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AEROELASTIC VIBRATIONS AND STABILITY OF A CONICAL SHELL STREAMLINED BY GAS FLOW WITH HIGH SUPERSONIC SPEED

Abstract

Many papers have been devoted to the panel flutter of shells [1-4], it is used the piston theory formula for pressure of the aerodynamical interactions between the flow and a shell. Inadequacy of such an approach is discussed in papers [5-7]; results of these studies were used in [8-9] for new statements of problems on the flutter of conical shells. In the proposed work, in elaborations of results of [6], it is considered a problem on the truncated conical shell flutter, it is adduced data on evaluative computations and comparison them with analogous ones, obtained by the piston theory.

In a lot of papers on panel flutter of shells [1-4] the piston theory formula for pressure of the aerodynamical interactions between flow and shell is used. Insufficiency of such an approach has been discussed in the papers [5-7], the results of these investigations have been used in [8-9] for new statements of the filter problems of conical shells. In the suggested paper in development of the results of [6], the flutter problem of truncated conical shell have been considered, the data of evaluation computations and their comparison with analogous ones obtained by the piston theory are cited.

1. Relation of gas dynamics. Consider a circular cone streamlined by supersonic speed without attack angle. Origin of rectangular system of coordinates is an vertex, the axis x is directed in velocity vector. In nondeformed state the equation generating $z_1 = kx$, $k = tg\alpha$, α is a semi-opening angle; assume $\alpha^2 << 1$. Shell occupies one part of the cone $x_1 \leq x \leq x_2$, denote by $w(x,t)$ its bending in axisymmetric case, and we'll have

$$x_1 \leq x \leq x_2, \quad z = kx - w(x,t) \tag{1.1}$$

In accordance with law of plane sections [10], gas state after shock wave is determined from the solution of a plane piston problem which moves by the law

$$\bar{z}(t) = kvt - w(vt,t)$$

where v is flow speed. The solution of this problem has been obtained in [6] by small parameter method [11] at additional suggestion $|w(x,t)/(kx)| << 1$ however it [6] hasn't been analyzed this analysis is cited below. We write the expression for pressure on the shell

$$\Delta p = \frac{2\rho^0 D^2}{\chi+1}\left[1+\varepsilon\frac{a(D)}{4} - \frac{\chi p^0(\chi+1)}{2\rho^0 D^2}\right] - \frac{4\rho^0 D^2}{\chi+1}\left[1+\frac{3\varepsilon}{4} - \varepsilon\frac{11a(D)}{8\chi}\right]\dot{w} -$$

$$-2\varepsilon\frac{\rho^0 D}{t}\left(1+\frac{5\chi+1}{\chi(\chi+1)}\frac{9a(D)}{8}\right)w - \varepsilon\frac{\rho^0 D}{2}\left(1-\varepsilon\frac{3a(D)}{2\chi(\chi+1)}\right)t\cdot\ddot{w} - \tag{1.2}$$

$$-2\varepsilon\frac{3-\chi}{\chi+1}\frac{\rho^0 D}{t^2}\int\limits_0^t w(\xi,t)\,d\xi-2\varepsilon\frac{1+\chi}{\chi}\frac{\rho^0 D}{t^4}\int\limits_0^t \xi^2 w(\xi,t)\,d\xi;$$

here: ρ^0, p^0, a^0 is density, pressure and sound speed in nondisturbed flow, χ is exponent of polytrop, $\varepsilon=(\chi-1)/(\chi+1)$; speed of shock wave D is found from the quadratic equation $\varepsilon Da(D)+2vtg\alpha=2D$, $a(D)=1+2a_0^2/\left((\chi-1)D^2\right)$, which after notation $u_0=Mtg\alpha$, $u=D/a_0=Mtg\beta$ is led to the following one

$$(3+\chi)u^2-2(\gamma+1)u_0u-2=0 \tag{1.3}$$

By passing to Euler coordinate system it should be assumed $x=vt$, $x_1=vt_1$, $\dot w=\partial w/\partial t+v\partial w/\partial x$.

