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ON A BOUNDARY VALUE PROBLEM FOR OSCILLATION EQUATION IN STRATIFIED LIQUID

Abstract

In the paper we prove a unique existence of a classic solution of a non-local boundary value problem for a partial differential equation of fourth order.

By studying the problem of small oscillations of exponentially stratified liquid there arises the equation

$$\frac{\partial^2}{\partial t^2}\Delta_3 u+\omega_0^2\Delta_2 u=0,$$

in many respects similar to S.L.Sobolev known equation [1], tere Δ_3 is Lapacian with respect to variables x_1,x_2,x_3,Δ_2 is Laplacian with respect to variables x_1,x_2,x_3, and ω_0 is a real number parameter. In the present paper we consider one-dimensional variant of this equation.

Let's consider the following boundary-value problem:

$$u_{ttxx}(x,t)+\alpha^2 u_{xx}(x,t)=f(x,t)\,,\quad (x,t)\in D_T=$$

$$=\{(x,t):0\le x\le 1,\quad 0\le t\le T\} \tag{1}$$

$$u(0,t)=,\quad u_x(1,t)=0,\quad 0\le t\le T, \tag{2}$$

$$u(x,0)+\delta u(x,T)=\varphi(x)\,,\quad u_t(x,0)+\delta u_t(x,T)=\psi(x)\,,\quad 0\le x\le 1, \tag{3}$$

where σ and δ are the given numbers $f(x,t)\,,\varphi(x)\,,\psi(x)$ are the given functions, $u(x,t)$ is the desired function. Under the classic solution of problem (1)-(3) we understand the function $u(x,t)$ continuous in closed domain D_T together with all its derivatives contained in equation (1) and satisfying all conditions of (1)-(3) in the ordinary sense.

Theorem 1. *If $\delta\neq\pm 1$, problem (1)-(3) may have at most one classic solution.*

Proof. The proof of this theorem is conducted by the following scheme [2].

Assume there exist two classic solutions of the considered problem $u_1(x,t)$ and $u_2(x,t)$, and consider the difference $\nu(x,t)=u_1(x,t)-u_2(x,t)$.

Obviously, the function $\nu(x,t)$ satisfies the homogeneous equation

$$\nu_{ttxx}(x,t)+\alpha^2\nu_{xx}(x,t)=0\qquad (0\le x\le 1,\quad 0\le t\le T) \tag{4}$$

and conditions

$$\nu(0,t)=0,\quad \nu_x(1,t)=0,\quad 0\le t\le T, \tag{5}$$

$$\nu(x,0)+\delta\nu(x,T)=0,\quad \nu_t(x,0)+\delta\nu_t(x,T)=0\quad 0\le x\le 1 \tag{6}$$

Prove that the function $\nu(x,t)$ in identically equals zero.

Multiply the both sides of equation (4) by the function $2\nu_t(x,t)$ and integrate the obtained equality with respect to x from 0 to 1 :

$$2\int\limits_0^1 \nu_{ttxx}(x,t)\,\nu_t(x,t)\,dx+2\alpha^2\int\limits_0^1 \nu_{xx}(x,t)\,\nu_t(x,t)\,dx=0. \tag{7}$$

Obviously

$$2\int_0^1 \nu_{ttxx}(x,t)\,\nu_t(x,t)\,dx = 2\left(\nu_{ttx}(1,t)\,\nu_t(1,t) - \nu_{ttx}(0,t)\,\nu_t(0,t)\right) -$$

$$-2\int_0^1 \nu_{ttx}(x,t)\,\nu_{tx}(x,t)\,dx = -\frac{d}{dt}\int_0^1 \nu_{tx}^2(x,t)\,dx;$$

$$2\int_0^1 \nu_{xx}(x,t)\,\nu_t(x,t)\,dx = 2\left(\nu_x(1,t)\,\nu_t(1,t) - \nu_x(0,t)\,\nu_t(0,t)\right) -$$

$$-2\int_0^1 \nu_x(x,t)\,\nu_{tx}(x,t)\,dx = -\frac{d}{dt}\int_0^1 \nu_x^2(x,t)\,dx.$$

Then from (7) $\forall t \in [0,T]$ we have:

$$\frac{d}{dt}\int_0^1 \nu_{tx}^2(x,t)\,dx + \alpha^2\frac{d}{dt}\int_0^1 \nu_x^2(x,t)\,dx = 0$$

or

$$y(t) = \int_0^1 \nu_{tx}^2(x,t)\,dx + \alpha^2\int_0^1 \nu_x^2(x,t)\,dx = C,$$

where $C = const$.

