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DISPERSION OF AXISYMMETRIC LONGITUDINAL WAVES IN A FINITE PRE-STRAINED COMPOUND CYLINDER MADE OF INCOMPRESSIBLE MATERIAL

Abstract

Within the framework of the piecewise homogeneous body model with the use of the three-dimensional linearized theory of elastic wave propagation in the initially stressed body the dispersion of the axisymmetric longitudinal wave in the finite pre-strained compound cylinder is investigated. The materials of the inner and outer cylinders are assumed to be incompressible. The stress-strain relations are given through the Treloar's potential. The numerical results regarding the influence of the problem parameters on the dispersion of the considered wave propagation are presented and discussed. The expressions for the calculation of the wave propagation velocity for long wavelength limit and for short wavelength limit are derived.

1. Introduction.

Up-to -date level of development of almost all fields of science demands to study nonlinear effects that occur in dynamical processes happening in deformable media. Wave propagation in prestressed (pre-strained) elastic bodies is one of such processes. Note that initial stresses occur in structural elements in their mounting, in eath core as a result of action of geostatic and geodynamical forces, in composite materials and etc. Moreover, in many cases elastic waves propagation in structural elements occurs in their exploitation (e.g. posts of buildings, bridges and etc.). Thus, study of corresponding dynamical problems are important not only from fundamental investigations point of view and also from the point of view of their applied assignment. In this connection many investigations were carried out in this field up to now. Therewith three-dimensional Linearized theory of elastic waves propagation in pre-stressed bodies $[TLTEWPPB]$ was applied. Derivation of principal equations and relations of $TLTEWPPB$ and also discussion of the results of concrete investigations are given in the monographs $[1-3]$. A brief review of the results obtained in the last years is in the papers $[4-12]$.

From the above-stated monographs and papers it follow that pre-stretched propagation of normal waves in inhomogeneous (laminated) stressed (pre-strained) cylinders and plates have not been investigated up to day. Notice that, here under "normal waves" we understand the waves that are propagated in bodies whose sizes in directions perpendicular do the direction of wave propation are finite. First attempts in this field for cylindrical bodies were made in the paper [4]. However, in [4] it was assumed that initial strains in the components of a compound cylinder are small and their effect on wave propagation velocity are insignificant. Consequently investigation of the mentioned effect for finite initial strains is specially important and urgent. Allowing for this state, in this paper, investigation [4] is developed for a finite pre-stretched compound cylinder made of incompressible material. Therewith, it is assumed that relation of elasticity materials of a component of a compound cylinder is described by the Treloar potential.

2. Problem Statement.
Main equations and relations.

Let's consider a compound cylinder with a circular cross-section. In natural state we denote a radius of inner perfect cylinder by R , the thickness of outer overlapping hollow cylinder by h. We determine the state of cylinders in natural state by Lagrangian coordinates in the Cartesian system of coordinates $Oy_1y_2y_3$ and also in the cylindrical system of coordinates $Or\theta y_3$. Accept that the length of cylinders in the direction of the axis Oy_3 is infinite and homogeneous axisymmetric (with respect to the axis Oy_3) finite initial strain holds at each component. Such an initial strain may occur in stretching cylinders along the axis Oy_3.

At the initial state we determine the position of the body points with Lagrangian coordinates in the cartesian system of coordinates $O'y'_1y'_2y'_3$ and in cylindrical system of coordinates $O'r'\theta'y'_3$. Example that elastic relation of material of cylinders is determined by the Treloar potential and the quantities belonging to inner perfect (outer hollow) cylinders we denote by the upper index (2) ((1)). Besides, the quantities belonging to the initial state-by an additional upper index. Thus, taking the above-mentioned ones into account, we can determine the quantities belonging to the initial state in the following form:

$$u_m^{(k),0} = \left(\lambda_m^{(k)} - 1\right) y_m,\ \ \lambda_1^{(k)} = \lambda_2^{(k)} \neq \lambda_3^{(k)},\ \ \lambda_m^{(k)} = const.$$

$$\lambda_1^{(k)}\lambda_2^{(k)}\lambda_3^{(k)} = 1, \quad m = 1,2,3; \quad k = 1,2. \tag{1}$$

where $u_m^{(k),0}$ are permutations vector components, $\lambda_m^{(k)}$ are the parameters extended in the direction of the axis Oy_m. Introduce the denotation

$$\lambda_3^{(k)} = \lambda^{(k)}, \ \ \lambda_1^{(k)} = \lambda_2^{(k)} = \left(\lambda^{(k)}\right)^{-1/2} \tag{2}$$

Assume that the above-mentioned initial strain in inner and outer cylinders is created before their joining.