Estimate the order of addends in (1.2), We'll have subsequently

$$\varepsilon\rho^0 D=\frac{w(vt,t)}{t}=\frac{w(x,t)}{x}\varepsilon\rho^0 Dv=\frac{w(x,t)}{kx}k\varepsilon\rho^0 Dv;$$

$$\varepsilon\rho^0 D\frac{1}{t^2}\int\limits_0^t w(\xi,t)\,d\xi=\varepsilon\rho^0 D\frac{1}{t^2}w\left(\tilde t,t\right)(t-t_1)=$$

$$=k\varepsilon\rho^0 Dv\frac{w(\tilde x,t)(x-x_1)}{kx},\quad x_1\le\tilde x\le x;$$

$$\varepsilon\rho^0 D\frac{1}{t^4}\int\limits_0^t \xi^2 w(\xi,t)\,d\xi=\frac{1}{3}\varepsilon\rho^0 Dv\frac{w\left(\approx{x},t\right)\left(t^3-t_1^3\right)}{t^4}=$$

$$=\frac{1}{3}k\varepsilon\rho^0 Dv\frac{w\left(\approx{x},t\right)}{kx}\left(1-\frac{x_1^3}{\approx{x}^3}\right);$$

The integrals are computed by the mean value theorem. Since $|w/(kx)|<<1$ and $k\sim\varepsilon$, we conclude that written addends are of order ε^2 or higher, therefore we can neglect them.

As is known from (1.2) Δp consists of sum of quasistatic and dynamic components q_0 and q_1. Assuming $w=w_0(x)+w_1(x,t)$, we obtain

$$q_0=\frac{2\rho^0 D^2}{\chi+1}\left(1+\varepsilon\frac{a(D)}{4}-\frac{\chi\rho^0}{2\rho^0 D^2}\right)-$$
$$-\frac{4\rho^0 Dv}{\chi+1}\left(1+\frac{3\varepsilon}{4}-\varepsilon\frac{11a(D)}{8\chi}\right)\frac{\partial w_0}{\partial x}-$$
$$-\frac{\rho^0 Dvx}{2}\left(1-\varepsilon\frac{3a(D)}{2\chi(\chi+1)}\right)\frac{\partial^2 w_0}{\partial x^2} \tag{1.4}$$

$$q_1=-\frac{4\rho^0 D}{\chi+1}\left(1+\frac{3\varepsilon}{4}-\varepsilon\frac{11a(D)}{8\chi}\left(\frac{\partial w_1}{\partial t}+v\frac{\partial w_1}{\partial x}\right)\right)-$$
$$-\frac{\rho^0 Dx}{2v}\left(1-\varepsilon\frac{3a(D)}{2\chi(\chi+1)}\right)\left(\frac{\partial^2 w_1}{\partial t^2}+2v\frac{\partial^2 w_1}{\partial t\partial x}+v^2\frac{\partial^2 w_1}{\partial x^2}\right) \tag{1.5}$$

We estimate the orders of quantities of addends in the second parenthesis of (1.5) attached mass of shell

$$\frac{\rho^0 Dx}{2v} \cdot \frac{1}{\rho h} \sim \frac{\rho^0 kvl}{2v\rho h} = \frac{k\rho^0 l}{2\rho h}, \quad l = x_2 - x_1;$$

compare the second addend with the third one

$$\left(2v\frac{\partial^2 w_1}{\partial t \partial x}\right) : v^2 \frac{\partial^2 w_1}{\partial x^2} \sim \frac{2vw_1}{t_0 l} \frac{l^2}{v^2 w_1} = \frac{2l}{t_0 v} \sim \frac{C_0 h}{vl}$$

here $C_0^2 = E/\rho$, E is a Young module of material of shell.

At ordinary values of parameters the both relations are quantities of order $10^{-1} := 10^{-2}$; therefore we neglect these addends. The last addend makes sense of chain efforts in median surface of a shell and can make remarkable influence on character of vibrations and their stability. Finally from (1.5) we obtain

$$q_1(\times, t) = -\frac{4\rho^0 D}{\chi + 1}\left(1 + \frac{3\varepsilon}{4} - \varepsilon\frac{11a(D)}{8\chi}\right)\left(\frac{\partial w_1}{\partial t} + v\frac{\partial w_1}{\partial x}\right) -$$

$$-\frac{\rho^0 Dvx}{2v}\left(1 - \varepsilon\frac{3a(D)}{2\chi(\chi+1)}\right)\frac{\partial^2 w_1}{\partial x^2}. \tag{1.6}$$

2. Statement of the flutter problem. The position of the point on conical surface is determined by the coordinates $s = x/\cos\alpha$ and $\theta = \psi \sin\alpha$, where ψ is a polar angle. We'll describe mode of deformation of the shell by the equation [12]

$$D_0 \Delta^2 w - \Delta_k F - L(w, F) = q(s, t)$$

$$2\Delta^2 F + 2Eh\Delta_k w + L(w, w) = 0 \tag{2.1}$$

where $D_0 = Eh^3/\left(12\left(1 - v^2\right)\right)$ is cylindrical rigidity, v is a Poison coefficient, F is a stress function.

