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ON THE SOLVABILITY OF THE EQUATIONS CONTAINING IN THE MAIN PART THE OPERATORS OF THE FORM $-\frac{d^3}{dt^3} + A^3$ IN THE WEIGHTED SPACE

Abstract

Sufficient conditions put on the operator coefficients of the investigated operator-differential equation of the third order, which provide its regular solvability in weighted space, are obtained in the paper. In this case the connection between the weigh parameter and lower bound of spectrum of the operator A is shown.

Let's consider in separable Hilbert space H the polynomial operator bunch

$$\Phi(\lambda) = -\lambda^3 E + \lambda^2 A_1 + \lambda A_2 + A_3 + A^3, \tag{1}$$

where E is identity operator, A_1, A_2, A_3, A are the linear operators in H, moreover A is a self-adjoint positively-defined operator with lower bound of spectrum λ_0 (i.e. $A = A^* \geq \lambda_0 E$, $\lambda_0 > 0$), $A_j A^{-j}$, $j = 1, 2, 3$ are bounded operators.

Let æ be some real number: æ$\in \mathbb{R} = (-\infty; +\infty)$. We define the following Hilbert spaces:

$$L_{2,æ}(\mathbb{R}; H) = \left\{ u(t) : \|u\|_{L_{2,æ}(\mathbb{R};H)} = \left(\int_{-\infty}^{+\infty} \|u(t)\|_H^2 e^{-æt} dt \right)^{1/2} < +\infty \right\},$$

$$W_{2,æ}^3(\mathbb{R}; H) =$$

$$= \left\{ u(t) : \|u\|_{W_{2,æ}^3(\mathbb{R};H)} = \left(\int_{-\infty}^{+\infty} \left(\left\| \frac{d^3 u(t)}{dt^3} \right\|_H^2 + \left\| A^3 u(t) \right\|_H^2 \right) e^{-æt} dt \right)^{1/2} < +\infty \right\}.$$

It is clear that for æ$=0$ we'll have the spaces $L_{2,0}(\mathbb{R}; H) = L_2(\mathbb{R}; H)$, $W_{2,0}^3(\mathbb{R}; H) = W_2^3(\mathbb{R}; H)$ about which in detail see [1, chapter 1].

We connect with the bunch (1) the operator-differential equation

$$\Phi(d/dt)\, u(t) = f(t), \quad t \in \mathbb{R}, \tag{2}$$

where $f(t) \in L_{2,æ}(\mathbb{R}; H)$, $u(t) \in W_{2,æ}^3(\mathbb{R}; H)$.

We note that many problems in mathematical physics and mechanic require studying the spectral properties of the operator bunches, which, in their turn, are connected with the solvability problems for the corresponding operator-differential equations. The aim of the present paper is investigation the solvability problem for the equation (2) in definite weighted space.

Definition 1. *If for* $f(t) \in L_{2,æ}(\mathbb{R}; H)$ *there is the vector-function* $u(t) \in W^3_{2,æ}(\mathbb{R}; H)$*, satisfying the equation (2) almost everywhere, then we'll call it the regular solution of the equation (2).*

Definition 2. *If for any* $f(t) \in L_{2,æ}(\mathbb{R}; H)$ *there is the regular solution of the equation (2), moreover, the inequality*

$$\|u\|_{W^3_{2,æ}(\mathbb{R};H)} \le const \, \|f\|_{L_{2,æ}(\mathbb{R};H)}$$

takes place, then we'll call the equation (2) regularly solvable.

In the given paper we'll obtain the conditions, expressed in terms of the operator coefficients of the bunch (1), providing the regular solvability of the equation (2).

Now we'll investigate the stated problem. Let's introduce the following notations:

$$\Phi_0 u(t) = -\frac{d^3 u(t)}{dt^3} + A^3 u(t), \quad u(t) \in W^3_{2,æ}(\mathbb{R}; H),$$

$$\Phi_1 u(t) = A_1 \frac{d^2 u(t)}{dt^2} + A_2 \frac{du(t)}{dt} + A_3 u(t), \quad u(t) \in W^3_{2,æ}(\mathbb{R}; H).$$

Substituting $u(t) = v(t) e^{\frac{æ}{2}t}$ in the equation $\Phi_0 u(t) = f(t)$, we obtain that

$$-\left(\frac{d}{dt} + \frac{æ}{2}\right)^3 v(t) + A^3 v(t) = g(t),$$

where $v(t) = u(t) e^{-\frac{æ}{2}t} \in W^3_2(\mathbb{R}; H), g(t) = f(t) e^{-\frac{æ}{2}t} \in L_2(\mathbb{R}; H)$.

