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SOLUTION OF A NONLOCAL BOUNDARY VALUE PROBLEM FOR A FOURTH ORDER PARTIAL DIFFERENTIAL EQUATION

Abstract

In this paper the existence of the classical solution of a nonlocal boundary value problem for a fourth order partial differential equation is proved.

For the equation [1,2]

$$\varepsilon^2 u_{tttt}(x,t) - u_{ttxx}(x,t) + u_{tt}(x,t) - u_{xx}(x,t) = f(x,t) \qquad (1)$$

in the domain $D_T = \{(x,t): \ 0 \le x \le T\}$ it is considered a problem under ordinary local boundary conditions

$$u(0,t) = 0, \ u_x(1,t) = 0, \quad 0 \le t \le T, \qquad (2)$$

and nonlocal boundary conditions

$$\begin{array}{l} u(x,0) + \delta u(x,T) = \varphi_0(x), \\ u_t(x,0) + \delta u_t(x,T) = \varphi_1(x), \\ u_{tt}(x,0) + \delta u_{tt}(x,T) = \varphi_2(x), \\ u_{ttt}(x,0) + \delta u_{ttt}(x,T) = \varphi_3(x), \quad 0 \le x \le 1 \end{array} \qquad (3)$$

where $0 < \varepsilon \le 1$, δ are the given numbers, $f(x,t)$, $\varphi_i(x)$ $\left(i = \overline{0,3}\right)$ are the given functions, and $u(x,t)$ is a desired function, and under the classical solution of problem (1) - (3) we understand the function $u(x,t)$ continuous in the domain D_T together with all its derivatives included in equation (1), and satisfying all conditions (1) - (3) in the ordinary sense.

Since the system $\left\{\sin\lambda_k x, \ \ \lambda_k = \frac{\pi}{2}(2k-1)\right\}_{k=1}^{\infty}$ is complete in $L_2(0,1)$, each classical solution of problem(1) - (3) is of the form:

$$u(x,t) = \sum_{k=1}^{\infty} u_k(t)\sin\lambda_k x, \quad \lambda_k = \frac{\pi}{2}(2k-1), \qquad (4)$$

where

$$u_k(t) = 2\int_0^1 u(x,t)\sin\lambda_k x dx.$$

Then, applying the formal scheme of Fourier method, from (1), (3) we have:

$$\varepsilon^2 u_k^{(4)}(t) + \left(1+\lambda_k^2\right) u_k''(t) + \lambda_k^2 u_k(t) = f_k(t), \quad 0 \le t \le T,$$

$$\begin{array}{c} u_k(0) + \delta u_k(T) = \varphi_{0k}, \quad u_k'(0) + \delta u_k'(T) = \varphi_{1k}, \\ u_k''(0) + \delta u_k''(T) = \varphi_{2k}, \quad u_k'''(0) + \delta u_k'''(T) = \varphi_{3k}, \end{array} \qquad (5)$$

where

$$f_k(t)=2\int_0^1 f(x,t)\sin\lambda_k x dx,\quad \varphi_{ik}=2\int_0^1 \varphi_i(x,t)\sin\lambda_k x dx \qquad \left(i=\overline{0,3}\right).$$

It is obvious, that

$$\left(\lambda_k^2+1\right)^2-4\varepsilon^2\lambda_k^2>0,\qquad \left(\lambda_k^2+1\right)-\sqrt{\left(\lambda_k^2+1\right)^2-2\varepsilon^2\lambda}>0.$$

Then the roots of the characteristic equation

$$\varepsilon^2\mu_k^4+\left(1+\lambda_k^2\right)\mu_k^2+\lambda_k^2=0$$

corresponding to (5) are defined by the formula:

$$\mu_{jk}=(-1)^j\alpha_k i \qquad (j=1,2),$$

$$\mu_{jk}=(-1)^j\beta_k i \qquad (j=3,4),$$

where

$$a_k=\frac{1}{\varepsilon}\sqrt{\frac{\lambda_k^2+1-\sqrt{\left(\lambda_k^2+1\right)^2-2\varepsilon^2\lambda_k}}{2}}, \tag{6}$$

$$\beta_k=\frac{1}{\varepsilon}\sqrt{\frac{\lambda_k^2+1+\sqrt{\left(\lambda_k^2+1\right)^2-2\varepsilon^2\lambda_k}}{2}}, \tag{7}$$

The general solution of equation (5), according to Lagrange method, we search in the form of

$$u_k(t)=\overline{c_{1k}}(t)\cos\alpha_k t+\overline{c_{2k}}(t)\sin\alpha_k t+\overline{c_{3k}}(t)\cos\beta_k t+\overline{c_{4k}}(t)\sin\beta_k t, \tag{8}$$

where $\overline{c_{ik}}(t)$ are until unknown functions.

For definition of the function $\overline{c_{ik}}(t)$ $\left(i=\overline{1,4}\right)$ we have the system

$$\begin{cases} \overline{c'_{1k}}(t)\cos\alpha_k t+\overline{c'_{2k}}(t)\sin\alpha_k t+\overline{c'_{3k}}(t)\cos\beta_k t+\overline{c'_{4k}}(t)\sin\beta_k t=0 \\ -a_k\overline{c'_{1k}}(t)\sin\alpha_k t+a_k\overline{c'_{2k}}(t)\cos\alpha_k t-\beta_k\overline{c'_{3k}}(t)\sin\beta_k t+ \\ \qquad +\beta_k\overline{c'_{4k}}(t)\cos\beta_k t=0 \\ -a_k^2\overline{c'_{1k}}(t)\cos\alpha_k t-a_k^2\overline{c'_{2k}}(t)\sin\alpha_k t-\beta_k^2\overline{c'_{3k}}(t)\cos\beta_k t+ \\ \qquad +\beta_k^2\overline{c'_{4k}}(t)\sin\beta_k t=0 \\ a_k^3\overline{c'_{1k}}(t)\sin\alpha_k t-a_k^3\overline{c'_{2k}}(t)\cos\alpha_k t+\beta_k^3\overline{c'_{3k}}(t)\sin\beta_k t- \\ \qquad -\beta_k^3\overline{c'_{4k}}(t)\cos\beta_k t=\frac{1}{\varepsilon^2}f(t) \end{cases}$$

hence we find:

$$\overline{c'_{1k}}(t)=\frac{1}{\varepsilon^2 a_k\left(\beta_k^2-\alpha_k^2\right)}f_k(t)\sin\alpha_k t,\quad \overline{c'_{2k}}(t)=\frac{1}{\varepsilon^2 a_k\left(\beta_k^2-\alpha_k^2\right)}f_k(t)\cos\alpha_k t,$$

$$\overline{c'_{3k}}(t)=\frac{1}{\varepsilon^2\beta_k\left(\beta_k^2-\alpha_k^2\right)}f_k(t)\sin\beta_k t,\quad \overline{c'_{4k}}(t)=\frac{1}{\varepsilon^2\beta_k\left(\beta_k^2-\alpha_k^2\right)}f_k(t)\cos\beta_k t.$$

