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OPTIMAL CONTROL OF DISCRETE DIFFERENCE INCLUSIONS

Abstract

In the paper we obtain the necessary extremum conditions in an optimal control problem of discrete difference inclusions. As distinct from the known papers ([1,2]) we consider a more general problem, we apply the Clark subdifferential, and optimal trajectory, in the general case, isn't inner.

Let X be a Banach space, $a_t : X^2 \to 2^X, t = 0, 1, ..., T-1,$ be a multivalued mapping, where we denote by 2^X a set of all subsets of X. The set $gra_t = \left\{(x_1, x_2, y) \in X^3 : y \in a_t(x_1, x_2)\right\}$ is called a graph of multivalued mapping.

Consider the discrete inclusion with the delay

$$x_{t+1} \in a_t(x_{t-\Delta}, x_t), \quad t = 0, 1, ..., T-1, \tag{1}$$

where Δ and T are fixed integers, $0 < \Delta < T-1$.

Under the trajectory (solution) $\{x(t)\}$ of discrete inclusion (1) we'll understand the sequence $x(t)$, $t = -\Delta, -\Delta+1, ..., 0, ..., T$, for which (1) is satisfied.

Let $g_t(\cdot) = g(\cdot, t) : X \to R$, $t \in \overline{1, T}$, C be a closed subset in X. Consider the minimization problem of the function

$$f(\omega) = \sum_{t=1}^{T} g(x_t, t) \tag{2}$$

on the trajectories of discrete inclusion (1) under phase constraints $x(t) = c(t)$, $t = -\Delta, ... -1, 0$ and $x(T) \in C$, where $\omega = (x_1, x_2, ...x_T)$.

Denote by D a set of solutions of problem (1) satisfying the condition $x(t) = c(t)$, $t = -\Delta, ... -1, 0$ and $x(T) \in C$.

The trajectory $\{\bar{x}_t\}_{t=-\Delta}^{T}$ is called optimal if $f(\bar{\omega}) \le f(\omega)$ for $\omega(x_1, x_2, ...x_T)$, where $\bar{\omega} = (\bar{x}_1, \bar{x}_2, ...\bar{x}_T)$, $\{x_t\}_{t=-\Delta}^{T} \in D$.

If we determine in X^T the sets

$$M_0 = \left\{\omega = (x_1, ..., x_T) \in X^T : \ x_1 \in a_0(c(-\Delta), c(0))\right\},$$
$$M_t = \left\{\omega = (x_1, ..., x_T) \in X^T : \ x_{t+1} \in a_t(c(t-\Delta), x_t)\right\}, \ t = \overline{1, \Delta},$$
$$M_t = \left\{\omega = (x_1, ..., x_T) \in X^T : \ x_{t+1} \in a_t(x_{t-\Delta}, x_t)\right\}, \ t = \overline{\Delta+1, T-1},$$
$$M_T = \left\{\omega = (x_1, ..., x_T) \in X^T : \ x_T \in C\right\},$$

then the posed problem is led to the minimization of the function $f(\omega)$ on the set $M = \bigcap_{t=0}^{T} M_t$.

Let Y be a Banach space, E be a nonempty subset of Y. Consider its displacement function, i.e. the function $d_E : Y \to R$ defined in the following form $d_E(x) = \inf\{\|x - y\| : y \in E\}$. The function d_E satisfies the global Lipschitz condition. The generalized derivative of the function φ at the point y_0 in the direction of y is defined as

$$\varphi^0(y_0;y)=\overline{\lim_{\substack{z\to y_0\\ \lambda\downarrow 0}}}\frac{\varphi(z+\lambda y)-\varphi(z)}{\lambda}.$$

If φ is a Lipschitz function near y_0, then $y\to\varphi^0(y_0;y)$ is a sublinear function. The generalized gradient of the function φ at the point y_0 denoted by $\partial\varphi(y_0)$ is a set of all linear continuous functionals $\xi\in X^*$ such that $\varphi^0(y_0;y)\geq<\xi,y>$ for all $y\in Y$.

Assume that y_0 is a point of E. The vector $y\in Y$ is said to be tangent to E at y_0, if $d_E^0(y_0;y)=0$. The set of all tangents to E at y_0 is denoted by $T_E(y_0)$, i.e. $T_E(y_0)=\{y:d_E^0(y_0;y)=0\}$. If $y_0\in int\ E$, then $T_E(y_0)=Y$.