Now let's denote by

$$\Phi_{0,æ} v(t) = -\left(\frac{d}{dt} + \frac{æ}{2}\right)^3 v(t) + A^3 v(t), \quad v(t) \in W^3_2(\mathbb{R}; H).$$

For solving the equation

$$\Phi_{0,æ} v(t) = g(t), \tag{3}$$

where $v(t) \in W^3_2(\mathbb{R}; H)$, $g(t) \in L_2(\mathbb{R}; H)$, we'll do Fourier transformation:

$$\left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3\right) \hat{v}(\zeta) = \hat{g}(\zeta),$$

where $\hat{v}(\zeta)$, $\hat{g}(\zeta)$ are Fourier transformations of the vector-functions $v(t)$, $g(t)$, correspondingly.

Let's prove that for æ$\in (-\lambda_0; 2\lambda_0)$ the operator bunch

$$\Phi_{0,æ}(-i\zeta; A) = -\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3 \tag{4}$$

is invertible. Really, let $\lambda \in \sigma(A)$ $(\lambda \geq \lambda_0)$, then the characteristical polynomial (4) has the following form:

$$\Phi_{0,æ}(-i\zeta;\lambda) = -\left(-i\zeta + \frac{æ}{2}\right)^3 + \lambda^3 = -\left(-i\zeta + \frac{æ}{2} - \lambda\right) \times$$
$$\times \left(-i\zeta + \frac{æ}{2} + \frac{\lambda}{2} - i\frac{\sqrt{3}}{2}\lambda\right)\left(-i\zeta + \frac{æ}{2} + \frac{\lambda}{2} + i\frac{\sqrt{3}}{2}\lambda\right).$$

From here we have

$$|\Phi_{0,æ}(-i\zeta;\lambda)| = \left|i\zeta - \frac{æ}{2} + \lambda\right| \left|-i\zeta + \frac{æ}{2} + \frac{\lambda}{2} - i\frac{\sqrt{3}}{2}\lambda\right| \left|-i\zeta + \frac{æ}{2} + \frac{\lambda}{2} + i\frac{\sqrt{3}}{2}\lambda\right| \geq$$
$$\geq \left(\lambda - \frac{æ}{2}\right)\left(\frac{\lambda}{2} + \frac{æ}{2}\right)^2 \geq \left(\lambda_0 - \frac{æ}{2}\right)\left(\frac{\lambda_0}{2} + \frac{æ}{2}\right)^2 > 0,$$

i.e. from the spectral decomposition of the operator A it follows that the operator bunch (4) is invertible for $æ \in (-\lambda_0; 2\lambda_0)$.

That's why,

$$\hat{v}(\zeta) = \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3\right)^{-1} \hat{g}(\zeta). \tag{5}$$

So,

$$v(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3\right)^{-1} \hat{g}(\zeta)\, e^{i\zeta t} d\zeta.$$

It is clear that $v(t)$ satisfies the equation (3) almost everywhere. Let's prove that $v(t) \in W_2^3(\mathbb{R}; H)$. Really, from Plancherel theorem it's sufficiently to show that $A^3\hat{v}(\zeta) \in L_2(\mathbb{R}; H)$ and $\zeta^3\hat{v}(\zeta) \in L_2(\mathbb{R}; H)$.