Integrating from 0 to τ we get:

$$\begin{aligned}
\overline{c_{1k}}(t)&=-\frac{1}{\varepsilon^2a_k\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^t f_k(\tau)\sin\alpha_k\tau d\tau+c_{1k},\\
\overline{c_{2k}}(t)&=\frac{1}{\varepsilon^2a_k\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^t f_k(\tau)\cos\alpha_k\tau d\tau+c_{2k},\\
\overline{c_{3k}}(t)&=\frac{1}{\varepsilon^2\beta_k\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^t f_k(\tau)\sin\beta_k\tau d\tau+c_{3k},\\
\overline{c_{4k}}(t)&=\frac{1}{\varepsilon^2\beta_k\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^t f_k(\tau)\cos\beta_k\tau d\tau+c_{4k}.
\end{aligned}\tag{9}$$

Substituting (9) in (8), we find the general solution $u_k(t)$ of equation (5):

$$u_k(t)=c_{1k}\cos\alpha_k t+c_{2k}\sin\alpha_k t+c_{3k}\cos\beta_k t+c_{4k}\sin\beta_k t+$$

$$+\frac{1}{\varepsilon^2a_k\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^t f_k(\tau)\left[\frac{1}{\alpha_k}\sin\alpha_k(t-\tau)-\frac{1}{\beta_k}\sin\beta_k(t-\tau)\right]d\tau,\tag{10}$$

where $c_{ik}\ \left(i=\overline{1,4}\right)$ are arbitrary constants, and α_k,β_k are determined by relations (6), (7), respectively.

Hence we find:

$$u'_k(t)=-a_kc_{1k}\sin\alpha_k t+\alpha_kc_{2k}\cos\alpha_k t-\beta_kc_{3k}\sin\beta_k t+\beta_kc_{4k}\sin\beta_k t+$$

$$+\frac{1}{\varepsilon^2a_k\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^t f_k(\tau)\left[\cos\alpha_k(t-\tau)-\cos\beta_k(t-\tau)\right]d\tau,$$

$$u''_k(t)=-a_k^2c_{1k}\cos\alpha_k t-\alpha_k^2c_{2k}\sin\alpha_k t-\beta_k^2c_{3k}\cos\beta_k t-\beta_k^2c_{4k}\sin\beta_k t+$$

$$+\frac{1}{\varepsilon^2a_k\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^t f_k(\tau)\left[-\alpha_k\sin\alpha_k(t-\tau)+\beta_k\sin\beta_k(t-\tau)\right]d\tau,$$

$$u'''_k(t)=a_k^3c_{1k}\sin\alpha_k t-\alpha_k^3c_{2k}\cos\alpha_k t+\beta_k^3c_{3k}\sin\beta_k t-\beta_k^3c_{4k}\sin\beta_k t+$$

$$+\frac{1}{\varepsilon^2a_k\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^t f_k(\tau)\left[-\alpha_k^2\cos\alpha_k(t-\tau)+\beta_k^2\cos\beta_k(t-\tau)\right]d\tau.$$

Further, using nonlocal conditions (6), we obtain:

$$\begin{cases} c_{1k}\left(1+\delta\cos\alpha_k T\right)+c_{2k}\delta\sin\alpha_k T+c_{3k}\left(1+\delta\cos\beta_k T\right)+ \\ \qquad +c_{4k}\delta\sin\beta_k T=F_{1k}\left(T\right) \\ -a_k c_{1k}\delta\sin\alpha_k T+c_{2k}a_k\left(1+\delta\cos\beta_k T\right)-c_{3k}\beta_k\sin\delta\beta_k T+ \\ \qquad +c_{4k}\beta_k\left(1+\delta\cos\beta_k T\right)=F_{2k}\left(T\right) \\ -a_k^2 c_{ik}\left(1+\delta\cos\alpha_k T\right)-c_{2k}a_k^2\delta\sin\alpha_k T-c_{3k}\left(1+\delta\cos\beta_k T\right)+ \\ \qquad +c_{4k}\beta_k^2\delta\sin\beta_k T=F_{3k}\left(T\right) \\ a_k^3 c_{1k}\delta\sin\alpha_k t-c_{2k}a_k^3\left(1+\delta\cos\beta_k t\right)+c_{3k}\beta_k^3\delta\sin\beta_k t- \\ \qquad -c_{4k}\beta_k^3\left(1+\delta\cos\beta_k t\right)=F_{4k}\left(T\right), \end{cases} \tag{11}$$

where

$$F_{1k}\left(T\right)=\varphi_{0k}-\frac{\delta}{\varepsilon^2\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^T f_k\left(\tau\right)\left[\frac{1}{\alpha_k}\sin\alpha_k\left(T-\tau\right)-\tfrac{1}{\beta_k}\sin\beta_k\left(T-\tau\right)\right]d\tau,$$

$$F_{2k}\left(T\right)=\varphi_{1k}-\frac{\delta}{\varepsilon^2\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^T f_k\left(\tau\right)\left[\cos\alpha_k\left(T-\tau\right)-\cos\beta_k\left(T-\tau\right)\right]d\tau,$$

$$F_{3k}\left(T\right)=\varphi_{2k}-\frac{\delta}{\varepsilon^2\left(\beta_k^2-\alpha_k^2\right)}\int\limits_0^T f_k\left(\tau\right)\left[-\alpha_k\sin\alpha_k\left(T-\tau\right)+\beta_k\sin\beta_k\left(T-\tau\right)\right]d\tau,$$

$$F_{4k}\left(T\right)=\varphi_{3k}-\frac{\delta}{\varepsilon^2\left(\beta_k^2-\alpha_k^2\right)}\times$$

$$\times\int\limits_0^T f_k\left(\tau\right)\left[-\alpha_k^2\cos\alpha_k\left(T-\tau\right)+\beta_k^2\cos\beta_k\left(T-\tau\right)\right]d\tau. \tag{12}$$