Hence, allowing for (6) we get :

$$y(0) - \delta^2 y(T) = \int_0^1 \left(\nu_{tx}^2(x,0) - \delta^2\nu_{tx}^2(x,T)\right)dx +$$

$$+\alpha^2\int_0^1 \left(\nu_x^2(x,0) - \delta^2\nu_x^2(x,T)\right)dx = \int_0^1 \left(\nu_{tx}(x,0) - \delta\nu_{tx}(x,T)\right) \times$$

$$\times\left(\nu_{tx}(x,0) + \delta\nu_{tx}(x,T)\right)dx + \alpha^2\int_0^1 \left(\nu_x(x,0) - \delta\nu_x(x,T)\right) \times$$

$$\times\left(\nu_x(x,0) + \delta\nu_x(x,T)\right)dx = 0.$$

Thus

$$y(0) - \delta^2 y(T) = C\left(1 - \delta^2\right) = 0.$$

Since $\delta \neq \pm 1$, then $C = 0$. Consequently, $\forall t \in [0,T]$:

$$\int_0^1 \nu_{tx}^2(x,t)\,dx + \alpha^2\int_0^1 \nu_x^2(x,t)\,dx = 0.$$

Hence, we deduce

$$\nu_{tx}(x,t) \equiv 0, \nu_x(x,t) \equiv 0.$$

$\nu(x,t) \equiv g(t)$ yields $\nu_x(x,t) \equiv 0$. And hence, by (5) it holds $g(t) \equiv 0$. Consequently $\nu(x,t) \equiv 0$.

Thus, if there exist two classic solutions $u_1 x, t$ and $u_2(x,t)$ of problem (1)-(3), then $u_1(x,t) \equiv u_2(x,t)$. Hence, if follows that if the klassic solution of problem (1)-(3) exist, it is unique. The theorem is proved.

Obviously, the necessary condition for the existence of continuous in D_T solution is the fulfillment of concordance conditions:

$$\varphi(0) = 0, \quad \varphi'(1) = 0, \quad \psi(0) = 0, \quad \psi'(1) = 0.$$

We'll search for the solution of classic problem (1)-(3) in the form

$$u(x,t) = \sum_{k=1}^{\infty} u_k(t) \sin \lambda_k x, \quad \lambda_k = \frac{\pi}{2}(2k-1), \tag{8}$$

where

$$u_k(t) = 2\int_0^1 u(x,t) \sin \lambda_k x dx.$$

Since the system $\{\sin \lambda_k x\}_{k=1}^{\infty}$ is complete in $L_2(0,1)$, then obviously, each classic solution $u(x,t)$ of problem (1)-(3) is really of the form of (8).

Applying Fourier formal method, from (1)-(3) we get:

$$\lambda_k^2 u''(t) + \alpha^2 \lambda_k^2 u_k(t) = -f_k(t), \quad 0 \le t \le T, \tag{9}$$

$$u_k(0) + \delta u_k(T) = \varphi_k, \; u_k'(0) + \delta u_k'(T) = \psi_k \quad (k = 1, 2, ...), \tag{10}$$

where

$$f_k(t) = 2\int_0^1 f(x,t) \sin \lambda_k x dx,$$

$$\varphi_k = 2\int_0^1 \varphi(x) \sin \lambda_k x dx, \quad \psi_k = 2\int_0^1 \psi(x) \sin \lambda_k x dx, \quad (k = 1, 2, ...)$$

Since the roots of the characteristic equation

$$\mu^2 + \alpha^2 = 0,$$

corresponding to (9) is defined by the formula:

$$\mu_j = (-1)^j \alpha i, \quad j = 1, 2$$

general solution of (9) according to Largange method is of the form:

$$u_k(t) = C_{1k}(t) \cos \alpha t + C_{2k}(t) \sin \alpha t, \tag{11}$$

where $C_{1k}(t)$ and $C_{2k}(t)$ are still unknown functions.