We define the normal cone to E at the point y_0 as dual cone to $T_E(y_0)$

$$N_E(y_0)=\{y^*\in Y^*:<y^*,y>\leq 0\ \text{ at }\ y\in T_E(y_0)\}.$$

If $c\in X$, then denote $gra_{tc}=\{(x,y)\in X^2:y\in a_t(c,x)\}$.

It follows from the corollary of theorem 2.4.5 [3] that

$$T_{M_0}(\bar{\omega})=\{(x_1,x_2,...,x_T)\in X^T:\ x_1\in T_{a0(c(-\Delta),c(0))}(\bar{x}_1)\},$$

$$T_{M_t}(\bar{\omega})=\left\{(x_1,x_2,...,x_T)\in X^T:\ (x_t,x_{t+1})\in T_{gra_{t(c(t-\Delta))}}(\bar{x}_t,\bar{x}_{t+1})\right\},$$

$$t=\overline{1,\Delta},$$

$$T_{M_t}(\bar{\omega})=\{(x_1,x_2,...,x_T)\in X^T:\ (x_{t-\Delta},x_t,x_{t+1})\in T_{gra_t}(\bar{x}_{t-\Delta},\bar{x}_t,\bar{x}_{t+1})\},$$

$$t=\overline{\Delta+1,T-1},$$

$$T_{M_T}(\bar{\omega})=\{(x_1,x_2,...,x_T)\in X^T:\ x_T\in T_C(\bar{x}_T)\}.$$

Besides

$$N_{M_0}(\bar{\omega})=\{(x_1^*,0,...,0)\in X^{*T}:\ x_1^*\in N_{a0(c(-\Delta),c(0))}(\bar{x}_1)\},$$

$$N_{M_t}(\bar{\omega})=\Big\{(x_1^*,x_2^*,...,x_T^*)\in X^{*T}:\ \left(x_t^*,x_{t+1}^*\right)\in N_{gra_{t(c(t-\Delta))}}(\bar{x}_t,\bar{x}_{t+1}),$$
$$x_i^*=0\ \text{ at } i\neq t,t+1\ \},\quad t=\overline{1,\Delta},$$

$$N_{M_t}(\bar{\omega})=\{(x_1^*,x_2^*,...,x_T^*)\in X^{*T}:\ \left(x_{t-\Delta}^*,x_t^*,x_{t+1}^*\right)\in N_{gra_t}(\bar{x}_{t-\Delta},\bar{x}_t,\bar{x}_{t+1}),$$

$$x_i^*=0\ \text{ at } i\neq\{t-\Delta,t,t+1\}\ \}\ \text{ at }\ t=\overline{\Delta+1,T-1},$$
$$N_{M_T}(\bar{\omega})=\{(0,0,...,0,x_T^*)\in X^{*T}:\ x_T^*\in N_C(\bar{x}_T)\}.$$

Further denote by $\{\overline{x}_t\}_{t=-\Delta}^T$ an optimal solution of problem (1),(2).

The following corollary follows from the corollary of proposition 2.4.3 [3].

Corollary 1. *If g_t is Lipschitzian function at the neighbourhood of $\overline{x}_t,\ t=\overline{1,T}$, then*

$$0\in\partial f(\overline{\omega})+N_M(\overline{\omega}).$$

Corollary 2. *If the condition*

$$T_{M_0}(\overline{\omega}) \cap intT_{M_1}(\overline{\omega}) \cap intT_{M_2}(\overline{\omega}) \cap ... \cap intT_{M_T}(\overline{\omega}) \neq \varnothing$$

is satisfied, there exists at least one hypertangent to M_t, $t = \overline{1,T}$, *and* g_t *is Lipschitzian function at the neighbourhood* $\overline{x}_t$, *then* $0 \in \partial f(\overline{\omega}) \cap N_{M_0}(\overline{\omega}) + ... + N_{M_T}(\overline{\omega})$.

The proof of the corollary follows from corollary 1 and corollary 1 [4].

Note that when M_t, $t = \overline{1,T}$, are closed and $X = R^n$, then the existence condition of hypertangent is superfluous.

By proposition 2.3.3 [3] we have that $\partial f(\omega) \subset \sum\limits_{t=1}^{T} \partial g(\overline{x}_t, t)$. Therefore, provided corollary 2 we obtain

$$0 \in \sum_{t=1}^{T} \partial g(\overline{x}_t, t) + \sum_{t=0}^{T} N_{M_t}(\overline{\omega}). \tag{3}$$

It's clear that the vector $\omega^* \in \partial f(\omega)$ has the form $\omega^* = (x_{10}^*, ..., x_{T0}^*)$, where $x_{t0}^* \in \partial g(\overline{x}_t, t)$.