Therefore, there may hold the cases for which $\lambda^{(1)} \neq \lambda^{(2)}$, and also the cases for which $\lambda^{(1)} = \lambda^{(2)}$. However, occurrence of initial strain in the form $\lambda^{(1)} = \lambda^{(2)}$ may also be at tension of a compound cylinder after joining its components. Since, we assume that the materials of cylinders are incompressible, therefore in the latter form, creation of initial strain, there will not be inhomogenneous stress acting in areas whose normal is perpendicular to the direction of the axis Oy_3.

By the above-stated one and relation (1) we have

$$y'_i = \lambda_i^{(k)} y_i \ , \ \ r' = \left(\lambda^{(k)}\right)^{-1/2} r \ , \ \ R' = \left(\lambda^{(2)}\right)^{-1/2} R \ , \ \ h' = h\left(\lambda^{(1)}\right)^{-1/2} ,$$

$$k = 1 \quad \text{for} \quad 0 \le r \le R \ , \ \ k = 2 \quad \text{for} \quad R < r \le R + h. \tag{3}$$

Below, the quantities connected with the system of coordinates $O'y'_1y'_2y'_3$ or $O'r'\theta'y'_3$ will be denoted by upper prime.

So, let's investigate propagation of axisymmetric longitudinal wave in the direction of the axis $O'y'_3$ in the above-mentioned compound cylinder involving a finite initial strain determined by the relations (1),(2). We carry out investigation in the frames of piece-wise-homogeneous body model with attracting $TLTEWPPB$ in coordinates connected with initial state. By [3] in the considered case in cylindrical system of coordinates we have the following motion equations

$$\frac{\partial}{\partial r'}Q'^{(k)}_{r'r'}+\frac{\partial}{\partial y'_3}Q'^{(k)}_{r'3}+\frac{1}{r'}\left(Q'^{(k)}_{r'r'}-Q'^{(k)}_{\theta'\theta'}\right)=\rho'^{(k)}\frac{\partial^2}{\partial t^2}u'^{(k)}_{r'}\ ,$$

$$\frac{\partial}{\partial r'}Q'^{(k)}_{3r'}+\frac{\partial}{\partial y'_3}Q'^{(k)}_{33}+\frac{1}{r'}Q'^{(k)}_{3r'}=\rho'^{(k)}\frac{\partial^2}{\partial t^2}u'^{(k)}_{3}\ , \tag{4}$$

and mechanical relations

$$Q'^{(k)}_{r'r'}=X'^{(k)}_{1111}\frac{\partial u'^{(k)}_{r'}}{\partial r'}+X'^{(k)}_{1122}\frac{u'^{(k)}_{r'}}{r'}+X'^{(k)}_{1133}\frac{\partial u'^{(k)}_{3}}{\partial y'_3}+p'^{(k)}\ ,$$

$$Q'^{(k)}_{\theta'\theta'}=X'^{(k)}_{2211}\frac{\partial u'^{(k)}_{r'}}{\partial r'}+X'^{(k)}_{2222}\frac{u'^{(k)}_{r'}}{r'}+X'^{(k)}_{2233}\frac{\partial u'^{(k)}_{3}}{\partial y'_3}+p'^{(k)}\ ,$$

$$Q'^{(k)}_{33}=X'^{(k)}_{3311}\frac{\partial u'^{(k)}_{r'}}{\partial r'}+X'^{(k)}_{3322}\frac{u'^{(k)}_{r'}}{r'}+X'^{(k)}_{3333}\frac{\partial u'^{(k)}_{3}}{\partial y'_3}+p'^{(k)}\ ,$$

$$Q'^{(k)}_{r'3}=X'^{(k)}_{1313}\frac{\partial u'^{(k)}_{r'}}{\partial y'_3}+X'^{(k)}_{1331}\frac{\partial u'^{(k)}_{3}}{\partial r'}$$

$$Q'^{(k)}_{3r'}=X'^{(k)}_{3113}\frac{\partial u'^{(k)}_{r'}}{\partial y'_3}+X'^{(k)}_{3131}\frac{\partial u'^{(k)}_{3}}{\partial r'} \tag{5}$$

In $[4],[5]$ the following denotation were accepted: $Q'^{(k)}_{r'r'},...,Q'^{(k)}_{3r'}$ are perturbations of Kirkhoff stress tensor components in the cylindrical system of coordinates: $u'^{(k)}_{r'}$, $u'^{(k)}_{3}$ are perturlations of permutation vector components; $p'^{(k)}=p'^{(k)}(r',\ y'_3\ ,t)$ is an unknown function (Lagrange multiplier); $X'^{(k)}_{1111},...,X'^{(k)}_{3333}$ are the constants whose values are calculated by mechanical constant and initial strain; $\rho'^{(k)}$ is density of the material of the k-th cylinder.