To determine the functions $C_{1k}(t)$ and $C_{2k}(t)$ we have the system:

$$\begin{cases} C_{1k}'(t) \cos \alpha t + C_{2k}'(t) \sin \alpha t = 0, \\ C_{1k}'(t) \sin \alpha t - C_{2k}'(t) \cos \alpha t = \frac{1}{\alpha \lambda_k^2} f_k(t). \end{cases}$$

Hence we find:

$$C'_{1k}(t) = \frac{1}{\alpha\lambda_k^2} f_k(t) \sin\alpha t$$

$$C'_{2k}(t) = -\frac{1}{\alpha\lambda_k^2} f_k(t) \cos\alpha t$$

Integrating from 0 to t we have:

$$\begin{cases} C_{1k}(t) = \frac{1}{\alpha\lambda_k^2} \int\limits_0^t f_k(\tau) \sin\alpha\tau d\tau + C_{1k}; \\ C_{2k}(t) = -\frac{1}{\alpha\lambda_k^2} \int\limits_0^t f_k(\tau) \cos\alpha\tau d\tau + C_{2k}, \end{cases} \tag{12}$$

where C_{1k} and C_{2k} are arbitrary constants.

Substituting (12) into (11) we get:

$$u_k(t) = C_{1k}(t) \cos\alpha t + C_{2k}(t) \sin + \frac{1}{\alpha\lambda_k^2} \int\limits_0^t f_k(\tau) \sin\alpha(t-\tau) d\tau. \tag{13}$$

From (13) we find:

$$u'_k(t) = -\alpha C_{1k}(t) \cos\alpha t + \alpha C_{2k}(t) \cos + \frac{1}{\lambda_k^2} \int\limits_0^t f_k(\tau) \cos\alpha(t-\tau) d\tau. \tag{14}$$

Now, using conditions of (10) we determine C_{1k} and C_{2k}:

$$\begin{cases} (1+\delta\cos\alpha T) C_{1k} + C_{2k}\delta\sin\alpha T = q_{1k}(T), \\ -C_{1k}\alpha\delta\sin\alpha T + \alpha(1+\delta\cos\alpha T) C_{2k} = q_{2k}(T), \end{cases} \tag{15}$$

where

$$\begin{aligned} q_{1k}(T) &= \varphi_k - \frac{\delta}{\alpha\lambda_k^2} \int\limits_0^T f_k(\tau) \sin\alpha(T-\tau) d\tau \\ q_{2k}(T) &= \psi_k - \frac{\delta}{\lambda_k^2} \int\limits_0^T f_k(\tau) \cos\alpha(T-\tau) d\tau \end{aligned} \tag{16}$$

Obviously

$$\Delta_k(T) \equiv \begin{vmatrix} 1+\delta\cos\alpha T & \delta\sin\alpha T \\ -\alpha\delta\sin\alpha T & \alpha(1+\delta\cos\alpha T) \end{vmatrix} = \left(1+2\delta\cos\alpha T+\delta^2\right) a,$$

$$\Delta_{1k}(T) \equiv \begin{vmatrix} q_{1k}(T) & \delta\sin\alpha T \\ q_{2k}(T) & \alpha(1+\delta\cos\alpha T) \end{vmatrix} =$$

$$= \alpha(1+\delta\cos\alpha T) q_{1k}(T) - \delta q_{2k}(T) \sin\alpha T,$$

$$\Delta_{2k}(T) \equiv \begin{vmatrix} 1+\delta\cos\alpha T & q_{1k}(T) \\ -\alpha\delta\sin\alpha T & q_{2k}(T) \end{vmatrix} =$$

$$= (1+\delta\cos\alpha T) q_{2k}(T) + \alpha\delta q_{1k}(T) \sin\alpha T.$$

Hence we have:

$$\begin{cases} C_{1k} = \frac{1}{\alpha\rho(T)}\left[\alpha\left(1+\delta\cos\alpha T\right) q_{1k}\left(T\right) - \delta q_{2k}\left(T\right)\sin\alpha T\right], \\ C_{2k} = \frac{1}{\alpha\rho(T)}\left[\left(1+\delta\cos\alpha T\right) q_{2k}\left(T\right) + \alpha\delta q_{1k}\left(T\right)\sin\alpha T\right], \end{cases} \tag{17}$$

where $\rho\left(T\right) = 1 + 2\delta\cos\alpha\delta + \delta^2$. Substituting value (17) into (13) we get:

$$u_k\left(t\right) = \frac{1}{\alpha\rho\left(T\right)}\left\{\left[\beta\left(1+\delta\cos\alpha T\right) q_{1k}\left(T\right) - \delta q_{2k}\left(T\right)\sin\alpha T\right]\cos\alpha t + \right.$$

$$\left. + \left[\left(1+\delta\cos\alpha T\right) q_{2k}\left(T\right) + \alpha\delta q_{1k}\left(T\right)\sin\alpha T\right]\sin\alpha t\right\} +$$

$$+\frac{1}{\alpha\lambda_k^2}\int\limits_0^t f_k\left(\tau\right)\sin\alpha\left(t-\tau\right)d\tau$$

or

$$u_k\left(t\right) = \frac{1}{\alpha\rho\left(T\right)}\left\{\alpha\left[\cos\alpha t + \delta\cos\alpha\left(T-t\right)\right] q_{1k}\left(T\right) + \right.$$

$$\left. + \left[\sin\alpha t - \delta\sin\alpha\left(T-t\right)\right] q_{2k}\left(T\right)\right\} + \frac{1}{\alpha\lambda_k^2}\int\limits_0^t f_k\left(\tau\right)\sin\alpha\left(t-\tau\right)d\tau.$$

Hence, allowing for (16), we find:

$$u_k\left(t\right) = \frac{1}{\alpha\rho\left(T\right)}\left\{\alpha\left(\cos\alpha t + \delta\cos\alpha\left(T-t\right)\right)\varphi_k + \left(\sin\alpha t - \delta\sin\alpha\left(T-t\right)\right)\psi_k - \right.$$

$$\left. -\frac{\delta}{\lambda_k^2}\int\limits_0^T f_k\left(\tau\right)\left(\sin\alpha\left(T+t-\tau\right) + \delta\sin\alpha\left(t-\tau\right)\right)d\tau\right\} +$$

$$+\frac{1}{\alpha\lambda_k^2}\int\limits_0^t f_k\left(\tau\right)\sin\alpha\left(t-\tau\right)d\tau$$

Thus, solving problem (9)(10) we find

$$u_k\left(t\right) = \frac{1}{\alpha\rho\left(T\right)}\left\{\alpha\left(\cos\alpha t + \delta\cos\alpha\left(T-t\right)\right)\varphi_k + \left(\sin\alpha t - \delta\sin\alpha\left(T-t\right)\right)\psi_k - \right.$$

$$\left. -\frac{\delta}{\lambda_k^2}\int\limits_0^T f_k\left(\tau\right)\left(\sin\alpha\left(T+t-\tau\right) + \delta\sin\alpha\left(t-\tau\right)\right)d\tau\right\} +$$

$$+\frac{1}{\alpha\lambda_k^2}\int\limits_0^t f_k\left(\tau\right)\sin\alpha\left(t-\tau\right)d\tau, \qquad \left(k = 1, 2, ...\right), \tag{18}$$

where

$$\rho\left(T\right) = 1 + 2\cos\alpha T + \delta^2.$$

Obviously,

$$u_k'(t)=\frac{1}{\rho(T)}\{\alpha(\cos\alpha t+\delta\cos\alpha(T-t))\varphi_k+(\cos\alpha t+\delta\cos\alpha(T-t))\psi_k-$$

$$-\frac{\delta}{\lambda_k^2}\int_0^T f_k(\tau)(\cos\alpha(T+t-\tau)+\delta\cos\alpha(t-\tau))d\tau\}+$$

$$+\frac{1}{\lambda_k^2}\int_0^t f_k(\tau)\cos\alpha(t-\tau)d\tau,\qquad(k=1,2,...),\tag{19}$$

$$u_k''(t)=\frac{1}{\lambda_k^2}f_k(t)-\frac{\alpha}{\rho(T)}\{\alpha(\cos\alpha t+\delta\cos\alpha(T-t))\varphi_k+$$

$$+(\sin\alpha t-\delta\sin\alpha(T-t))\psi_k-$$

$$-\frac{\delta}{\lambda_k^2}\int_0^T f_k(\tau)(\sin\alpha(T+t-\tau)+\delta\sin\alpha(t-\tau))d\tau\}-$$

$$-\frac{\alpha}{\lambda_k^2}\int_0^t f_k(\tau)\sin\alpha(t-\tau)d\tau,\qquad(k=1,2,...)\tag{20}$$

