MECHANICS

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ON STABILITY OF INHOMOGENEOUS ORTHOTROPIC RECTANGULAR PLATE LYING ON AN INHOMOGENEOUS BASES

Abstract

A stability problem of a rectangular inhomogeneous orthotropic plate lying on an inhomogeneous base is solved. By solving the stated problem we carried out calculations for supported plate under one-sided compression for a cylindrical form stability loss.

As is known, at present the bars, plates and shells of different figurations are bearing elements of engineering complexes, mechanisms, devices and so on.

Recent years natural and artificial inhomogeneous anisotropic (orthotropic) materials are widely used in various fields of machine-building and civil engineering and in projecting arterial railways, different assignment of pipelines and in many other fields, and therefore there arises necessity to create methods for stability, strength and etc. analysis allowing for external medium resistance [1,2].

In the given paper it is supposed that elasticity modules E_1 and E_2 and shear modulus G depend on lenth coordinate and a plate is in biaxial uniform compression of identity P and Q and lies on an inhomogeneous base of Fouss-Winkler type [3]

$$F = -k\Pi\left(x, y\right) w \tag{1}$$

here k is a linear resistance coefficient, $\Pi\left(x, y\right)$ is a continuous function, W is deflection (coordinate system is chosen in the following way: the axes X and Y are situated on the median plane, and the axis Z is perpendicular to them). The quantities E_1, E_2 and G are accepted in the form

$$E_1 = E_1^0 f\left(x\right), \quad E_2 = E_2^0 f\left(x\right); \quad G = G^0 f\left(x\right) \tag{2}$$

here E_1^0, E_2^0, G^0 correspond to the homogeneous isotropic case, the function $f\left(x\right)$ with its derivatives is a continuous function, ν_1 and ν_2 are the Poisson coefficients, the relation between stress and deformations is of the form:

$$\begin{gathered}\sigma_1 = \frac{E^0 f\left(x\right)}{12\left(1 - v_1 v_2\right)}\left(\varepsilon_1 + v_2\varepsilon_2\right)\\ \sigma_2 = \frac{E_2^0 f\left(x\right)}{12\left(1 - v_1 v_2\right)}\left(\varepsilon_2 + v_1\varepsilon_1\right)\\ \tau = G_0 f\left(x\right)\gamma\end{gathered} \tag{3}$$

Bending stresses in the arbitrary layer of the plate are written as follows:

$$\sigma_{1,n} = -\frac{E_1^0 f\left(x\right)}{1 - v_1 v_2} z\left(\frac{\partial^2 w}{\partial x^2} + v_1\frac{\partial^2 w}{\partial y^2}\right)$$

$$\sigma_{2,n}=-\frac{E_2^0 f(x)}{1-v_1v_2}z\left(\frac{\partial^2 w}{\partial y^2}+v_1\frac{\partial^2 w}{\partial x^2}\right) \tag{4}$$

$$\tau=-2G_0 f(x)\,Z\frac{\partial^2 w}{\partial x\partial y}$$

Using (4) we find the bending moment and torque

$$M_1=-D_1^0 f(x)\left(\frac{\partial^2 w}{\partial x^2}+v_2\frac{\partial^2 w}{\partial y^2}\right)$$

$$M_2=-D_2^0 f(x)\left(\frac{\partial^2 w}{\partial y^2}+v_1\frac{\partial^2 w}{\partial x^2}\right) \tag{5}$$

$$H=-2D_K^0\frac{\partial^2 W}{\partial x\partial y}f(x)$$

Here $D_1^0=\dfrac{E_1^0 h^3}{12(1-v_1v_2)}$; $D_2^0=\dfrac{E_2^0 h^3}{12(1-v_1v_2)}$; $D_K^0=\dfrac{1}{12}G_0h^3$;
We write the stability equation in the form

$$\frac{\partial^2 M_1}{\partial x^2}+2\frac{\partial H}{\partial x\partial y}+\frac{\partial^2 M_2}{\partial y^2}+k\prod(x,y)\,w-P\left(\frac{\partial^2 w}{\partial x^2}+\beta\frac{\partial^2 w}{\partial y^2}\right)=0 \tag{6}$$