As it was noticed above, mechanical relation of materials of a cylinder is described by the Treloar potential that has the following expression;

$$\Phi^{(k)}=C^{(k)}_{10}\left(I^{(k)}_1-3\right),\quad I^{(k)}_1=3+2A^{(k)}_1,\quad A^{(k)}_1=\epsilon'^{(k)}_{r'r'}+\epsilon'^{(k)}_{\theta'\theta'}+\epsilon'^{(k)}_{33}, \tag{6}$$

where $C^{(k)}_{10}$ is a material constant; $\epsilon'^{(k)}_{r'r'},\epsilon'^{(k)}_{\theta'\theta'},\epsilon'^{(k)}_{33}$ are corresponding components of Green's strain tensor in a cylindrical system of coordinates.

And it holds the relation

$$\varepsilon^{(k)}_{rr}=\frac{\partial u^{(k)}_r}{\partial r}+\frac{1}{2}\left(\frac{\partial u^{(k)}_r}{\partial r}\right)^2+\frac{1}{2}\left(\frac{\partial u^{(k)}_3}{\partial r}\right)^2,\quad \varepsilon^{(k)}_{\theta\theta}=\frac{u^{(k)}_r}{r}+\frac{1}{2}\left(\frac{\partial u^{(k)}_r}{\partial r}\right)^2,$$

$$\varepsilon^{(k)}_{33}=\frac{\partial u^{(k)}_3}{\partial y_3}+\frac{1}{2}\left(\frac{\partial u^{(k)}_r}{\partial y_3}\right)^2+\frac{1}{2}\left(\frac{\partial u^{(k)}_3}{\partial y_3}\right)^2. \tag{7}$$

Notice that the relations (6) and (7) are written for any system of coordinates, not connecting it neither with initial nor with natural states.

Performing corresponding mathematical procedures stated for example in [3] we get the following expressions for the constants $X'^{(k)}_{1111}, ..., X'^{(k)}_{3333}$

$$X'^{(k)}_{1111} = X'^{(k)}_{2222} = 4C^{(k)}_{10}\left(\lambda^{(k)}\right)^{-1},$$

$$X'^{(k)}_{1122} = X'^{(k)}_{1133} = X'^{(k)}_{2233} = X'^{(k)}_{3311} = X'^{(k)}_{2211} = X'^{(k)}_{3322} = 0,$$

$$X'^{(k)}_{1331} = 2C^{(k)}_{10}\left(\lambda^{(k)}\right)^{-1}, \quad X'^{(k)}_{1221} = 2C^{(k)}_{10}\left(\lambda^{(k)}\right)^{-1},$$

$$X'^{(k)}_{3333} = 2C^{(k)}_{10}\left(1+\left(\lambda^{(k)}\right)^{-3}\right)\left(\lambda^{(k)}\right)^{2}$$

$$X'^{(k)}_{1313} = X'^{(k)}_{3131} = 2C^{(k)}_{10}\left(\lambda^{(k)}\right)^{-1}, \quad X'^{(k)}_{3113} = 2C^{(k)}_{10}\left(\lambda^{(k)}\right)^{2}. \tag{8}$$

Incompressibility conditions

$$\frac{\partial u'^{(k)}_{r'}}{\partial r'} + \frac{u'^{(k)}_{r'}}{r'} + \frac{\partial u'^{(k)}_{3}}{\partial y'_3} = 0. \tag{9}$$

should ne added to the above reduced equations.

We also write boundary and contact conditions in whose frames frances we'll study wave propagation

$$Q'^{(1)}_{r'r'}\Big|_{r'=R'} = Q'^{(2)}_{r'r'}\Big|_{r'=R'}, \quad Q'^{(1)}_{r'3}\Big|_{r'=R'} = Q'^{(2)}_{r'3}\Big|_{r'=R'},$$

$$u'^{(1)}_{r'}\Big|_{r'=R'} = u'^{(2)}_{r'}\Big|_{r'=R'}, \quad u'^{(1)}_{3}\Big|_{r'=R'} = u'^{(2)}_{3}\Big|_{r'=R'},$$

$$Q'^{(1)}_{r'3}\Big|_{r'=R'+h'} = 0, \quad Q'^{(1)}_{r'r'}\Big|_{r'=R'+h'} = 0. \tag{10}$$

Thus, the statement of the problem is completely settled by the above-stated one. Notice that in the case $\lambda^{(k)} = 1.0, \quad (k = 1, 2)$ we pass to the statement of corresponding problems of classic (linear) theory of elastodynamics for a compound cylinder, i.e. for initial strainless compound cylinder.