Theorem 2. *Let*
1. $\varphi(x)\in C^2[0,1],\varphi^{(3)}(x)\in L_2(0,1)$ and $\varphi(0)=\varphi'(1)=\varphi''(0)=0$;
2. $\psi(x)\in C^2[0,1],\psi^{(3)}(x)\in L_2(0,1)$ and $\psi(0)=\psi'(1)=\psi''(0)=0$;
3. $f(x,t)\in C(D_T)$ and $f_x(x,t)\in L_2(D_T)$ and $f(0,t)=0,\forall t\in[0,T]$.
Then the function

$$u(x,t)=\sum_{k=1}^{\infty}\left\{\frac{1}{\alpha\rho(T)}[\alpha(\cos\alpha t+\delta\cos\alpha(T-t))\varphi_k+\right.$$

$$+(\sin\alpha t-\delta\sin\alpha(T-t))\psi_k-\left(\frac{\delta}{\lambda_k^2}\int_0^T f_k(\tau)(\sin\alpha(T+t-\tau)\times\right.$$

$$\left.\times\ -\delta\sin\alpha(t-\tau))d\tau]-\frac{1}{\alpha\lambda_k^2}\int_0^t f_k(\tau)\sin\alpha(t-\tau)d\tau\right\}\sin\lambda_k x\tag{21}$$

is a classic solution of problem (1)-(3).

Proof. From (18), (19) and (20) we find respectively:

$$|u_k(t)|\le\frac{1}{\rho(T)}(1+|\delta|)|\varphi_k|+\frac{1}{\rho(T)\alpha}(1+|\delta|)|\psi_k|+$$

$$+\frac{1}{\alpha}\left(1+\frac{1}{\rho(T)}|\delta|(1+|\delta|)\right)\sqrt{T}\lambda_k^{-2}\left(\int_0^T|f_k(\tau)|^2d\tau\right)^{\frac{1}{2}},$$

$$\left|u_k'(t)\right| \le \frac{\alpha}{\rho(T)}(1+|\delta|)|\varphi_k| + \frac{1}{\rho(T)}(1+|\delta|)|\psi_k| +$$

$$+\left(1+\frac{|\delta|}{\rho(T)}(1+|\delta|)\right)\sqrt{T}\lambda_k^{-2}\left(\int_0^T |f_k(\tau)|^2 d\tau\right)^{\frac{1}{2}},$$

$$\left|u_k''(t)\right| \le \lambda_k^2 |f_k(t)| \frac{\alpha^2}{\rho(T)}(1+|\delta|)|\varphi_k| + \frac{\alpha}{\rho(T)}(1+|\delta|)|\psi_k| +$$

$$+\alpha\left(1+\frac{|\delta|}{\rho(T)}(1+|\delta|)\right)\sqrt{T}\lambda_k^{-2}\left(\int_0^T |f_k(\tau)|^2 d\tau\right)^{\frac{1}{2}}.$$

Hence, we have respectively:

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3 |u_k(t)|\right)^2\right)^{\frac{1}{2}} \le \frac{\sqrt{3}}{\rho(T)}(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|\varphi_k|\right)^2\right)^{\frac{1}{2}} +$$

$$+\frac{\sqrt{3}}{\rho(T)\alpha}(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|\psi_k|\right)^2\right)^{\frac{1}{2}} + \frac{\sqrt{3}}{\alpha}\left(1+\frac{1}{\rho(T)}|\delta|(1+|\delta|)\right)\times$$

$$\times\sqrt{T}\left(\int_0^T\sum_{k=1}^{\infty}\left(\lambda_k|f_k\tau|\right)^2 d\tau\right)^{\frac{1}{2}} \le \frac{\sqrt{3}}{\rho(T)}(1+|\delta|)\left\|\varphi^{(3)}(x)\right\|_{L_2(0,1)} +$$

$$+\frac{\sqrt{3}}{\alpha}(1+|\delta|)\left\|\psi^{(3)}(x)\right\|_{L_2(0,1)} +$$

$$+\frac{\sqrt{3}}{\alpha}\left(1+\frac{1}{\rho(T)}|\delta|(1+|\delta|)\right)\sqrt{T}\left\|f_x(x,t)\right\|_{L_2(D_T)}, \tag{22}$$