Here $\beta=Q\cdot P^{-1}$.
Substituting (5) in equation (6) we get:

$$D_1^0\frac{\partial^2}{\partial x^2}\left[f(x)\left(\frac{\partial^2 w}{\partial x^2}+v_2\frac{\partial^2 w}{\partial y^2}\right)\right]+2D_K^0\frac{\partial^2}{\partial x\partial y}\left[f(x)\frac{\partial^2 w}{\partial x\partial y}\right]+$$

$$+D_2^0\frac{\partial^2}{\partial y^2}\left[f(x)\left(\frac{\partial^2 w}{\partial y^2}+v_1\frac{\partial^2 w}{\partial x^2}\right)\right]-$$

$$-K_0\prod(x,y)\,w+P\left(\frac{\partial^2 w}{\partial x^2}+\beta\frac{\partial^2 w}{\partial y^2}\right)=0$$

Making some transformations we finally get:

$$f(x)\left[D_1^0\frac{\partial^4 w}{\partial x^2}+\left(D_k^0v_1+v_2D_1^0\right)\left(\frac{\partial^4 W}{\partial x^2\partial y^2}+D_2\frac{\partial^4 w}{\partial y^4}\right)\right]+$$

$$+2D_1^0\frac{df}{dx}\left(\frac{\partial^3 w}{\partial x^3}+v_2\frac{\partial^3 w}{\partial x\partial y^2}\right)+$$

$$+D_1^0\frac{\partial^2 f}{\partial x^2}\left(\frac{\partial^2 w}{\partial y^2}+v_2\frac{\partial^2 w}{\partial x^2}\right)-K\prod(x,y)\,w+f\left(\frac{\partial^2 w}{\partial x^2}+\beta\frac{\partial^2 w}{\partial y^2}\right)=0 \tag{7}$$

As we see equation (7) is complicated and it is unlikely to get exact solution and moreover complexity degree of equations essentially depends on the functions $f(x)$ and $\prod(x,y)$.

Therefore, it is appropriate to use approximate analytic methods. As is noted in A.A.Alkhutov's monograph [4] and many other scientists researches in this, case Bubnov-Galerkin's method is very approved. By solving this problem we'll use the above-mentioned orthogonalization method.

We'll take the approximating functions in the form:

$$W(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}A_{ij}\varphi_i(x)\psi_j(y) \tag{8}$$

Here each of $\varphi_i(x)$ and $\psi_j(y)$ must satisfy the appropriate boundary conditions. For convergence of analysis we take the following denotation

$$Q_1=f(x)\left[D_1^0\frac{\partial^4 w}{\partial x^4}+\left(D_k v_1+v_2D_1^0\right)\frac{\partial^4 w}{\partial x^2\partial y^2}+D_2^0\frac{\partial^4 w}{\partial x^4}\right]$$

$$Q_2=2D_1^0\frac{df}{dx}\left(\frac{\partial^3 w}{\partial x^3}+v_2\frac{\partial^3 w}{\partial x\partial y^2}\right)$$

$$Q_4=\frac{\partial^2 w}{\partial x^2}+\beta\frac{\partial^2 w}{\partial y^2} \tag{9}$$

Then, allowing for (9), equation (7) will take the form:

$$Q_1(x,y)+Q_2(x,y)+Q_3(x,y)-K\prod(x,y)\,w+PQ_4=0$$

Or, allowing for (8) on the basis of Bubnov-Galerkin method:

$$\sum_{i=1}^{n}\sum_{j=1}A_{ij}\int_0^a\int_0^b[Q_1(x,y)+Q_2(x,y)+Q_3(x,y)-K\prod(x,y)\varphi_i(x)\psi_j(y)+$$

$$+P\left(\frac{d^2\varphi_i}{dx^2}\psi_j(y)+P\frac{\partial^2\psi_j(y)}{dy^2}\varphi_i(x)\right)]\varphi_k\psi_q dxdy=0$$

For engineering calculating it sufficient to neglect the first approximation, since at the given moment the solution is not difficult [4].