Obviously,

$$\|v\|^2_{W_2^3(\mathbb{R};H)} = \left\|\frac{d^3 v}{dt^3}\right\|^2_{L_2(\mathbb{R};H)} + \left\|A^3 v\right\|^2_{L_2(\mathbb{R};H)} =$$
$$= \left\|\zeta^3 \hat{v}(\zeta)\right\|^2_{L_2(\mathbb{R};H)} + \left\|A^3 \hat{v}(\zeta)\right\|^2_{L_2(\mathbb{R};H)}.$$

According to

$$\left\|A^3 \hat{v}(\zeta)\right\|_{L_2(\mathbb{R};H)} = \left\|A^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3\right)^{-1} \hat{g}(\zeta)\right\|_{L_2(\mathbb{R};H)} \leq$$
$$\leq \sup_{\zeta \in \mathbb{R}} \left\|A^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3\right)^{-1}\right\| \|\hat{g}(\zeta)\|_{L_2(\mathbb{R};H)}$$

and

$$\left\|\zeta^3 \hat{v}(\zeta)\right\|_{L_2(\mathbb{R};H)} = \left\|\zeta^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3\right)^{-1} \hat{g}(\zeta)\right\|_{L_2(\mathbb{R};H)} \leq$$

$$\leq \sup_{\zeta\in\mathbb{R}} \left\| \zeta^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3 \right)^{-1} \right\| \|\hat{g}(\zeta)\|_{L_2(\mathbb{R};H)}$$

we estimate the norms $\left\| A^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3 \right)^{-1} \right\|$ and $\left\| \zeta^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3 \right)^{-1} \right\|$ for $\zeta \in \mathbb{R}$. Applying the spectral theory of selfadjoint operators, we can determine that for $æ\in(-\lambda_0; 2\lambda_0)$

$$\left\| A^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3 \right)^{-1} \right\| = \sup_{\lambda\in\sigma(A)} \left| \lambda^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 + \lambda^3 \right)^{-1} \right| \leq c_0(æ),$$

where

$$c_0(æ) = \frac{4\lambda_0^3}{\left(\lambda_0 - \frac{æ}{2}\right)(\lambda_0 + æ)^2}, \tag{6}$$

and

$$\left\| \zeta^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3 \right)^{-1} \right\| = \sup_{\lambda\in\sigma(A)} \left\| \zeta^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 + \lambda^3 \right)^{-1} \right\| \leq$$

$$\leq \frac{\left((æ+\lambda_0)^2 + 3\lambda_0^2\right)^2}{(æ+\lambda_0)^2\left((æ+\lambda_0)^2 + 3\lambda_0^2\right)}.$$

So,

$$\left\| A^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3 \right)^{-1} \hat{g}(\zeta) \right\|_{L_2(\mathbb{R};H)} \leq c_0(æ) \|\hat{g}(\zeta)\|_{L_2(\mathbb{R};H)}$$

and

$$\left\| \zeta^3 \left(-\left(-i\zeta + \frac{æ}{2}\right)^3 E + A^3 \right)^{-1} \hat{g}(\zeta) \right\|_{L_2(\mathbb{R};H)} \leq$$

$$\leq \frac{\left((æ+\lambda_0)^2 + 3\lambda_0^2\right)^2}{(æ+\lambda_0)^2\left((æ+\lambda_0)^2 + 3\lambda_0^2\right)} \|\hat{g}(\zeta)\|_{L_2(\mathbb{R};H)}.$$

Consequently, $v(t) \in W_2^3(\mathbb{R};H)$.

From the other side, $\Phi_{0,æ} : W_2^3(\mathbb{R};H) \to L_2(\mathbb{R};H)$ is bounded. Really, for $v(t) \in W_2^3(\mathbb{R};H)$

$$\|\Phi_{0,æ} v\|_{L_2(\mathbb{R};H)} = \left\| -\left(\frac{d}{dt} + \frac{æ}{2}\right)^3 v + A^3 v \right\|_{L_2(\mathbb{R};H)} \leq$$

$$\leq \left\| \left(\frac{d}{dt} + \frac{æ}{2}\right)^3 v \right\|_{L_2(\mathbb{R};H)} + \|A^3 v\|_{L_2(\mathbb{R};H)} \leq$$

$$\leq \left\| \frac{d^3v}{dt^3} \right\|_{L_2(\mathbb{R};H)} + 3\frac{|æ|}{2} \left\| \frac{d^2v}{dt^2} \right\|_{L_2(\mathbb{R};H)} + 3\frac{æ^2}{4} \left\| \frac{dv}{dt} \right\|_{L_2(\mathbb{R};H)} +$$