3. Solution method.

Substituting expression (5) into equation (4) we derive motion equations in permutations. We join equation (9) to these equations and get a closed system of equations for the unknown functions $u'^{(k)}_{r'}$, $u'^{(k)}_{3}$ and $\rho'^{(k)}$. By [3] we write the following representation for these unknowns:

$$u'^{(k)}_{r'} = -\frac{\partial^2}{\partial r' \partial y'_3} X'^{(k)}, \; u'^{(k)}_{3} = \Delta'_1 X'^{(k)},$$

$$p'^{(k)} = \left[\left(X'^{(k)}_{1111} - X'^{(k)}_{1133} - X'^{(k)}_{3131}\right)\Delta'_1 + X'^{(k)}_{3113}\frac{\partial^2}{\partial y'^2_3} - \rho'^{(k)}\frac{\partial^2}{\partial t^2}\right]\frac{\partial}{\partial y'_3}X'^{(k)},$$

$$\Delta'_1 = \frac{\partial^2}{\partial r'^2} + \frac{1}{r'}\frac{\partial}{\partial r'}, \tag{11}$$

where the function $X'^{(k)}$ satisfies the following equation

$$\left[\left(\Delta_1' + \left(\xi_2'^{(k)}\right)^2 \frac{\partial^2}{\partial y_3'^2}\right)\left(\Delta_1' + \left(\xi_3'^{(k)}\right)^2 \frac{\partial^2}{\partial y_3'^2}\right) - \right.$$

$$\left. -\frac{\rho'^{(k)}}{X_{1331}'^{(k)}}\left(\Delta_1' + \frac{\partial^2}{\partial y_3'^2}\right)\frac{\partial^2}{\partial t^2}\right] X'^{(k)} = 0 \qquad (12)$$

For the considered case we have:

$$\left(\xi_2'^{(k)}\right)^2 = \left(\lambda^{(k)}\right)^3, \ \left(\xi_3'^{(k)}\right)^2 = 1, \ X_{1331}'^{(k)} = 2C_{10}^{(k)}\left(\lambda^{(k)}\right)^{-1} \qquad (13)$$

By the problem statement, we can represent the function

$$X'^{(k)} = X'^{(k)}\left(r' y_3' \ , \ t\right)$$

in the form

$$X'^{(k)} = X_1'^{(k)}\left(r'\right)\cos\left(k y_3' - \omega t\right). \qquad (14)$$

From $(11), (13)$ we get the following equation for $X_1'\left(r'\right)$

$$\left(\Delta_1' + \left(\zeta_2'^{(m)}\right)^2\right)\left(\Delta_1' + \left(\zeta_3'^{(m)}\right)^2\right) X_1'^{(m)}\left(r'\right) = 0, \qquad (15)$$

where

$$\left(\zeta_2'^{(m)}\right)^2 = \lambda^{(m)}\left(\left(s^{(m)}\right)^2 - \left(\lambda^{(m)}\right)^2\right), \quad \left(\zeta_3'^{(m)}\right)^2 = -1,$$

$$s^{(m)} = \frac{C}{C_2^{(m)}} \ , \ \left(C_2^{(m)}\right)^2 = \frac{2C_{10}^{(m)}}{\rho^{(m)}} \ , \quad C = \frac{\omega}{k}. \qquad (16)$$

After some transformations we determine the solution of equations $(15), (16)$ in the following way

$$X_1'^{(2)} = A^{(2)} I_0\left(kr'\right) + B^{(2)} \times$$

$$\times \begin{cases} J_0\left(\sqrt{\lambda^{(2)}\left(\left(s^{(2)}\right)^2 - \left(\lambda^{(2)}\right)^2\right)} kr'\right) & \text{if } \ s^{(2)} > \lambda^{(2)}, \\ I_0\left(\sqrt{\lambda^{(2)}\left(\left(\lambda^{(2)}\right)^2 - \left(s^{(2)}\right)^2\right)} kr'\right) & \text{if } \ s^{(2)} < \lambda^{(2)}, \end{cases}$$

$$X_1'^{(1)} = A^{(1)} I_0\left(kr'\right) + B^{(1)} \times$$

$$\times \begin{cases} J_0\left(\sqrt{\lambda^{(1)}\left(\left(s^{(1)}\right)^2 - \left(\lambda^{(1)}\right)^2\right)} kr'\right) & \text{if } \ s^{(1)} > \lambda^{(1)}, \\ I_0\left(\sqrt{\lambda^{(1)}\left(\left(\lambda^{(1)}\right)^2 - \left(s^{(1)}\right)^2\right)} kr'\right) & \text{if } \ s^{(1)} < \lambda^{(1)} \end{cases} +$$