The operators introduced in (2.1) have the form

$$L(u, v) = \left(\frac{1}{s}\frac{\partial u}{\partial s} + \frac{1}{s^2}\frac{\partial^2 u}{\partial \theta^2}\right)\frac{\partial^2 v}{\partial s^2} +$$

$$+\left(\frac{1}{s}\frac{\partial v}{\partial s} + \frac{1}{s^2}\frac{\partial^2 v}{\partial \theta^2}\right)\frac{\partial^2 u}{\partial s^2} - 2\left(\frac{\partial}{\partial s}\frac{1}{s}\frac{\partial u}{\partial \theta}\right)\left(\frac{\partial}{\partial s}\frac{1}{s}\frac{\partial v}{\partial \theta}\right),$$

$$\Delta = \frac{\partial^2}{\partial s^2} + \frac{1}{s}\frac{\partial}{\partial s} + \frac{1}{s^2}\frac{\partial^2}{\partial \theta^2}; \qquad \Delta_k = \frac{1}{stg\alpha}\frac{\partial^2}{\partial s^2}.$$

Analogously to the representation $w = w_0(s) + w_1(s, t)$ we assume $F = F_0(s) + F_1(s, t)$. Let's introduce dimensionless quantities, keeping their previous notation:

$$s \Longrightarrow s/s_2, \quad w_0 \Longrightarrow w_0/h, \quad F_0 \Longrightarrow F_0/\left(Eh^2 s_2\right),$$

$$w_1 = w_1/h, \quad F_1 = F_1/\left(Eh^2 s_2\right).$$

Substitute all in (2.1), linerize by perturbations w_1, F_1 and carry out the obvious simplifications. For the functions of basic state we obtain

$$\frac{tg\alpha}{12\left(1-v^2\right)}\frac{h^2}{s_2^2}\Delta^2 w_0-\frac{1}{s}\frac{\partial^2 F_0}{\partial s^2}=q_0^*$$

$$tg\alpha\Delta^2 F_0+\frac{1}{s}\frac{\partial^2 w_0}{\partial s^2}=0 \qquad (2.2)$$

here it is denoted

$$q_0^*=\frac{2\chi p^0 s_2^2 u^2 tg\alpha}{(\chi+1)\,Eh^2}\left(1+\frac{\varepsilon}{4}a^*(u)-\frac{1}{2u^2}\right);\ a^*(u)=1+\frac{2}{(\chi-1)\,u^2}$$

We complete system (2.2) by hinge support conditions

$$s=s_1,\ s=1: w_0=0,\ \frac{\partial^2 w_0}{\partial s^2}+\frac{v}{s}\frac{\partial w_0}{\partial s}=0$$

$$\frac{\partial F_0}{\partial s}=0,\ \frac{\partial^2 F_0}{\partial s^2}=0 \qquad (2.3)$$

For the functions of disturbed state, we obtain

$$tg\alpha\Delta^2 F_1+\frac{1}{s}\frac{\partial^2 w_1}{\partial s^2}=0$$

$$\frac{tg\alpha}{12\left(1-v^2\right)}\frac{h^2}{s_2^2}\Delta^2 w_1-\frac{1}{s}\frac{\partial^2 F_1}{\partial s^2}-tg\alpha\frac{h}{s_2}\frac{1}{s}\frac{\partial F_0}{\partial s}\frac{\partial^2 w_1}{\partial s^2}-$$

$$-tg\alpha\frac{h}{s_2}\frac{\partial^2 F_0}{\partial s^2}\left(\frac{1}{s}\frac{\partial w_1}{\partial s}+\frac{1}{s^2}\frac{\partial^2 w_1}{\partial\theta^2}\right)=q_1\frac{tg\alpha}{E}\frac{s_2^2}{h^2} \qquad (2.4)$$