$$+\frac{|æ|^3}{8} \|v\|_{L_2(\mathbb{R};H)} + \left\|A^3v\right\|_{L_2(\mathbb{R};H)} \leq$$

$$\leq \left\| \frac{d^3v}{dt^3} \right\|_{L_2(\mathbb{R};H)} + \frac{3\,|æ|}{2\lambda_0} \left\| A\frac{d^2v}{dt^2} \right\|_{L_2(\mathbb{R};H)} +$$

$$+3\frac{æ^2}{4\lambda_0^2} \left\| A^2\frac{dv}{dt} \right\|_{L_2(\mathbb{R};H)} + \left(\frac{|æ|^3}{8\lambda_0^3} + 1 \right) \left\|A^3v\right\|_{L_2(\mathbb{R};H)}. \qquad (7)$$

Then from the theorem on intermediate derivatives [1, chapter 1]

$$\left\| A^j\frac{d^{3-j}v}{dt^{3-j}} \right\|_{L_2(\mathbb{R};H)} \leq d_{3-j} \|v\|_{W_2^3(\mathbb{R};H)}, \quad j = 0, 1, 2, 3,$$

because of it from the inequality (7) it follows that

$$\|\Phi_{0,æ}v\|_{L_2(\mathbb{R};H)} \leq const \|v\|_{W_2^3(\mathbb{R};H)}.$$

So, the operator $\Phi_{0,æ} : W_2^3(\mathbb{R};H) \to L_2(\mathbb{R};H)$ is mutually one-valued and bounded. Then from Banach theorem on the inverse operator we obtain that there exists the bounded inverse $\Phi_{0,æ}^{-1} : L_2(\mathbb{R};H) \to W_2^3(\mathbb{R};H)$, i.e. the operator $\Phi_{0,æ}$ isomorphically maps the space $W_2^3(\mathbb{R};H)$ on $L_2(\mathbb{R};H)$.

As the mappings $v(t) \to u(t)\,e^{-\frac{æ}{2}t}$, $g(t) \to f(t)\,e^{-\frac{æ}{2}t}$ are isomorphisms between the spaces $W_2^3(\mathbb{R};H)$ and $W_{2,æ}^3(\mathbb{R};H)$, $L_2(\mathbb{R};H)$ and $L_{2,æ}(\mathbb{R};H)$, correspondingly, then the following theorem takes place.

Theorem 1. *Let the number* $æ\in \mathbb{R}$ *satisfies the condition* $æ\in (-\lambda_0, 2\lambda_0)$. *Then the operator* Φ_0 *makes isomorphism between* $W_{2,æ}^3(\mathbb{R};H)$ *and* $L_{2,æ}(\mathbb{R};H)$.

This theorem shows that the norm $\|\Phi_0 u\|_{L_{2,æ}(\mathbb{R};H)}$ is equivalent to the norm $\|u\|_{W_{2,æ}^3(\mathbb{R};H)}$ in the space $W_{2,æ}^3(\mathbb{R};H)$. As it is known, the operators of intermediate derivatives

$$A^{3-j}\frac{d^j}{dt^j} : W_{2,æ}^3(\mathbb{R},H) \to L_{2,æ}(\mathbb{R};H), \quad j = 0, 1, 2,$$

are continuous [1, chapter 1]. Because of it the norms of these operators can be estimated through $\|\Phi_0 u\|_{L_{2,æ}(\mathbb{R};H)}$. To do it, as $v(t) \to u(t)\,e^{-\frac{æ}{2}t}$ is the isomorphism, it is sufficiently to estimate below-mentioned norms through $\|\Phi_{0,æ}v\|_{L_{2,æ}(\mathbb{R};H)}$:

$$\left\|A^3v\right\|_{L_2(\mathbb{R};H)}, \quad \left\| A^2\left(\frac{d}{dt} + \frac{æ}{2}\right) v \right\|_{L_2(\mathbb{R};H)},$$

$$\left\| A\left(\frac{d}{dt} + \frac{æ}{2}\right)^2 v \right\|_{L_2(\mathbb{R};H)}.$$