Solving system (11), we get:

$$c_{1k}=\frac{1}{\alpha_k\left(\beta_k^2-\alpha_k^2\right)\rho_{1k}\left(T\right)}\left[\alpha_k\beta_k^2F_{1k}\left(T\right)\left(1+\delta\cos\alpha_k T\right)-\right.$$

$$\left.-\beta_k^2F_{2k}\left(T\right)\delta\sin\alpha_k T+\alpha_k F_{3k}\left(T\right)\left(1+\delta\cos\alpha_k T\right)-F_{4k}\left(T\right)\delta\sin\alpha_k T\right],$$

$$c_{2k}=\frac{1}{\alpha_k\left(\beta_k^2-\alpha_k^2\right)\rho_{2k}\left(T\right)}\left[\alpha_k\beta_k^2F_{1k}\left(T\right)\delta\sin\alpha_k T+\right.$$

$$\left.+\beta_k^2F_{2k}\left(T\right)\left(1+\delta\cos\alpha_k T\right)+\alpha_k F_{3k}\left(T\right)\delta\sin\alpha_k T+F_{4k}\left(T\right)\left(1+\delta\cos\alpha_k T\right)\right],$$

$$c_{3k}=\frac{1}{\beta_k\left(\beta_k^2-\alpha_k^2\right)\rho_{2k}\left(T\right)}\left[-\alpha_k^2\beta_k F_{1k}\left(T\right)\left(1+\delta\cos\beta_k T\right)+\right.$$

$$\left.+\alpha_k^2F_{2k}\left(T\right)\delta\sin\beta_k T-\beta_k F_{3k}\left(T\right)\left(1+\delta\cos\beta_k T\right)+F_{4k}\left(T\right)\delta\sin\beta_k T\right],$$

$$c_{4k}=\frac{1}{\beta_k\left(\beta_k^2-\alpha_k^2\right)\rho_{2k}\left(T\right)}\left[\alpha_k^2\beta_k F_{1k}\left(T\right)\delta\sin\beta_k T+\alpha_k^2F_{2k}\left(T\right)\left(1+\delta\cos\beta_k T\right)+\right.$$

$$+\beta_k F_{3k}(T)\,\delta\sin\beta_k T + F_{4k}(T)\,(1+\delta\cos\beta_k T)\Big], \tag{13}$$

where

$$\rho_{1k}(T) = 1+2\delta\cos\alpha_k T+\delta^2, \qquad \rho_{2k}(T) = 1+2\delta\cos\beta_k T+\delta^2. \tag{14}$$

Substituting c_{1k}, c_{2k}, c_{3k}, c_{4k} from (13) in (10), taking into account (12), we find:

$$u_k(t) = \frac{1}{\beta_k^2-\alpha_k^2}\left\{\left[\frac{\beta_k^2}{\rho_{1k}(T)}\left(\cos\alpha_k t+\delta\cos\alpha_k(T-\tau)\right)-\right.\right.$$
$$\left.-\frac{\alpha_k^2}{\rho_{2k}(T)}\left(\cos\beta_k t+\delta\cos\beta_k(T-t)\right)\right]\varphi_{0k}+$$
$$+\left[\frac{\beta_k^2}{\alpha_k\rho_{1k}(T)}\left(\sin\alpha_k t-\delta\sin\alpha_k(T-t)\right)-\right.$$
$$\left.-\frac{\alpha_k^2}{\beta_k\rho_{2k}(T)}\left(\sin\beta_k t-\delta\sin\beta_k(T-t)\right)\right]\varphi_{1k}+$$
$$+\left[\frac{1}{\rho_{1k}(T)}\left(\cos\alpha_k t+\delta\cos\alpha_k(T-t)\right)-\right.$$
$$\left.-\frac{1}{\rho_{2k}(T)}\left(\cos\beta_k t+\delta\cos\beta_k(T-t)\right)\right]\varphi_{2k}+$$
$$+\left[\frac{1}{\alpha_k\rho_{1k}(T)}\left(\sin\alpha_k t-\delta\sin\alpha_k(T-t)\right)-\right.$$
$$\left.-\frac{1}{\beta_k\rho_{2k}(T)}\left(\sin\beta_k t-\delta\sin\beta_k(T-t)\right)\right]\varphi_{3k}-$$
$$-\frac{\delta}{\varepsilon^2}\int_0^t f_k(\tau)\left[\frac{1}{\alpha_k\rho_{1k}(T)}\left(\sin\alpha_k(T+t-\tau)-\delta\sin\alpha_k(t-\tau)\right)-\right.$$
$$\left.-\frac{1}{\beta_k\rho_{2k}(T)}\left(\sin\beta_k(T+t-\tau)-\delta\sin\beta_k(t-\tau)\right)\right]d\tau-$$
$$\left.+\frac{1}{\varepsilon^2}\int_0^t f_k(\tau)\frac{1}{\alpha_k}\left[\left(\sin\alpha_k(t-\tau)-\frac{1}{\beta_k}\sin\beta_k(t-\tau)\right)\right]d\tau\right\}. \tag{15}$$