$$+C^{(1)} K_0\left(kr'\right) + D^{(1)} \times$$

$$\times\begin{cases} Y_0\left(\sqrt{\lambda^{(1)}\left(\left(s^{(1)}\right)^2-\left(\lambda^{(1)}\right)^2\right)}kr'\right) & \text{if } s^{(1)}>\lambda^{(1)}, \\ K_0\left(\sqrt{\lambda^{(1)}\left(\left(\lambda^{(1)}\right)^2-\left(s^{(1)}\right)^2\right)}kr'\right) & \text{if } s^{(1)}<\lambda^{(1)} \end{cases} \tag{17}$$

In (17) the following denotation were used; $J_0(x)$ and $Y_0(x)$ are Bessel functions of first and second kind, relatively, of zero order: $I_0(x)$ and $K_0(x)$ are Bessel functions of purely imaginary argument and McDonald function, relatively, of zero order; k is a wave number. Notice that in cases $s^{(m)}=\lambda^{(m)}$ the expression for the function $X_1'(kr')$ is determined by means of the known mathematical procedures that we don't cite here.

Thus, substituting (17) into expression (11), from (5) and (10) we get a dispersion equation in the form

$$\det\|\alpha_{ij}\|=0,\quad i;j=\overline{1,6}, \tag{18}$$

where α_{ij} are the coefficients of unknowns in the equations obtained from (10) for the constants A^2, $B^{(2)}$, $A^{(1)}$, $B^{(1)}$, $C^{(1)}$ and $D^{(1)}$ that are contained in (17). Because of their awkwardness we don't cite here expressions for α_{ij} . Solving equations (18) with respect to $C/C_2^{(2)}$ we construct dispersive curves. Therewith the indicated solution is conducted with attraction of PK by means of the algorithm "bisect the segment". The corresponding computer programs are created by the author of the paper.

4. Numerical results and their discussion.

Below we'll mark the quantities belonging to the inner cylinder by the upper index (f), and the quantities belonging to the outer hollow cylinder by the upper index (m) (instead of the indices (2) and (1), respectively). First of all we consider the case when there are no initial strains in the components of a compound cylinder, i.e. $\lambda^{(m)}=\lambda^{(f)}=1.0$. Accept $\rho^{(f)}/\rho^{(m)}=0.7$ and investigate a dispersing curve corresponding to the first (fundamental) and second modes. In figures 1 and 2 dispersing curves,i.e. graph of dependencies between $C/C_2^{(f)}$ and kR for different values of $C_{10}^{(f)}/C_{10}^{(m)}$ in the case $h/R=0.5$, are reduced for the first and second modes, respectively. If follows from these dependencies that for each fixed value of kR, with growth of $C_{10}^{(f)}/C_{10}^{(m)}$ the wave propagation velocity, i.e. the value of $C/C_2^{(f)}$ decreases. The noticed decrease is explained with growth of $C_2^{(f)}$ (in comparison with $C_{10}^{(m)}$) with increase of $C_{10}^{(f)}/C_{10}^{(m)}$ for the fixed relation of $\rho^{(f)}/\rho^{(m)}$. Besides, these results show that for the first mode $C/C_2^{(f)}$ has a finite limit as $kR\to 0$ (long-wave approach) and as $kR\to\infty$ (long-wave approach). However, for the second mode the value of $C/C_2^{(f)}$ has a finite limit only for short-wave approach $(kR\to\infty)$, for long-wave approach $(kR\to 0)$ the value $C/C_2^{(f)}$ has no finite limits. The stated one remains valid for higher modes.

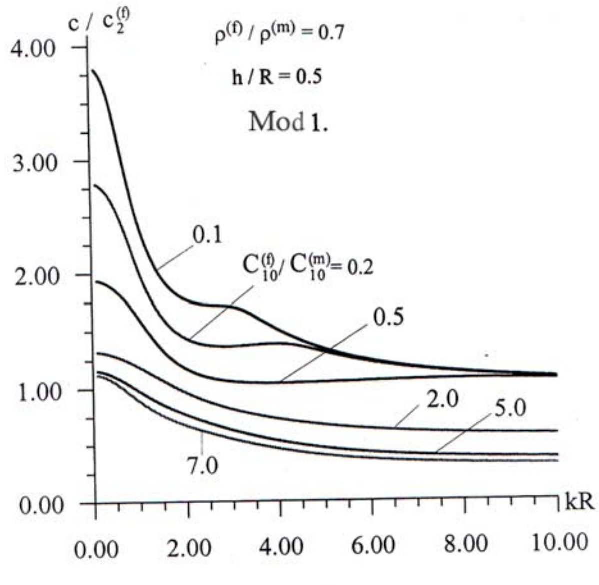

Fig. 1.