From the equality (5) it follows that $(\hat{g}(\zeta) = \Phi_{0,æ}\hat{v}(\zeta))$

$$\left\|A^3\hat{v}(\zeta)\right\|_{L_2(\mathbb{R};H)} \le c_0(æ)\left\|\Phi_{0,æ}\,\hat{v}(\zeta)\right\|_{L_2(\mathbb{R};H)},$$

equivalent to the inequality

$$\left\|A^3 v\right\|_{L_2(\mathbb{R};H)} \le c_0(æ)\left\|\Phi_{0,æ}\,v\right\|_{L_2(\mathbb{R};H)}.$$

For estimation $\left\|A^2\left(\frac{d}{dt}+\frac{æ}{2}\right)v\right\|_{L_2(\mathbb{R};H)}$ through $\left\|\Phi_{0,æ}v\right\|_{L_2(\mathbb{R};H)}$ after substitution $\Phi_{0,æ}v(t) = g(t)$ and applying Fourier transformation we have:

$$\left\|A^2\left(-i\zeta+\frac{æ}{2}\right)\Phi^{-1}_{0,æ}(-i\zeta;A)\,\hat{g}(\zeta)\right\|_{L_2(\mathbb{R};H)} \le$$

$$\le \sup_{\zeta\in\mathbb{R}}\left\|A^2\left(-i\zeta+\frac{æ}{2}\right)\left(-\left(-i\zeta+\frac{æ}{2}\right)^3 E + A^3\right)^{-1}\right\|\left\|\hat{g}(\zeta)\right\|_{L_2(\mathbb{R};H)}.$$

That's why estimating the following norm for $\zeta\in\mathbb{R}$ and $æ\in(-\lambda_0;2\lambda_0)$:

$$\left\|A^2\left(-i\zeta+\frac{æ}{2}\right)\left(-\left(-i\zeta+\frac{æ}{2}\right)^3 E + A^3\right)^{-1}\right\| =$$

$$= \sup_{\lambda\in\sigma(A)}\left|\lambda^2\left(-i\zeta+\frac{æ}{2}\right)\left(-\left(-i\zeta+\frac{æ}{2}\right)^3 + \lambda^3\right)^{-1}\right| \le c_1(æ)$$

where

$$c_1(æ) = \frac{2\lambda_0^2\left(6\lambda_0^2+æ^2\right)^{1/2}}{(æ+\lambda_0)^2\left(\lambda_0-\frac{æ}{2}\right)}, \tag{8}$$

we obtain that

$$\left\|A^2\left(-i\zeta+\frac{æ}{2}\right)\Phi^{-1}_{0,æ}(-i\zeta;A)\,\hat{g}(\zeta)\right\|_{L_2(\mathbb{R};H)} \le c_1(æ)\left\|\hat{g}(\zeta)\right\|_{L_2(\mathbb{R};H)},$$

equivalent to the inequality

$$\left\|A^2\left(\frac{d}{dt}+\frac{æ}{2}\right)v\right\|_{L_2(\mathbb{R};H)} \le c_1(æ)\left\|\Phi_{0,æ}\,v\right\|_{L_2(\mathbb{R};H)}.$$

Analogously we obtain that for $æ\in(-\lambda_0;2\lambda_0)$

$$\left\|A\left(\frac{d}{dt}+\frac{æ}{2}\right)^2 v\right\|_{L_2(\mathbb{R};H)} \le c_2(æ)\left\|\Phi_{0,æ}\,v\right\|_{L_2(\mathbb{R};H)},$$

where

$$c_2(æ) = \frac{\lambda_0\left(6\lambda_0^2+æ^2\right)}{\left(\lambda_0-\frac{æ}{2}\right)(æ+\lambda_0)^2}. \tag{9}$$

Thus, the following theorem is true.