Differentiating (15) four times, we obtain:

$$u_k'(t) = \frac{1}{\beta_k^2-\alpha_k^2}\left\{\left[\frac{\alpha_k\beta_k^2}{\rho_{1k}(T)}\left(-\sin\alpha_k t+\delta\sin\alpha_k(T-\tau)\right)-\right.\right.$$
$$\left.-\frac{\alpha_k^2\beta_k}{\rho_{2k}(T)}\left(-\sin\beta_k t+\delta\sin\beta_k(T-t)\right)\right]\varphi_{0k}+$$
$$+\left[\frac{\beta_k^2}{\rho_{1k}(T)}\left(\cos\alpha_k t+\delta\cos\alpha_k(T-t)\right)-\right.$$
$$\left.-\frac{\alpha_k^2}{\rho_{2k}(T)}\left(\cos\beta_k t+\delta\cos\beta_k(T-t)\right)\right]\varphi_{1k}+$$

$$+\left[\frac{\alpha_k}{\rho_{1k}(T)}\left(-\sin\alpha_k t+\delta\sin\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{\beta_k}{\rho_{2k}(T)}\left(-\sin\beta_k t+\delta\sin\beta_k(T-t)\right)\right]\varphi_{2k}+$$

$$+\left[\frac{1}{\rho_{1k}(T)}\left(\cos\alpha_k t+\delta\cos\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{1}{\rho_{2k}(T)}\left(\cos\beta_k t+\delta\cos\beta_k(T-t)\right)\right]\varphi_{3k}-$$

$$-\frac{\delta}{\varepsilon^2}\int_0^t f_k(\tau)\left[\frac{1}{\rho_{1k}(T)}\left(\cos\alpha_k(T+t-\tau)-\delta\sin\alpha_k(t-\tau)\right)-\right.$$

$$\left.-\frac{1}{\rho_{2k}(T)}\left(\cos\beta_k(T+t-\tau)-\delta\sin\beta_k(t-\tau)\right)\right]d\tau-$$

$$\left.+\frac{1}{\varepsilon^2}\int_0^t f_k(\tau)\left[\left(\cos\alpha_k(t-\tau)-\frac{1}{\beta_k}\cos\beta_k(t-\tau)\right)\right]d\tau\right\}; \tag{16}$$

$$u_k''(t)=\frac{1}{\beta_k^2-\alpha_k^2}\left\{\left[\frac{\alpha_k^2\beta_k^2}{\rho_{1k}(T)}\left(-\cos\alpha_k t-\delta\cos\alpha_k(T-\tau)\right)-\right.\right.$$

$$\left.-\frac{\alpha_k^2\beta_k^2}{\rho_{2k}(T)}\left(-\cos\beta_k t-\delta\cos\beta_k(T-t)\right)\right]\varphi_{0k}+$$

$$+\left[\frac{\alpha_k^2\beta_k^2}{\alpha_k\rho_{1k}(T)}\left(-\sin\alpha_k t+\delta\sin\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{\beta_k^2\alpha_k^2}{\beta_k\rho_{2k}(T)}\left(-\sin\beta_k t+\delta\sin\beta_k(T-t)\right)\right]\varphi_{1k}+$$

$$+\left[\frac{\alpha_k^2}{\rho_{1k}(T)}\left(-\cos\alpha_k t-\delta\cos\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{\beta_k^2}{\rho_{2k}(T)}\left(-\cos\beta_k t-\delta\cos\beta_k(T-t)\right)\right]\varphi_{2k}+$$

$$+\left[\frac{\alpha_k}{\rho_{1k}(T)}\left(-\sin\alpha_k t+\delta\sin\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{\beta_k}{\rho_{2k}(T)}\left(-\sin\beta_k t+\delta\sin\beta_k(T-t)\right)\right]\varphi_{3k}-$$

$$-\frac{\delta}{\varepsilon^2}\int_0^t f_k(\tau)\left[\frac{\alpha_k}{\rho_{1k}(T)}\left(-\sin\alpha_k(T+t-\tau)+\delta\sin\alpha_k(t-\tau)\right)-\right.$$

$$\left.-\frac{\beta_k}{\rho_{2k}(T)}\left(-\sin\beta_k(T+t-\tau)+\delta\sin\beta_k(t-\tau)\right)\right]d\tau-$$

$$+\frac{1}{\varepsilon^2}\int_0^t f_k(\tau)\left[\left(-\alpha_k\sin\alpha_k(t-\tau)+\beta_k\sin\beta_k(t-\tau)\right)\right]d\tau\Bigg\};\qquad(17)$$

$$u_k'''(t)=\frac{1}{\beta_k^2-\alpha_k^2}\Bigg\{\left[\frac{\alpha_k^3\beta_k^2}{\rho_{1k}(T)}\left(\sin\alpha_k t-\delta\sin\alpha_k(T-\tau)\right)-\right.$$

$$\left.-\frac{\alpha_k^2\beta_k^3}{\rho_{2k}(T)}\left(\sin\beta_k t-\delta\sin\beta_k(T-t)\right)\right]\varphi_{0k}+$$

$$+\left[\frac{\alpha_k^2\beta_k^2}{\rho_{1k}(T)}\left(-\cos\alpha_k t-\delta\cos\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{\beta_k^2\alpha_k^2}{\rho_{2k}(T)}\left(-\cos\beta_k t-\delta\cos\beta_k(T-t)\right)\right]\varphi_{1k}+$$

$$+\left[\frac{\alpha_k^3}{\rho_{1k}(T)}\left(\sin\alpha_k t-\delta\sin\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{\beta_k^3}{\rho_{2k}(T)}\left(\sin\beta_k t-\delta\sin\beta_k(T-t)\right)\right]\varphi_{2k}+$$

$$+\left[\frac{\alpha_k^2}{\rho_{1k}(T)}\left(-\cos\alpha_k t-\delta\cos\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{\beta_k^2}{\rho_{2k}(T)}\left(-\cos\beta_k t-\delta\cos\beta_k(T-t)\right)\right]\varphi_{3k}-$$

$$-\frac{\delta}{\varepsilon^2}\int_0^t f_k(\tau)\left[\frac{\alpha_k^2}{\rho_{1k}(T)}\left(-\cos\alpha_k(T+t-\tau)+\delta\sin\alpha_k(t-\tau)\right)-\right.$$

$$\left.-\frac{\beta_k^2}{\rho_{2k}(T)}\left(-\cos\beta_k(T+t-\tau)+\delta\sin\beta_k(t-\tau)\right)\right]d\tau-$$

$$+\frac{1}{\varepsilon^2}\int_0^t f_k(\tau)\left[\left(-\alpha_k^2\cos\alpha_k(t-\tau)+\beta_k^2\cos\beta_k(t-\tau)\right)\right]d\tau\Bigg\};\qquad(18)$$

$$u_k^{(4)}(t)=\frac{1}{\beta_k^2-\alpha_k^2}\Bigg\{\left[\frac{\alpha_k^4\beta_k^2}{\rho_{1k}(T)}\left(\cos\alpha_k t+\delta\cos\alpha_k(T-\tau)\right)-\right.$$

$$\left.-\frac{\alpha_k^2\beta_k^4}{\rho_{2k}(T)}\left(\cos\beta_k t+\delta\cos\beta_k(T-t)\right)\right]\varphi_{0k}+$$