As it was noted above, the solution of the problem is simplified in the case of simple dependencies on $f(x)$ and $\prod(x,y)$ in the form:

$$f=1+\varepsilon x\cdot a^{-1};\qquad \prod(x,y)=1+\mu XYa^{-1}b^{-1};$$

$$where\quad \varepsilon\in[0,1)\qquad \mu\in[0,1); \tag{10}$$

For the first approximation the critical load solution is determined from the following condition:

$$\int_0^a\int_0^b[Q_1(x,y)+Q_2(x,y)+Q_3(x,y)-K\prod(x,y)\varphi_1(x)\psi_1(y)+$$

$$+P\left(\frac{d^2\varphi_1}{dx^2}\psi_1(y)+\beta\frac{\partial^2\psi_1(y)}{dy^2}\varphi_1(x)\right)]\varphi_1\psi_1 dxdy=0 \tag{11}$$

For the case (10) equation (11) take the following form:

$$\int_0^a\int_0^b\left(1+\varepsilon xa^{-1}\right)\left[D_1^0\frac{d^4\varphi_1}{dx^4}\psi_1^2\varphi_1+\left(D_k\nu_1+\nu_2D_1^0\right)\frac{d^2\varphi_1}{dx^2}\frac{\partial^2\psi_1}{dy^2}\varphi_1\psi_1+\right.$$

$$+2\varepsilon a^{-1}D_1^0\left(\frac{d^3\varphi_1}{dx^3}\psi_1^2\varphi_1+v_2\frac{d^2\psi d\varphi_1}{dydx}\varphi_1\psi_1\right)-$$

$$-k\int_0^a\int_0^b\left(1+\mu xya^{-1}b^{-1}\right)\varphi_1^2\left(x\right)\psi_1^2\left(y\right)dxdy+$$

$$+P\int_0^a\int_0^b\left(\frac{d^2\varphi_1}{dx^2}\psi_1^2\left(y\right)\varphi_1+\beta\frac{d^2\psi}{dy^2}\varphi_1^2+\psi_1\left(y\right)\right)dxdy=0$$

We can construct the solution for comprehensive hinge joint and comprehensive rigit fixation.

In the first case the approximating functions should satisfy the following conditions:

$$\varphi_1=0;\quad x=0;\quad \psi_1=0;\quad y=0;$$

$$\frac{d^2\varphi_1}{dx^2}=0\quad for\quad x=a;\qquad \frac{d^2\psi_1}{dy^2}=0\quad for\quad y=b \tag{12}$$

For the second case the following conditions should be satisfied:

$$\varphi_1=0;\quad x=0;\quad \psi_1=0;\quad y=0;$$

$$\frac{d\varphi_1}{dx}=0\quad for\quad x=a;\qquad \frac{d\psi_1}{dy}=0\quad for\quad y=b \tag{13}$$

For the first case we can take W in the form

$$w=f_0\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{14}$$

For the second case

$$w=f_0\left(1-\cos\frac{2\pi x}{a}\right)\left(1-\cos\frac{2\pi y}{b}\right)$$

or

$$w=x^2y^2\left(x-a\right)^2\left(y-b\right)^2$$

It should be noted that we can select other approximations as well, however for calculations bilateral hinge support is convenient, though other variants are not difficult.

Stated problem analysis is sufficiently simplified at one sided compression when stability loss get a cylindrical form of bending.

Accepting

$$\prod=1+\mu xa^{-1},\qquad \beta=1,\quad \psi=1$$

we get that critical load is determined from the following relation

$$\int_0^a\left(1+\varepsilon xa^{-1}\right)\left[D_1^0\frac{d^4\varphi_1}{dx^4}\varphi_1\right]dx+2\varepsilon a^{-1}D_1^0\frac{d^3\varphi_1}{dx^3}\varphi_1dx+$$

$$+k\int_0^a\left(1+\mu xa^{-1}\right)\varphi_1^2dx-p\int_0^a\frac{d^2\varphi_1}{dx^2}\varphi_1dx=0 \tag{15}$$

From (15) we find:

$$P^k = D_1^0 \int_0^a \left(1+\varepsilon x a^{-1}\right)\left(\frac{d^4\varphi_1}{dx^4}\varphi_1\right)dx +$$

$$+2\varepsilon\alpha^{-1}D_1^0\int_0^a \frac{d^3\varphi_1}{dx^3}\varphi_1 dx + k\int_0^a \left(1+\mu x a^{-1}\right)\varphi_1^2 dx$$

$$\int_0^a \frac{d^2\varphi_1}{dx^2}\varphi_1 dx \qquad (16)$$

In the case when a plate is homogeneous, 2 orthotropic relation (16) accepts the form:

$$P_0^k = \frac{D_1^0 \int_0^a \left(\frac{d^4\varphi_1}{dx^4}\varphi_1\right)dx + k\int_0^a \left(1+\mu x a^{-1}\right)\varphi_1^2 dx}{\int_0^a \frac{d^2\varphi_1}{dx^2}\varphi_1 dx} \qquad (17)$$