Theorem 2. *For any* $u(t) \in W_{2,æ}^{3}(\mathbb{R};H)$ *the following inequalities take place for* $æ \in (-\lambda_0; 2\lambda_0)$:

$$\left\| A^{3-j} \frac{d^j u}{dt^j} \right\|_{L_{2,æ}(\mathbb{R};H)} \leq c_j(æ) \left\| \Phi_0 u \right\|_{L_{2,æ}(\mathbb{R};H)}, \quad j = 0,1,2,$$

where $c_j(æ)$, $j = 0,1,2,$ *are defined in (6), (8), (9).*

Remark 1. For $æ = -\lambda_0$ and $æ = 2\lambda_0$ the operator Φ_0 is not invertible.

Further, taking into account theorem on the intermediate derivatives [1, chapter 1], it is easy to prove the following statement.

Theorem 3. *Operator* Φ_1 *is continuous from the space* $W_{2,æ}^{3}(\mathbb{R};H)$ *into* $L_{2,æ}(\mathbb{R};H)$.

As result, we determine the conditions, providing the regular solvability of the equation (2).

Theorem 4. *Let* $æ \in (-\lambda_0; 2\lambda_0)$ *and it takes place the inequality*

$$\sum_{j=1}^{3} q_j(æ) \left\| A_j A^{-j} \right\| < 1,$$

where the numbers $q_j(æ) = c_{3-j}(æ)$, $j = 1,2,3,$ *are defined in theorem 2. Then the equation (2) is regularly solvable.*

Proof. We present the equation (2) in the form of the following operator equation:

$$\Phi_0 u(t) + \Phi_1 u(t) = f(t), \tag{10}$$

where $f(t) \in L_{2,æ}(\mathbb{R};H)$, $u(t) \in W_{2,æ}^{3}(\mathbb{R};H)$.

From theorem 1 the equation $\Phi_0 u(t) = f(t)$ is regularly solvable.

Doing substitution $\Phi_0 u(t) = w(t)$, the equation (10) can be written in the form $\left(E + \Phi_1 \Phi_0^{-1}\right) w(t) = f(t)$. Then for any $w(t) \in L_{2,æ}(\mathbb{R};H)$ from theorem 2 we have:

$$\left\| \Phi_1 \Phi_0^{-1} w \right\|_{L_{2,æ}(\mathbb{R};H)} = \left\| \Phi_1 u \right\|_{L_{2,æ}(\mathbb{R};H)} \leq$$

$$\leq \sum_{j=1}^{3} \left\| A_j A^{-j} \right\| q_j(æ) \left\| \Phi_0 u \right\|_{L_{2,æ}(\mathbb{R};H)} =$$

$$= \sum_{j=1}^{3} q_j(æ) \left\| A_j A^{-j} \right\| \left\| w \right\|_{L_{2,æ}(\mathbb{R};H)}.$$

According to $\sum\limits_{j=1}^{3} q_j(æ) \left\| A_j A^{-j} \right\| < 1$ the operator $E + \Phi_1 \Phi_0^{-1}$ is invertible in the space $L_{2,æ}(\mathbb{R};H)$. Then $u(t)$ can be determined by the following formula:

$$u(t) = \Phi_0^{-1} \left(E + \Phi_1 \Phi_0^{-1}\right)^{-1} f(t).$$

From here it follows that

$$\|u\|_{W^3_{2,æ}(\mathbb{R};H)} \le \left\|\Phi_0^{-1}\right\|_{L_{2,æ}(\mathbb{R};H)\to W^3_{2,æ}(\mathbb{R};H)} \times$$

$$\times \left\|\left(E+\Phi_1\Phi_0^{-1}\right)^{-1}\right\|_{L_{2,æ}(\mathbb{R};H)\to L_{2,æ}(\mathbb{R};H)} \|f\|_{L_{2,æ}(\mathbb{R};H)} \le const\, \|f\|_{L_{2,æ}(\mathbb{R};H)}\,.$$

Theorem is proved.

Remark 2. These investigations analogously can be done and in the case, when the remainder part of the equation (2) has the variable operator coefficients.

We note that in the weighted space the boundary-value problem for the equation (2) on semi-axis $\mathbb{R}_+ = [0;+\infty)$ is studied in the paper [2]. Number of questions, considered in the given paper, is considered in the paper [3] for one class elliptic operator-differential equations of the second order on the whole axis $\mathbb{R}$.

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