$$+\left[\frac{\alpha_k^3\beta_k^2}{\rho_{1k}(T)}\left(\sin\alpha_k t-\delta\sin\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{\beta_k^3\alpha_k^2}{\beta_k\rho_{2k}(T)}\left(\sin\beta_k t-\delta\sin\beta_k(T-t)\right)\right]\varphi_{1k}+$$

$$+\left[\frac{\alpha_k^4}{\rho_{1k}(T)}\left(\cos\alpha_k t+\delta\cos\alpha_k(T-t)\right)-\right.$$

$$-\frac{\beta_k^4}{\rho_{2k}(T)}\left(\cos\beta_k t+\delta\cos\beta_k(T-t)\right)\Big]\varphi_{2k}+$$

$$+\left[\frac{\alpha_k^3}{\rho_{1k}(T)}\left(\sin\alpha_k t-\delta\sin\alpha_k(T-t)\right)-\right.$$

$$-\frac{\beta_k^3}{\rho_{2k}(T)}\left(\sin\beta_k t\_\delta\sin\beta_k(T-t)\right)\Big]\varphi_{3k}-$$

$$-\frac{\delta}{\varepsilon^2}\int_0^t f_k(\tau)\left[\frac{\alpha_k^3}{\rho_{1k}(T)}\left(-\sin\alpha_k(T+t-\tau)+\delta\sin\alpha_k(t-\tau)\right)-\right.$$

$$\left.-\frac{\beta_k^3}{\rho_{2k}(T)}\left(-\sin\beta_k(T+t-\tau)+\delta\sin\beta_k(t-\tau)\right)\right]d\tau-$$

$$\left.+\frac{1}{\varepsilon^2}\int_0^t f_k(\tau)\left[-\alpha_k^3\sin\alpha_k^3(t-\tau)+\beta_k^3\sin\beta_k(t-\tau)\right]d\tau\right\}+\frac{1}{\varepsilon^2}f_k(t); \tag{19}$$

It is easy to see, that

$$\lambda_k^2-1<\sqrt{\left(\lambda_k^2+1\right)^2-4\varepsilon^2\lambda_k^2}<\lambda_k^2+1,$$

$$\alpha_k=\frac{\lambda_k}{\varepsilon\sqrt{2\beta_k}},\quad \frac{1}{\varepsilon}\lambda_k\le\beta_k\le\frac{\sqrt{2}}{\varepsilon}\lambda_k,\quad \frac{1}{2}\le\alpha_k\le\frac{1}{\sqrt{2}},$$

$$\beta_k^2-\alpha_k^2=\frac{\sqrt{\left(\lambda_k^2+1\right)^2-4\varepsilon^2\lambda_k^2}}{\varepsilon^2}>\frac{\lambda_k^2-1}{\varepsilon^2}>\frac{1}{2\varepsilon^2}\lambda_k^2,$$

$$|\rho_{1k}(T)|^{-1}\le\left|1+\delta^2-2|\delta|\right|^{-1}=(1-|\delta|)^{-2}\equiv\rho(T),$$

$$|\rho_{2k}(T)|^{-1}=\left|1+\delta^2-2|\delta|\right|^{-1}=(1-|\delta|)^{-2}\equiv\rho(T). \tag{20}$$

Now, from (15) - (19), taking into account (20), we have, respectively:

$$|u_k(t)|\le\frac{2}{\pi}\left(2\pi+\varepsilon^2\right)\rho(T)(1+|\delta|)|\varphi_{0k}|+\frac{2}{\pi^3}\left(\pi^3+4\varepsilon^2\right)\rho(T)(1+|\delta|)|\varphi_{1k}|+$$

$$+4\varepsilon^2\rho(T)(1+|\delta|)\frac{1}{\lambda_k^2}|\varphi_{2k}|+\left(2+\frac{\sqrt{2}\varepsilon}{\pi}\right)\rho(T)(1+|\delta|)\frac{2\varepsilon^2}{\lambda_k^2}|\varphi_{3k}|+$$

$$+\left(1+\frac{\varepsilon}{\pi}\right)\rho(T)(1+|\delta|)\sqrt{T}\frac{1}{\lambda_k^2}\left(\int_0^T|f_k(\tau)|^2d\tau\right)^{1/2},$$

$$\left|u_k'(t)\right|\le 2\rho(T)(1+|\delta|)|\varphi_{0k}|+\frac{2\sqrt{2}}{\pi}(\pi+\varepsilon)\rho(T)(1+|\delta|)|\varphi_{1k}|+$$

$$+\frac{2\sqrt{2}}{\pi}(\pi+\varepsilon)\rho(T)(1+|\delta|)\frac{1}{\lambda_k}|\varphi_{2k}|+4\varepsilon^2\rho(T)(1+|\delta|)\frac{1}{\lambda_k^2}|\varphi_{3k}|+$$

$$+4\rho(T)(1+|\delta|)\frac{1}{\lambda_k^2}\sqrt{T}\left(\int_0^T|f_k(\tau)|^2d\tau\right)^{1/2},$$

$$\left|u_k''(t)\right|\leq 4\rho(T)(1+|\delta|)|\varphi_{0k}|+2\sqrt{2}\left(1+\frac{\varepsilon}{\pi}\right)\rho(T)(1+|\delta|)|\varphi_{1k}|+$$

$$+4\left(1+\frac{\varepsilon^2}{\pi^2}\right)\rho(T)(1+|\delta|)|\varphi_{2k}|+2\sqrt{2}\left(\frac{\varepsilon^2}{\pi}+\varepsilon\right)\rho(T)(1+|\delta|)\frac{1}{\lambda_k}|\varphi_{3k}|+$$

$$+2\sqrt{2}\left(\frac{1}{\pi}+\frac{1}{\varepsilon}\right)(1+|\delta|\rho(T)(1+|\delta|))\sqrt{T}\frac{1}{\lambda_k}\left(\int_0^T|f_k(\tau)|^2d\tau\right)^{1/2},$$

$$\left|u_k'''(t)\right|\leq 2\sqrt{2}\left(\frac{1}{\pi^2}+\frac{1}{\varepsilon}\right)\rho(T)(1+|\delta|)\lambda_k|\varphi_{0k}|+$$