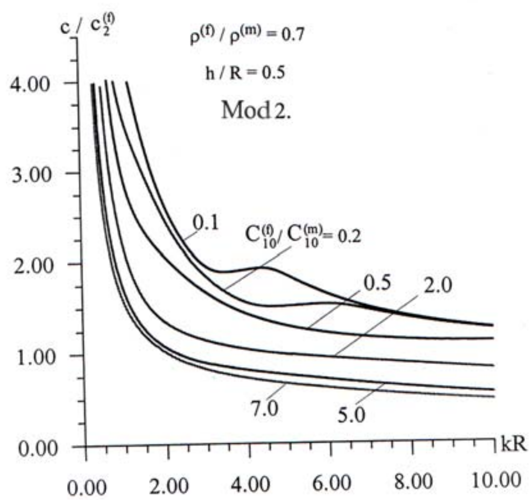

Fig. 2.

Let's consider determination of analytic expressions for calculating the indicate limits. using asymptotic representation for the functions $I_n(x)$, $Y_n(x)$, $J_n(x)$ and $K_n(x)$ for small values of the argument, and carrying out analytic solution of dispersing equation (18) as $kR \to 0$ for the first mode we obtain;

$$\frac{C}{C_2^{(f)}} = \sqrt{3}\left(\frac{C_{10}^{(m)}}{C_{10}^{(f)}}\eta^{(m)} + \eta^{(f)}\right)^{1/2}\left(\frac{\rho^{(m)}}{\rho^{(f)}}\eta^{(m)} + \eta^{(f)}\right)^{-1/2},$$

$$\eta^{(f)} = \left(1 + \frac{h}{R}\right)^{-2}, \quad \eta^{(m)} = \left(2\frac{h}{R} + \left(\frac{h}{R}\right)^2\right)\left(1 + \frac{h}{R}\right)^{-2}. \tag{19}$$

Using asymptotic representation for the functions $I_n(x)$, $Y_n(x)$, $J_n(x)$ and $K_n(x)$ for great values of the argument from equation (18) we define limit value of $C/C_2^{(f)}$ for the case $kR \to \infty$. Analysis shows that the value of this limit depends on $C_{10}^{(f)}/C_{10}^{(m)}$, since for the considered case we have:

$$C/C_2^{(f)} \underset{kR\to\infty}{\to} 1 \quad \text{for} \quad C_{10}^{(f)}/C_{10}^{(m)} < 1.$$

$$C/C_2^{(f)} \underset{kR\to\infty}{\to} C_R^{(m)}/C_2^{(f)} \quad \text{for} \quad C_{10}^{(f)}/C_{10}^{(m)} > 1. \tag{20}$$

where $C_R^{(m)}$ is the Rayleigh wave velocity in the material of the outer hollow cylinder and it is determined from he expression

$$C_R^{(m)}/C_2^{(f)} = \sqrt{(1-x_*^2)\, C_{10}^{(m)} \rho^{(f)} / C_{10}^{(f)} \rho^{(m)}}, \quad x_* \approx 0.2916$$

In the second mode we get

$$C/C_2^{(f)} \underset{kR\to\infty}{\to} C_R^{(m)}/C_2^{(f)} \quad \text{for} \quad C_{10}^{(f)}/C_{10}^{(m)} < 1.$$

$$C/C_2^{(f)} \underset{kR\to\infty}{\to} C_2^{(m)}/C_2^{(f)} \quad \text{for} \quad C_{10}^{(f)}/C_{10}^{(m)} > 1. \tag{21}$$

Notice has the results obtained above completely agree with the known positions of elastic bodies dynamics in classic (linear) statement where there is no initial strains therein. Besides, the obtained results confirm the validity and correctness of algorithms and computer programs used for obtaining these results.

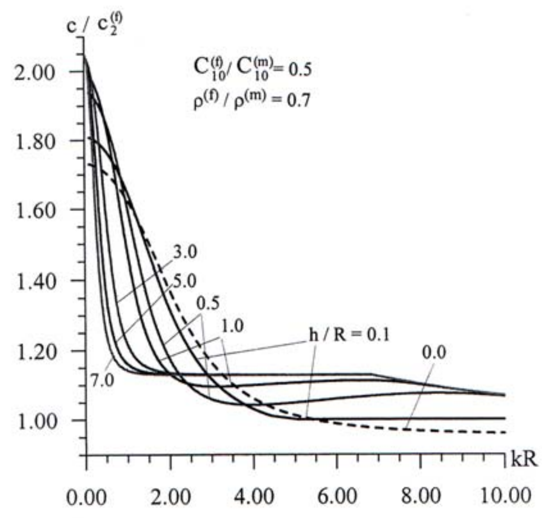

Fig. 3.