Assume $w_1=w(s)\exp(\omega t)\cos n\psi$, $F_1=\Phi(s)\exp(\omega t)\cos n\psi$,then from (2.4) it follows

$$tg\Delta_n^2\Phi+\frac{1}{s}W''=0$$

$$\frac{tg\alpha}{12\left(1-v^2\right)}\frac{h^2}{s_2^2}\Delta_n^2 W-\frac{1}{s}\Phi''-tg\alpha\frac{h}{s_2}\frac{1}{s}F_0'W''-$$

$$-tg\alpha\cdot\frac{h}{s_2}F_0^{"}\left(\frac{1}{s}W'-\frac{n^2}{s^2\sin^2\alpha}W\right)+A_3 sW''+A_2W'=\lambda W \qquad (2.5)$$

here the following notation are introduced

$$\Delta_n=\frac{\partial^2}{\partial s^2}+\frac{1}{s}\frac{\partial}{\partial s}-\frac{n^2}{s^2\sin^2\alpha};\quad A_4\Omega^2+A_1\Omega+\lambda=0;\ \Omega=\frac{s_2\omega}{C_0}$$

$$A_1=\frac{4\chi p^0 s_2 C_0}{(\chi+1)\,Eha_0}utg\alpha\left(1+\frac{3\varepsilon}{4}-\frac{11\varepsilon}{8\xi}a^*(u)\right),\quad A_4=tg\alpha;$$

$$A_2=\frac{a_0u_0}{C_0tg\alpha}A_1;\quad A_3=\frac{\chi p^0 s_2}{2Eh}uu_0\left(1-\frac{3\varepsilon}{2\chi(\chi+1)}a^*(u)\right).$$

We add the following boundary conditions to system (2.5):

$$s = s_1, \quad s = 1 : W = 0; \quad W'' + \frac{v}{s} W' = 0 \tag{2.6}$$

$$\Phi' = \frac{n^2}{s^2 \sin^2 \alpha} \Phi = 0, \;\; \Phi'' = 0. \tag{2.7}$$

By the conditions of the problem $\sin^2 \alpha << 1$, $n \geq 1$, $s - 1$, therefore instead of conditions (2.7) we can approximately accept:

$$s = s_1, \quad s = 1 : \Phi = 0, \quad \Phi'' = 0 \tag{2.8}$$

System (2.5)-(2.7) is an eigen-values problem. The movement is stable if $\operatorname{Re} \Omega_k < 0$, $\forall k$, is unstable, if at least for one value of k_0 there will be $\operatorname{Re} \Omega_{k_0} > 0$; The domains of stable and unstable vibrations is separated by parabola of stability $A_4 (\operatorname{Im} \lambda_k)^2 = A_1^2 \operatorname{Re} \lambda_k$, whose equation defines the critic values of parameters.

Remark 1. 1) If the critical rate of flatter is defined, then the solution will be $M_{cr} = M_{cr}(n)$, $M_{cr}(n_{cr}) = \min\limits_{n} M_{cr}(n)$ should be assumed the true critical rate of flatter. 2) In the domain of stable vibrations the static stability of basic state $w_0 F_0$ must be checked.

3. Some estimates. In the first point it was taken into account the last addend in formula (16), which has sense of chain stresses in median surface of shell. In order to clarify qualitatively influence of this addend, we construct an approximated solution of problem (2.5), (2.6), (2.8) basing on the following suggestions.

1) Following the results of the paper [4], for the basic state we accept the known expressions [12]

$$w_0 = 0, \quad \frac{\partial^2 F_0}{\partial s^2} = q_0^* s \cdot tg\alpha; \quad \frac{1}{s}\frac{\partial F_0}{\partial s} = \frac{1}{2} q_0^* s tg\alpha \tag{3.1}$$

2) For shell of small prolongation and small conicity we can substitute relative coordinates within shell (in integrals which will be met below) by its some mean value.

3) We find the solution in Bubnov-Galerkin binomial approximation

$$W = C_1 \sin \beta\pi y + C_2 \sin 2\beta\pi y; \quad \beta (1 - s_1) = 1; \quad 0 \leq y \leq 1 - s_1$$

$$\Phi = B_1 \sin \beta\pi y + B_2 \sin 2\beta\pi y$$

at that from the first equation of (2.4) we find

$$B_1 = \frac{(\beta\pi)^2 C_1}{R_1 B_{11}} = \delta_1 C_1; \quad B_2 = \frac{(2\beta\pi)^2 C_2}{R_1 B_{22}} \equiv \delta_2 C_2$$

$$B_{11} = (\beta\pi)^4 + \frac{2n^2 (\beta\pi)^2}{R_2^2} + \frac{n^4}{R_4^4}$$

parameter B_{22} is obtained from B_{11} by substitution $2\beta, R_1, R_2^2, ...$ for B these are mean values of the quantity $s \cdot \sin \alpha$, $(s \cdot \sin \alpha)^2$,.... The boundary conditions are satisfied in the sense that on end surfaces of the shell the reactive moments proportional to rotation angle are applied.