$$+2\sqrt{2}\varepsilon\left(\frac{1}{\pi}+\frac{1}{\varepsilon}\right)\rho(T)(1+|\delta|)|\varphi_{1k}|+$$

$$+2\sqrt{2}\left(\frac{\varepsilon^2}{\pi}+\frac{2}{\varepsilon}\right)\rho(T)(1+|\delta|)\lambda_k|\varphi_{2k}|+2\left(\frac{\varepsilon^2}{\pi}+2\right)\rho(T)(1+|\delta|)|\varphi_{3k}|+$$

$$+4\left(\frac{1}{\pi^2}+\frac{1}{\varepsilon^2}\right)(1+|\delta|\rho(T)(1+|\delta|))\sqrt{T}\left(\int_0^T|f_k(\tau)|^2d\tau\right)^{1/2},$$

$$\left|u_k^{(4)}(t)\right|\leq 4\left(\frac{1}{\pi^2}+\frac{1}{\varepsilon^2}\right)\rho(T)(1+|\delta|)\lambda_k^2|\varphi_{0k}|+$$

$$+2\sqrt{2}\varepsilon\left(\frac{1}{\pi}+\frac{1}{\varepsilon}\right)\rho(T)(1+|\delta|)\lambda_k|\varphi_{1k}|+$$

$$+8\left(\frac{\varepsilon^2}{\pi^4}+\frac{1}{\varepsilon^2}\right)\rho(T)(1+|\delta|)\lambda_k^2|\varphi_{2k}|+$$

$$+4\sqrt{2}\left(\frac{1}{\pi^3}+\frac{1}{\varepsilon^3}\right)\varepsilon^3\rho(T)(1+|\delta|)\lambda_k|\varphi_{3k}|+$$

$$+4\sqrt{2}\left(\frac{1}{\pi^3}+\frac{1}{\varepsilon^3}\right)(1+|\delta|\rho(T)(1+|\delta|))\times$$

$$\times\lambda_k\sqrt{T}\left(\int_0^T|f_k(\tau)|^2d\tau\right)^{1/2}+\frac{1}{\varepsilon^2}|f_k(t)|, \tag{21}$$

Hence, we have:

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\|u_k(t)\|_{C[0,T]}\right)^2\right)^{1/2}\leq$$

$$\leq\frac{2}{\pi}\left(2\pi+\varepsilon^2\right)\rho(T)(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|\varphi_{0k}|\right)^2\right)^{1/2}+$$

$$+\frac{2}{\pi^3}\left(\pi^3+4\varepsilon^2\right)\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left|\varphi_{1k}\right|\right)^2\right)^{1/2}+$$

$$+4\varepsilon^2\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k\left|\varphi_{2k}\right|\right)^2\right)^{1/2}+$$

$$+2\varepsilon^2\left(2+\frac{\sqrt{2}\varepsilon}{\pi}\right)\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k\left|\varphi_{3k}\right|\right)^2\right)^{1/2},$$

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k'\left(t\right)\right\|_{C[0,T]}\right)^2\right)^{1/2}\leq 2\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left|\varphi_{0k}\right|\right)^2\right)^{1/2}+$$

$$+\frac{2\sqrt{2}}{\pi}\left(\pi+\varepsilon\right)\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left|\varphi_{1k}\right|\right)^2\right)^{1/2}+$$

$$+\frac{2\sqrt{2}}{\pi}\left(\pi+\varepsilon\right)\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k\left|\varphi_{2k}\right|\right)^2\right)^{1/2}+$$

$$+4\varepsilon^2\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k\left|\varphi_{3k}\right|\right)^2\right)^{1/2}+$$

$$+4\left(1+\left|\delta\right|\rho\left(T\right)\left(1+\left|\delta\right|\right)\right)\sqrt{T}\left(\int_0^T\sum_{k=1}^{\infty}\left(\lambda_k\left|f_k\left(\tau\right)\right|^2d\tau\right)^2\right)^{1/2}; \qquad (22)$$

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k''\left(t\right)\right\|_{C[0,T]}\right)^2\right)^{1/2}\leq 4\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left|\varphi_{0k}\right|\right)^2\right)^{1/2}+$$

$$+2\sqrt{2}\left(1+\frac{\varepsilon}{\pi}\right)\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left|\varphi_{1k}\right|\right)^2\right)^{1/2}+$$

$$+4\left(1+\frac{\varepsilon^2}{\pi^2}\right)\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left|\varphi_{2k}\right|\right)^2\right)^{1/2}+$$

$$+2\sqrt{2}\left(\frac{\varepsilon^2}{\pi}+\varepsilon\right)\rho\left(T\right)\left(1+\left|\delta\right|\right)\left(\sum_{k=1}^{\infty}\left(\lambda_k^2\left|\varphi_{3k}\right|\right)^2\right)^{1/2}+$$

$$+2\sqrt{2}\left(\frac{1}{\pi}+\frac{1}{\varepsilon}\right)\left(1+\left|\delta\right|\rho\left(T\right)\left(1+\left|\delta\right|\right)\right)\sqrt{T}\left(\int_0^T\sum_{k=1}^{\infty}\left(\lambda_k^2\left|f_k\left(\tau\right)\right|^2\right)d\tau\right)^{1/2}; \qquad (23)$$

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k'''\left(t\right)\right\|_{C[0,T]}\right)^2\right)^{1/2}\leq$$

$$\leq 2\sqrt{2}\left(\frac{1}{\pi^2}+\frac{1}{\varepsilon}\right)\rho(T)(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^2|\varphi_{0k}|\right)^2\right)^{1/2}+$$

$$+2\sqrt{2}\varepsilon\left(\frac{1}{\pi}+\frac{1}{\varepsilon}\right)\rho(T)(1+|\delta|)\left(\sum_{k=1}^{\infty}(\lambda_k|\varphi_{1k}|)^2\right)^{1/2}+$$

$$+2\sqrt{2}\left(\frac{\varepsilon^2}{\pi}+\frac{2}{\varepsilon}\right)\rho(T)(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^2|\varphi_{2k}|\right)^2\right)^{1/2}+$$

$$+2\left(\frac{\varepsilon^2}{\pi}+2\right)\rho(T)(1+|\delta|)\left(\sum_{k=1}^{\infty}(\lambda_k|\varphi_{3k}|)^2\right)^{1/2}+$$

$$+4\left(\frac{1}{\pi^2}+\frac{1}{\varepsilon^2}\right)(1+|\delta|\rho(T)(1+|\delta|))\sqrt{T}\left(\int_0^T\sum_{k=1}^{\infty}\left(\lambda_k|f_k(\tau)|^2\right)d\tau\right)^{1/2}; \qquad (24)$$