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3 |u_k'(t)|\right)^2\right)^{\frac{1}{2}} \le \frac{\sqrt{3}}{\rho(T)}(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|\varphi_k|\right)^2\right)^{\frac{1}{2}} +$$

$$+\frac{\sqrt{3}}{\rho(T)}(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|\psi_k|\right)^2\right)^{\frac{1}{2}} + \sqrt{3}\left(1+\frac{|\delta|}{\rho(T)}(1+|\delta|)\right)\times$$

$$\times\sqrt{T}\left(\int_0^T\sum_{k=1}^{\infty}\left(\lambda_k|f_k(\tau)|\right)^2 d\tau\right)^{\frac{1}{2}} \le \frac{\sqrt{3}}{\rho(T)}(1+|\delta|)\left\|\varphi^{(3)}(x)\right\|_{L_2(0,1)} +$$

$$+\frac{\sqrt{3}}{\rho(T)}(1+|\delta|)\left\|\psi^{(3)}(x)\right\|_{L_2(0,1)} +$$

$$+\sqrt{3}\left(1+\frac{1}{\rho(T)}(1+|\delta|)\right)\sqrt{T}\left\|f_x(x,t)\right\|_{L_2(D_T)}, \tag{23}$$

$$\left(\sum_{k=1}^{\infty}\left(\lambda_k^3 |u_k''(t)|\right)^2\right)^{\frac{1}{2}} \le 2\left(\sum_{k=1}^{\infty}\left(\lambda_k|f_k(t)|\right)^2\right)^{\frac{1}{2}} +$$

$$+\frac{2\alpha^2}{\rho(T)}(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|\varphi_k|\right)^2\right)^{\frac{1}{2}}+$$

$$+\frac{2\alpha}{\rho(T)}(1+|\delta|)\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|\psi_k|\right)^2\right)^{\frac{1}{2}}+2\alpha\left(1+\frac{|\delta|}{\rho(T)}(1+|\delta|)\right)\times$$

$$\times\sqrt{T}\left(\int_0^T\sum_{k=1}^{\infty}\left(\lambda_k|f_k(\tau)|\right)^2d\tau\right)^{\frac{1}{2}}\le 2\left\|\|f_x(x,t)\|_{L_2(0,1)}\right\|_{C[0,T]}+$$

$$+\frac{2\alpha^2}{\rho(T)}(1+|\delta|)\left\|\varphi^{(3)}(x)\right\|_{L_2(0,1)}+\frac{2\alpha}{\rho(T)}(1+|\delta|)\left\|\psi^{(3)}(x)\right\|_{L_2(0,1)}+$$

$$+2\alpha\left(1+\frac{|\delta|}{\rho(T)}(1+|\delta|)\right)\sqrt{T}\,\|f_x(x,t)\|_{L_2(D_T)}. \tag{24}$$

Obviously

$$|u(x,t)|\le\left(\sum_{k=1}^{\infty}\lambda_k^{-6}\right)^{\frac{1}{2}}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|u_k(t)|\right)^2\right)^{\frac{1}{2}}, \tag{25}$$

$$|u_t(x,t)|\le\left(\sum_{k=1}^{\infty}\lambda_k^{-6}\right)^{\frac{1}{2}}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|u_k'(t)|\right)^2\right)^{\frac{1}{2}}, \tag{26}$$

$$|u_{tt}(x,t)|\le\left(\sum_{k=1}^{\infty}\lambda_k^{-6}\right)^{\frac{1}{2}}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|u_k''(t)|\right)^2\right)^{\frac{1}{2}}, \tag{27}$$

$$|u_{xx}(x,t)|\le\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{\frac{1}{2}}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|u_k(t)|\right)^2\right)^{\frac{1}{2}}, \tag{28}$$

$$|u_{ttxx}(x,t)|\le\left(\sum_{k=1}^{\infty}\lambda_k^{-2}\right)^{\frac{1}{2}}\left(\sum_{k=1}^{\infty}\left(\lambda_k^3|u_k''(t)|\right)^2\right)^{\frac{1}{2}}. \tag{29}$$

Allowing for (22)-(24) if follows from (25)-(29) that the functions $u(x,t)$, $u_t(x,t)$, $u_{tt}(x,t)$, $u_{xx}(x,t)$, $u_{ttxx}(x,t)$ are continuous in D_T. By immediate verification we can easily see that the function $u(x,t)$ satisfies equation (1) and conditions (2), (3) in the ordinary sense. The theorem is proved.

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