Let's investigate the influence of change of the parameter h/R on dispersing curves. The indicated influence is illustrated by the graphs cited in figure 3 and 4 for the first and second modes, respectively, that where constructed for $C_{10}^{(f)}/C_{10}^{(m)} =$

0.5 . Notice that in these figures, the graph of dependence between $C/C_2^{(f)}$ and kR for a hollow cylinder (punctured lines) is also given. It follows from these figures that in the first mode the character of the influence of the parameter h/R on the value of $C/C_2^{(f)}$ depends on kR. Moreover, for the second mode the value of $C/C_2^{(f)}$ decreases with decreasing h/R for any fixed kR. However for all the considered h/R the formulas (19)-(21) for limiting values of $C/C_2^{(f)}$ remain valid.

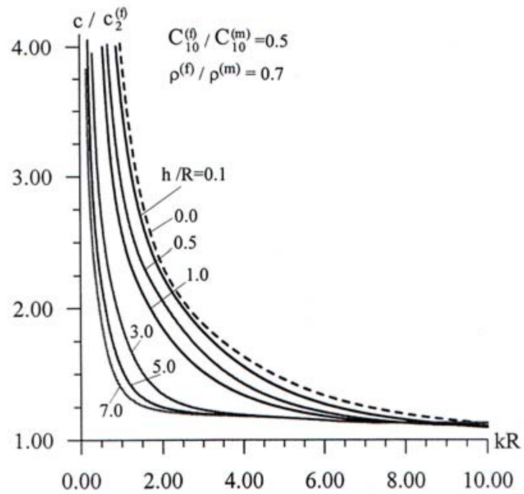

Fig. 4.

Now, let's analyze the results concerning the influence of initial strains in the component of a compound cylinder on dependence between $C/C_2^{(f)}$ and kR. Consider the case when $C_{10}^{(f)}/C_{10}^{(m)} = 0.5$; $h/R = 0.5$. The graphs of the indicated dependencies are cited in figures 5 and 6 for the first and second modes, respectively. It follows from these graphs that in all cases the presence of initial extension in the components of a cylinder leads to increase of propagation velocity of the considered wave.Moreover, with growth of this extension, the wave velocity monotonically increases.

Let's consider influence of pre-strains in the limiting value of $C/C_2^{(f)}$. Acting on the same way as in the pre-strainless case we get the following expression for $C/C_2^{(f)}$ as $kR \to 0$.

$$\frac{C}{C_2^{(f)}} = \left[\frac{C_{10}^{(m)}}{C_{10}^{(f)}} \eta^{(m)} \left(\left(\lambda^{(m)}\right)^2 + \frac{2}{\lambda^{(m)}} \right) + \right.$$

$$\left. + \eta^{(f)} \left(\left(\lambda^{(f)}\right)^2 + \frac{2}{\lambda^{(f)}} \right) \right]^{1/2} \left(\frac{\rho^{(m)}}{\rho^{(f)}} \eta^{(m)} + \eta^{(f)} \right)^{-1/2}, \qquad (22)$$

In case $\lambda^{(f)} = \lambda^{(m)} = 1.0$, expression (22) coincides with expression (19). Notice that the velocity of (22) ((19)) is "a bar velocity" for a pre-strained compound beam (pre-strain less). Thus, it follows from (22) that presense of initial extension in any component of compound cylinder causes increase of its "bar" velocity. i.e. limiting velocity as $kR \to 0$. However, influence of initial strains on the limiting value of $C/C_2^{(f)}$ as $kR \to \infty$ is of more complicated character. Analyses of results show that the following limits hold:

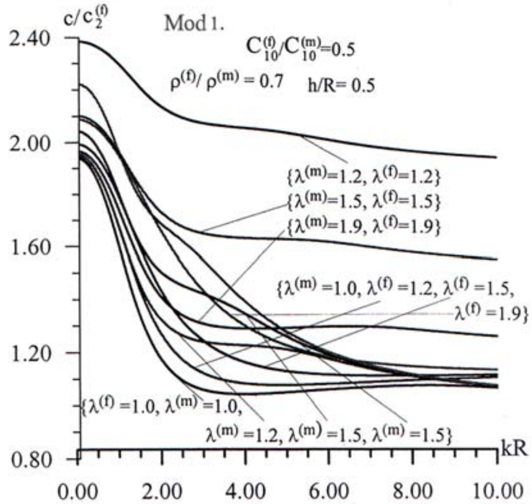

Fig. 5.