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k^{(4)}(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}\leq$$

$$\leq 4\left(\frac{1}{\pi^2}+\frac{1}{\varepsilon^2}\right)\rho(T)(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|\varphi_{0k}|\right)^2\right)^{1/2}+$$

$$+2\sqrt{2}\left(\frac{1}{\pi}+\frac{1}{\varepsilon}\right)\rho(T)(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^2|\varphi_{1k}|\right)^2\right)^{1/2}+$$

$$+8\left(\frac{\varepsilon^2}{\pi^4}+\frac{1}{\varepsilon^2}\right)\rho(T)(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|\varphi_{2k}|\right)^2\right)^{1/2}+$$

$$+4\sqrt{2}\left(\frac{1}{\pi^3}+\frac{1}{\varepsilon^3}\right)\rho(T)(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^2|\varphi_{3k}|\right)^2\right)^{1/2}+$$

$$+4\sqrt{2}\left(\frac{1}{\pi^3}+\frac{1}{\varepsilon^3}\right)(1+|\delta|\rho(T)(1+|\delta|))\sqrt{T}\left(\int_0^T\sum_{k=1}^{\infty}\left(\lambda_k^2|f_k(\tau)|^2\right)d\tau\right)^{1/2}+$$

$$+\frac{1}{\varepsilon^2}\left(\sum_{k=1}^{\infty}\left(\lambda_k|f_k(\tau)|^2\right)d\tau\right)^{1/2}. \qquad (25)$$

So we can prove the following theorem.

Theorem. *Let*

1. $\varphi_i(x)\in C^2[0,1]$, $\varphi_i'''(x)\in L_2(0,1)$ *and* $\varphi_i(0)=\varphi_i'(1)=\varphi_i''(1)=0$ $(i=0,1,2)$.

2. $\varphi_3(x)\in C^2[0,1]$, $\varphi_3''(x)\in L_2(0,1)$ *and* $\varphi_3(0)=\varphi_3'(1)$.

3. $f(x,t)\in C_{x,t}^{1,0}(D_T)$, $f_{xx}(x,t)\in L_2(D_T)$ *and* $f(0,t)=f_x(1,t)=0$ $(0\leq t\leq T)$.

4. $|\delta|\neq 1$, $0<\varepsilon<1$. .

Then the function

$$u(x,t)=\sum_{k=1}^{\infty}\frac{1}{\beta_k^2-\alpha_k^2}\left\{\left[\frac{\beta_k^2}{\rho_{1k}(T)}\left(\cos\alpha_k t+\delta\cos\alpha_k(T-\tau)\right)-\right.\right.$$

$$\left.-\frac{\alpha_k^2}{\rho_{2k}(T)}\left(\cos\beta_k t+\delta\cos\beta_k(T-t)\right)\right]\varphi_{0k}+$$

$$+\left[\frac{\beta_k^2}{\alpha_k\rho_{1k}(T)}\left(\sin\alpha_k t-\delta\sin\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{\alpha_k^2}{\beta_k\rho_{2k}(T)}\left(\sin\beta_k t-\delta\sin\beta_k(T-t)\right)\right]\varphi_{1k}+$$

$$+\left[\frac{1}{\rho_{1k}(T)}\left(\cos\alpha_k t+\delta\cos\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{1}{\rho_{2k}(T)}\left(\cos\beta_k t+\delta\cos\beta_k(T-t)\right)\right]\varphi_{2k}+$$

$$+\left[\frac{1}{\alpha_k\rho_{1k}(T)}\left(\sin\alpha_k t-\delta\sin\alpha_k(T-t)\right)-\right.$$

$$\left.-\frac{1}{\beta_k\rho_{2k}(T)}\left(\sin\beta_k t-\delta\sin\beta_k(T-t)\right)\right]\varphi_{3k}-$$

$$-\frac{\delta}{\varepsilon^2}\int_0^T f_k(\tau)\left[\frac{1}{\alpha_k\rho_{1k}(T)}\left(\sin\alpha_k(T+t-\tau)-\delta\sin\alpha_k(t-\tau)\right)-\right.$$

$$\left.-\frac{1}{\beta_k\rho_{2k}(T)}\left(\sin\beta_k(T+t-\tau)-\delta\sin\beta_k(t-\tau)\right)\right]d\tau-$$

$$\left.+\frac{1}{\varepsilon^2}\int_0^t f_k(\tau)\frac{1}{\alpha_k}\left[\left(\sin\alpha_k(t-\tau)-\frac{1}{\beta_k}\sin\beta_k(t-\tau)\right)\right]d\tau\right\}\sin\lambda_k x \qquad (26)$$

is a classical solution of problem (1) - (3).

Proof. Under theorem's conditions, from inequalities (21) - (25), we find, respectively:

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\|u_k(t)\|_{C[0,T]}\right)^2\right)^{1/2}\le\frac{2\sqrt{2}}{\pi}\left(2\pi+\varepsilon^2\right)\rho(T)(1+|\delta|)\left\|\varphi_0'''(x)\right\|_{L_2(0,1)}+$$

$$+\frac{2\sqrt{2}}{\pi^3}\left(\pi^3+4\varepsilon^2\right)\rho(T)(1+|\delta|)\left\|\varphi_1'''(x)\right\|_{L_2(0,1)}+$$

$$+4\sqrt{2}\varepsilon^2\rho(T)(1+|\delta|)\left\|\varphi_2'(x)\right\|_{L_2(0,1)}+$$

$$+2\sqrt{2}\varepsilon^2\left(2+\frac{\sqrt{2}}{\pi}\varepsilon\right)\rho(T)(1+|\delta|)\left\|\varphi_3'(x)\right\|_{L_2(0,1)}+$$

$$+4\sqrt{2}\left(1+\frac{\varepsilon}{\pi}\right)\rho(T)\left(1+|\delta|\rho(T)(1+\delta)\right)\sqrt{T}\left\|f_x(x)\right\|_{L_2(D_T)}, \qquad (27)$$