It follows from the definition of $N_{M_t}(\bar{\omega})$ that $\omega_t^* \in N_{M_t}(\bar{\omega})$ has the form

$$\omega_t^* = \left(0, ..., 0, x_t^*(t), x_{t+1}^*(t), 0, ..., 0\right), \ \left(x_t^*(t), x_{t+1}^*(t)\right) \in N_{gra_{t(c(t-\Delta))}}(\bar{x}_t, \bar{x}_{t+1}),$$

at $t = \overline{1, \Delta}$,

$$\begin{gathered} \omega_t^* = \left(0, ..., 0, x_{t-\Delta}^*(t), 0, ..., 0, x_t^*(t), x_{t+1}^*(t), 0, ..., 0\right), \\ \left(x_{t-\Delta}^*(t), x_t^*(t), x_{t+1}^*(t)\right) \in N_{gra_t}(\bar{x}_{t-\Delta}, \bar{x}_t, \bar{x}_{t+1}), \quad \text{at } t = \overline{\Delta+1, T-1}, \\ \omega_0^* = \left(x_1^*(0), 0, ..., 0\right), \quad x_1^*(0) \in N_{a0(c(-\Delta), c(0))}(\bar{x}_1), \\ \omega_T^* = \left(0, ..., 0, x_T^*(T)\right), \quad x_T^*(T) \in N_C(\bar{x}_T). \end{gathered}$$

Then it follows from (3) that there exist $x_{t0}^* \in \partial g(\overline{x}_t, t)$ at $t = \overline{1,T}$

$$\begin{gathered} \left(x_t^*(t), x_{t+1}^*(t)\right) \in N_{gra_{t(c(t-\Delta))}}(\bar{x}_t, \bar{x}_{t+1}) \quad \text{at} \quad t = \overline{1, \Delta}, \\ \left(x_{t-\Delta}^*(t), x_t^*(t), x_{t+1}^*(t)\right) \in N_{gra_t}(\bar{x}_{t-\Delta}, \bar{x}_t, \bar{x}_{t+1}) \quad \text{at} \quad t = \overline{\Delta+1, T-1}, \\ x_1^*(0) \in N_{a0(c(-\Delta), c(0))}(\bar{x}_1) \ \text{ and } \ x_T^*(T) \in N_C(\bar{x}_T) \end{gathered}$$

such that

$$\begin{gathered} (x_{10}^*, x_{20}^*, ..., x_{T0}^*) + (x_1^*(0), 0, ..., 0) + (x_1^*(1), x_2^*(1), 0, ..., 0) + \\ + (0, x_2^*(2), x_3^*(2), 0, ..., 0) + (0, 0, x_3^*(3), x_4^*(3), 0, ..., 0) + \\ + (0, 0, 0, x_4^*(4), x_5^*(4), 0, ..., 0) + ... + \\ + \left(0, 0, ..., 0, x_{\Delta-1}^*(\Delta-1), x_\Delta^*(\Delta-1), 0, ..., 0\right) + \\ + \left(0, 0, ..., 0, x_\Delta^*(\Delta), x_{\Delta+1}^*(\Delta), 0, ..., 0\right) + \\ + \left(x_1^*(\Delta+1), 0, ..., 0, x_{\Delta+1}^*(\Delta+1), x_{\Delta+2}^*(\Delta+1), 0, ..., 0\right) + \\ + \left(0, 0, x_2^*(\Delta+2), 0, ..., 0, x_{\Delta+2}^*(\Delta+2), x_{\Delta+3}^*(\Delta+2), 0, ..., 0\right) + \\ + \left(0, x_3^*(\Delta+3), 0, ..., 0, x_{\Delta+3}^*(\Delta+3), x_{\Delta+4}^*(\Delta+3), 0, ..., 0\right) + ... + \\ + \left(0, ..., 0, x_{T-2-\Delta}^*(T-2), 0, ..., 0, x_{T-2}^*(T-2), x_{T-1}^*(T-2), 0\right) + \\ + \left(0, ..., 0, x_{T-1-\Delta}^*(T-1), 0, ..., 0, x_{T-1}^*(T-1), x_T^*(T-1)\right) + \\ + (0, ..., 0, x_T^*(T)) = 0. \end{gathered}$$