We divide the equation into $tg\alpha$, denote

$$A_1' tg\alpha = A_1, ..., A_0 = h^2 / \left(12\left(2-v^2\right)s_2^2\right)$$

and carry out the Bubnov-Galrkin procedure, as a result we obtain

$$C_1\left\{A_0\left[(\beta\pi)^4+\frac{2n^2(\beta\pi)^2}{R_2^2}+\frac{n^4}{R_4^4}\right]+\right.$$

$$+\frac{\delta_1}{R_1}(\beta\pi)^2-\left(\frac{R_1h}{2s_2}q_0^*+\frac{R_1A_3'}{tg\alpha}\right)(\beta\pi)^2-$$

$$\left.-\frac{hR_1n^2}{s_2R_2^2}q_0^*-\lambda\right\}-C_2\frac{8}{3}\beta A_2'=0 \tag{3.2}$$

$$C_1\frac{8}{3}\beta A_2'+C_2\left\{A_0\left[(2\beta\pi)^4+\frac{2n^2(2\beta\pi)^2}{R_2^2}+\frac{n^4}{R_4^4}\right]+\right.$$

$$\left.+\frac{\delta_2}{R_1}(2\beta\pi)^2-\left(\frac{R_1h}{2s_2}q_0^*+\frac{R_1A_3'}{tg\alpha}\right)(2\beta\pi)^2-\frac{hR_1n^2}{s_2R_2^2}q_0^*-\lambda\right\}=0$$

We rewrite this system in the standard form: $(A_{ij}-\lambda\partial_{ij})C_j=0$, it is easily established the structure of the coefficients $A_{ii}=\beta_{ii}-\bar\beta_{ii}(M)M^2$; $A_{12}=-A_{21}=\bar\beta_{12}(M)M^2$. Now characteristic equation (3.2) is written in the form: $\lambda^2-(A_{22}+A_{11})\lambda+A_{22}A_{11}-A_{12}^2=0$, for its roots we obtain

$$\lambda=\frac{1}{2}\left(A_{22}+A_{11}\pm\left[(A_{22}A_{11})^2-4A_{12}^2\right]^{1/2}\right) \tag{3.3}$$

Discriminant of (3.3) is equal to

$$\delta=\left[\beta_{22}-\beta_{11}-\left(\bar\beta_{22}-\bar\beta_{11}\right)M^2\right]^2-4\bar\beta_{12}M^4. \tag{3.4}$$

If M is small, then $\delta>0$ (3.3) has two different positive roots, vibrations are stable. By growth of the M discriminant decreases, and at some M_0 vanishes, from (3.4) we find

$$M_0^2=\frac{\beta_{22}-\beta_{11}}{2\bar\beta_{12}+\left(\bar\beta_{22}-\bar\beta_{11}\right)}. \tag{3.5}$$

At $M>M_0$ the discriminant is negative, and λ are complex-adjoint; at some $M=M_{cr}>M_0$, they are on parabola of stability. Numerous calculations of flutter of plate [13-16] show that M_{cr} exceeds M_0 a little (approximate stability condition), therefore conclusion which we can make on the basis of (3.5), may be usefull. (Note that the parameters $\bar\beta_{ij}(M)$ depend on M only by $\varepsilon\cdot a^*(u)$, where ε is a small parameter of the problem, therefore can be substituted by some mean values $\left(\bar\beta_{ij}\right)_{mn}$).

The addend at the beginning of item is contained in (3.5) with coefficient A_3':

$$\bar\beta_{22}-\bar\beta_{11}=3(\beta\pi)^2\left(\frac{R_1h}{2s_2}q_0^*+\frac{R_1A_3'}{tg\alpha}\right).$$

From (3.5) subject to this expression the following obvious inequality follows

$$M_0^2\left(A_3' \neq 0\right) < M_0^2\left(A_3' = 0\right);$$

thereby a new important mechanical effect - decrease of critical flutter speed of conical shell calculated by approximated criterion $\delta = 0$ is discovered.

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Received Yanuary 13, 2006; Revised May 15, 2006.
Translated by Mamadzada K.S.