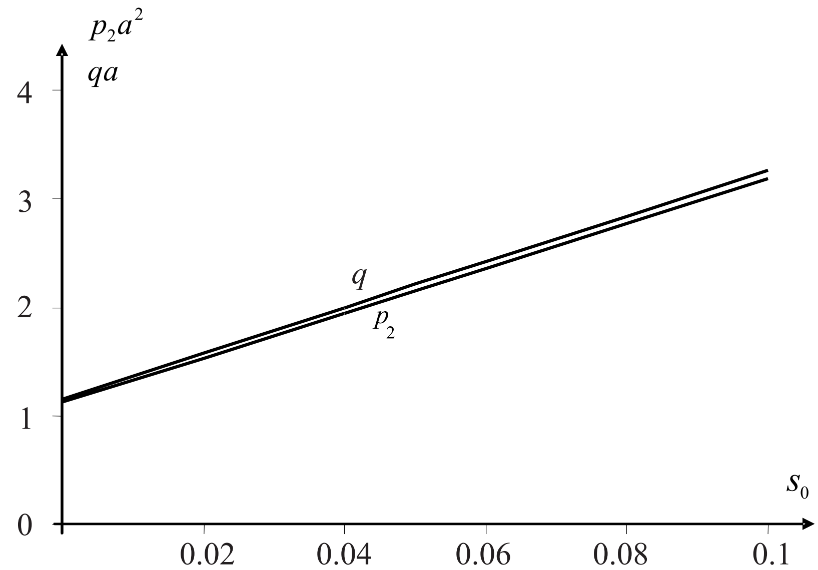

Fig. 4.

In figures 3 and 4 the graps of dependence of p_1, p_2 and q on s_0 for

$$m_{02}^{+} = m_{02}^{-} = m_{02}\ , l = 2$$

for different values of n are given. In the case when

$$k = 0,8; \mu = 0,8;\ d_i'' = 0,4$$

we have

$$m_{02} = 0,888 + 0,556s_0 - 0,044s_0^2$$

$$m_{01}^{*} = -m_{02} + (a_1 - a_2) = 5,248s_0 + 0,044s_0^2$$

Notice that if an annular plate is built in on internal or external contours or on both contours, then in the domains adjoint to these contours there arises plastic state for which radial bending moment has a negative value. Such cases will be studied in detail in another paper by the author.

Finally, in conclusion notice that load-carrying capacity of similar annular plates made of homogeneous material for different yield conditions were considered in [3] and [4], for the plate made of composite material under the action of lateral load of constant intensity - in [5]

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