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u'_k(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}\leq 2\sqrt{2}\rho(T)(1+|\delta|)\left\|\varphi'''_0(x)\right\|_{L_2(0,1)}+$$

$$+\frac{4}{\pi}(\pi+\varepsilon)\rho(T)(1+|\delta|)\left\|\varphi'''_1(x)\right\|_{L_2(0,1)}+$$

$$+\frac{4}{\pi}(\pi+\varepsilon)\rho(T)(1+|\delta|)\left\|\varphi''_2(x)\right\|_{L_2(0,1)}+$$

$$+4\sqrt{2}\varepsilon^2\rho(T)(1+|\delta|)\left\|\varphi'_3(x)\right\|_{L_2(0,1)}+$$

$$+4\sqrt{2}\left(1+|\delta|\rho(T)(1+|\delta|)\right)\sqrt{T}\left\|f_x(x)\right\|_{L_2(D_T)}; \qquad (28)$$

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u''_k(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}\leq 4\sqrt{2}\rho(T)(1+|\delta|)\left\|\varphi'''_0(x)\right\|_{L_2(0,1)}+$$

$$+4\left(1+\frac{\varepsilon}{\pi}\right)\rho(T)(1+|\delta|)\left\|\varphi'''_1(x)\right\|_{L_2(0,1)}+$$

$$+4\sqrt{2}\left(1+\frac{\varepsilon^2}{\pi^2}\right)\rho(T)(1+|\delta|)\left\|\varphi'''_2(x)\right\|_{L_2(0,1)}+$$

$$+4\left(\frac{\varepsilon^2}{\pi}+\varepsilon\right)\rho(T)(1+|\delta|)\left\|\varphi''_3(x)\right\|_{L_2(0,1)}+$$

$$+4\left(\frac{1}{\pi}+\frac{1}{\varepsilon}\right)\left(1+|\delta|\rho(T)(1+|\delta|)\right)\sqrt{T}\left\|f_{xx}(x)\right\|_{L_2(D_T)}; \qquad (29)$$

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u'''_k(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}\leq 4\left(\frac{1}{\pi^2}+\frac{1}{\varepsilon}\right)\rho(T)(1+|\delta|)\left\|\varphi''_0(x)\right\|_{L_2(0,1)}+$$

$$+4\varepsilon\left(\frac{1}{\pi}+\frac{1}{\varepsilon}\right)\rho(T)(1+|\delta|)\left\|\varphi'_1(x)\right\|_{L_2(0,1)}+$$

$$+4\left(\frac{\varepsilon^2}{\pi}+\frac{2}{\varepsilon}\right)\rho(T)(1+|\delta|)\left\|\varphi''_2(x)\right\|_{L_2(0,1)}+$$

$$+2\sqrt{2}\left(\frac{\varepsilon^2}{\pi}+2\right)\rho(T)(1+|\delta|)\left\|\varphi'_3(x)\right\|_{L_2(0,1)}+$$

$$+4\sqrt{2}\left(\frac{1}{\pi^2}+\frac{1}{\varepsilon^2}\right)\left(1+|\delta|\rho(T)(1+|\delta|)\right)\sqrt{T}\left\|f_{xx}(x)\right\|_{L_2(D_T)}; \qquad (30)$$

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k^{(4)}(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}\leq$$

$$\leq 4\sqrt{2}\left(\frac{1}{\pi^2}+\frac{1}{\varepsilon^2}\right)\rho(T)(1+|\delta|)\left\|\varphi'''_0(x)\right\|_{L_2(0,1)}+$$

$$+4\left(\frac{1}{\pi}+\frac{1}{\varepsilon}\right)\rho(T)(1+|\delta|)\left\|\varphi''_1(x)\right\|_{L_2(0,1)}+$$

$$+8\sqrt{2}\left(\frac{\varepsilon^2}{\pi^4}+\frac{1}{\varepsilon^2}\right)\rho(T)(1+|\delta|)\left\|\varphi'''_2(x)\right\|_{L_2(0,1)}+$$

$$+8\left(\frac{1}{\pi^3}+\frac{1}{\varepsilon^3}\right)\rho(T)(1+|\delta|)\left\|\varphi_3''(x)\right\|_{L_2(0,1)}+$$

$$+8\left(\frac{1}{\pi^3}+\frac{1}{\varepsilon^3}\right)(1+|\delta|\rho(T)(1+|\delta|))\sqrt{T}\left\|f_{xx}(x)\right\|_{L_2(D_T)}+$$

$$+\frac{\sqrt{2}}{\varepsilon^2}\left\|\left\|f_x(x)\right\|_{L_2(0,1)}\right\|_{L_2(D_T)}. \tag{31}$$

It is obvious, that, (32)

$$|u(x,t)|\leq\left(\sum_{k=1}^{\infty}\lambda_k^{-6}\right)^{1/2}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}, \tag{32}$$

$$|u_t(x,t)|\leq\left(\sum_{k=1}^{\infty}\lambda_k^{-6}\right)^{1/2}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k'(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}, \tag{33}$$

$$|u_{tt}(x,t)|\leq\left(\sum_{k=1}^{\infty}\lambda_k^{-6}\right)^{1/2}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k''(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}, \tag{34}$$

$$|u_{ttxx}(x,t)|\leq\left(\sum_{k=1}^{\infty}\lambda_k^{-6}\right)^{1/2}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k''(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}, \tag{35}$$

$$|u_{ttt}(x,t)|\leq\left(\sum_{k=1}^{\infty}\lambda_k^{-6}\right)^{1/2}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k'''(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}, \tag{36}$$

$$|u_{tttt}(x,t)|\leq\left(\sum_{k=1}^{\infty}\lambda_k^{-6}\right)^{1/2}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3\left\|u_k^{(4)}(t)\right\|_{C[0,T]}\right)^2\right)^{1/2}. \tag{37}$$

From (32) - (37), taking into account (27) - (31), it follows, that the functions $u(x,t)$, $u_t(x,t)$, $u_{tt}(x,t)$, $u_{ttxx}(x,t)$, $u_{ttt}(x,t)$, $u_{tttt}(x,t)$ are continuous in D_T . By immediate verification, it is easy to see, that the function $u(x,t)$ satisfies equation (1) and conditions (2), (3) in the ordinary sense. The theorem is proved.

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