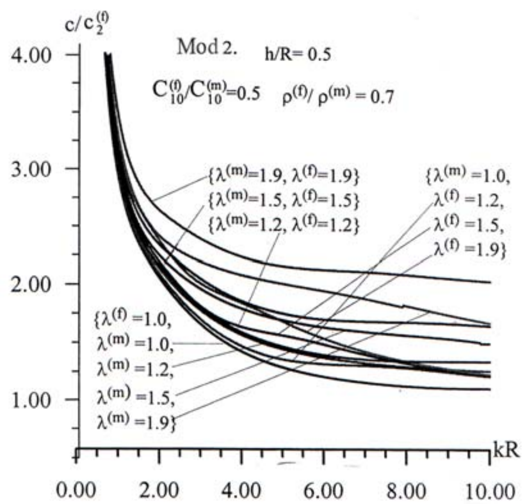

Fig. 6.

For the first mode:

$$C/C_2^{(f)} \underset{kR\to\infty}{\to} 1 \quad \text{for the cases} \quad \lambda^f = 1.0, \quad \lambda^m \geq 1.0,$$

$$C/C_2^{(f)} \underset{kR\to\infty}{\to} C_R^{(m)}/C_2^{(f)} \quad \text{for the cases} \quad \lambda^m = 1.0, \quad \lambda^f \geq 1.0,$$

$$C/C_2^{(f)} \underset{kR\to\infty}{\to} C_2^{(f)}\left(\lambda^{(f)}\right)/C_2^{(f)} \quad \text{for the cases} \quad \lambda^m = \lambda^f > 1.0, \qquad (23)$$

for the second mode:

$$C/C_2^{(f)}\overrightarrow{kR\to\infty}C_R^{(m)}\left(\lambda^{(m)}\right)/C_2^{(f)}\quad \text{for the cases}\quad \lambda^f=1.0,\quad \lambda^m\geq 1.0,$$

$$C/C_2^{(f)}\overrightarrow{kR\to\infty}C_2^{(m)}/C_2^{(f)}\quad \text{for the cases}\quad \lambda^m=1.0,\quad \lambda^f\geq 1.0,$$

$$C/C_2^{(f)}\overrightarrow{kR\to\infty}C_2^{(m)}\left(\lambda^{(m)}\right)/C_2^{(f)}\quad \text{for the cases}\quad \lambda^{(m)}=\lambda^{(f)}>1.0, \tag{24}$$

where $C_2^{(f)}\left(\lambda^{(f)}\right)$ $\left(C_2^{(m)}\left(\lambda^{(m)}\right)\right)$ is wave propagation velocity in the material of pre-strained inner (outer) perfect (hollow) cylinder; $C_R^{(m)}\left(\lambda^{(m)}\right)$ are strain velocities that by [3] are determined by the expression

$$C_R^{(m)}/C_2^{(f)}=\lambda^{(m)}\sqrt{C_{10}^{(m)}\rho^{(f)}\left(1-x_*^2\left(\lambda^{(m)}\right)^4\right)/\left(C_{10}^{(f)}\rho^{(m)}\right)}$$

where $x_*\approx 0.2916$.

Thus, presence of initial extensions essentially influence on the limiting value of $C/C_2^{(f)}$ as $kR\to\infty$. In this connection, the character of dependence between $C/C_2^{(f)}$ and kR strongly changes that is shown in the graphs of figures 5 and 6 . Notice that similar results on the influence of pre-strains on dispersing curve are obtained in other values of the parameters h/R and $C_{10}^{(m)}/C_{10}^{(f)}$ as well.

Conclusion.

Thus, in the frames of the piecewise homogeneous body model with the use of $TLTEWPPB$ the dispersion of the axisymmetric longitudinal wave in the finite pre-strained, compound cylinder made of incompressible material is studied. Therewith relation of elasticity for the materials of a cylinder is described by the Treloar potential. The numerical results illustrating the influence of problem parameters on the dispersing curves, are given. From the analysis of these results we conclude:

-presence of initial extension in the components of a compound cylinder reduces to increase of wave propagation velocity:

- to determine limiting values of this velocity as $kR\to\infty$ and as $kR\to 0$ analytic expressions are obtained in the form of the formulae (22)-(24).

-the obtained results pass to corresponding results obtained at pre-strain less cases that are also investigated in the present paper.

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