For the Winkler's basis we get:

$$P_0^k \frac{D_1^0 \int_0^a \left(\frac{d^4\varphi_1}{dx^4}\varphi_1\right)dx + k\int_0^a \varphi_1^2 dx}{\int_0^a \frac{d^2\varphi_1}{dx^2}\varphi_1 dx} \qquad (18)$$

From relations (17) and (18) we get:

$$\bar{P} = \frac{P_0^k}{p_b^k}\frac{D_1^0 \int_0^a \left(\frac{d^4\varphi_1}{dx^4}\varphi_1\right)dx + k\int_0^a \left(1+\mu x a^{-1}\right)\varphi_1^2 dx}{D_1^0 \int_0^a \left(\frac{d^4\varphi_1}{dx^4}\varphi_1\right)dx + k\int_0^a \varphi_1^2 dx} \qquad (19)$$

In the case when external medium resistance is ignored, from (16) we get:

$$P_b^k = \frac{D_1^0 \int_0^a \left(1+\varepsilon x a^{-1}\right)\left(\frac{d^4\varphi_1}{dx^4}\varphi_1\right)dx + 2\varepsilon\alpha^{-1}D_1^0\int_0^a \frac{d^3\varphi_1}{dx^3}\varphi_1 dx}{\int_0^a \frac{d^2\varphi_1}{dx^2}\varphi_1 dx} \qquad (20)$$

The following relation is convenient for calculations

$$\frac{p_b^k}{p_0^k} = \frac{D_1^0 \int_0^a \left(1+\varepsilon x a^{-1}\right)\left(\frac{d^4\varphi_1}{dx^4}\varphi dx\right) + 2\varepsilon\alpha^{-1}D_1^0\int_0^a \frac{d^3\varphi_1}{dx^3}\varphi_1 dx}{D_1^0 \int_0^a \left(\frac{d^4\varphi_1}{dx^4}\varphi\right)dx + k\int_0^a \left(1+\mu x a\right)\varphi_1^2 dx} \qquad (21)$$

For the case $\varphi_1 = \sin\dfrac{m\pi x}{a}$; $\quad \int_0^a \dfrac{d^3\varphi_1}{dx^3}\varphi_1 dx = 0.$

Hence we find:

$$\bar{p}_1 = \frac{p_0^k}{p_b^k} = \frac{\int_0^1 \sin^2 m\pi x dx}{\int (1+\varepsilon\bar{x})^2 \sin^2 \pi n\bar{x} dx}$$

or

$$\bar{p}_1 = \frac{1}{1+0,5\varepsilon} \tag{22}$$

Dependence $\bar{p}_1\tilde{}\varepsilon$ is represented in fig.1 and table.

Obviously, for different kinds of in homogeneity we'll get another curves that reflect influence of inhomogeneity we'll get another curves that reflect influence of inhomogeneity and orthotropy on critical parameters quantities

E	$\overline{P}$
0	1
0,4	0,8333
0,6	0,7669
0,8	0,7142
1,0	0,6666

Table

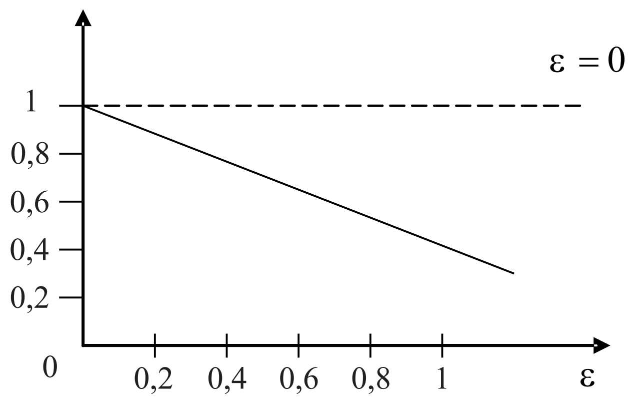

Fig.1.

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