Hence we obtain

$$\begin{gathered} x_{t0}^* + x_t^*(t-1) + x_t^*(t) + x_t^*(\Delta+t) = 0 \ \text{ at } t = \overline{1, T-1-\Delta}, \\ x_{t0}^* + x_t^*(t-1) + x_t^*(t) = 0 \ \text{ at } \ t = \overline{T-\Delta, T}. \end{gathered} \tag{4}$$

Denote

$$a_t^*(z^*;\nu_1) = \{(x^*, y^*) : (-x^*, -y^*, z^*) \in N_{gra_t}(\nu_1)\},$$
$$a_{tc(t-\Delta)}^*(y^*;\nu_2) = \left\{x^* : (-x^*; y^*) \in N_{gra_{tc(t-\Delta)}}(\nu_2)\right\}.$$

Then we can write relation (4) in the following form

1) $x_{t0}^* + x_t^*(t-1) + x_t^*(\Delta + t) \in a_{t(c(t-\Delta))}^*\left(x_{t+1}^*(t);\ (\bar{x}_t, \bar{x}_{t+1})\right)$ at $t = \overline{1,\Delta}$,

2) $\left(-x_{t-\Delta}^*(t),\ x_{t0}^* + x_t^*(t-1) + x_t^*(\Delta + t)\right) \in a_t^*\left(x_{t+1}^*(t); (\bar{x}_{t-\Delta}, \bar{x}_t, \bar{x}_{t+1})\right)$ at $t = \overline{\Delta + 1, T - 1 - \Delta}$,

3) $\left(-x_{t-\Delta}^*(t),\ x_{t0}^* + x_t^*(t-1)\right) \in a_t^*\left(x_{t+1}^*(t); (\bar{x}_{t-\Delta}, \bar{x}_t, \bar{x}_{t+1})\right)$ at $t = \overline{T - \Delta, T - 1}$,

4) $x_{T0}^* + x_T^*(T-1) + x_T^*(T) = 0$.

Theorem 1. *Let $a_t : X^2 \to 2^X$ be a multivalued mapping, $g(x,t)$ be functions with respect to x at $t = 1, ..., T$, $x(t) = c(t), (\ t = -\Delta, ..., -1, 0)$ be fixed vectors, $\bar{x}_t$ be an optimal trajectory and the following condition be satisfied*

$$T_{M_0}(\overline{\omega}) \cap intT_{M_1}(\overline{\omega}) \cap intT_{M_2}(\overline{\omega}) \cap ... \cap intT_{M_T}(\overline{\omega}) \neq \varnothing, \tag{5}$$

there exists at least one hypertangent to M_t, $t = \overline{1,T}$, the function $g(x,t)$ satisfies the Lipschithz condition with respect to x at the neghbourhood of $\bar{x}_t$.

Then in order that the trajectory $\{\bar{x}_t\}_{t=-\Delta}^T$ beginning from $\bar{x}(t) = c(t)$, $t = -\Delta, ..., -1, 0$, and finishing at $\bar{x}_T \in C$, minimize the function $f(\omega) = \sum\limits_{t=1}^{T} g(x_t, t)$ at all admissible trajectories, it's necessary that there would be found the vectors $x_t^*(t-1)$, $t = \overline{1,T}$, $x_s^*(\Delta + s)$, $s = \overline{1, T - 1 - \Delta}$ and $x^* \in N_C(\bar{x}_T)$ that

1) $x_1^*(0) \in N_{a0(c(-\Delta),c(0))}(\bar{x}_1)$,

2) $x_t^*(t-1) + x_t^*(t+\Delta) \in a_{t(c(t-\Delta))}^*\left(x_{t+1}^*(t);\ (\bar{x}_t, \bar{x}_{t+1})\right) - \partial g(\overline{x}_t, t)$, $t = \overline{1,\Delta}$,

3) $\left(-x_{t-\Delta}^*(t),\ x_t^*(t-1) + x_t^*(\Delta + t)\right) \in a_t^*\left(x_{t+1}^*(t); (\bar{x}_{t-\Delta}, \bar{x}_t, \bar{x}_{t+1})\right) - (0, \partial g(\overline{x}_t, t))$, $t = \overline{\Delta + 1, T - 1 - \Delta}$,

4) $\left(-x_{t-\Delta}^*(t), x_t^*(t-1)\right) \in a_t^*\left(x_{t+1}^*(t); (\bar{x}_{t-\Delta}, \bar{x}_t, \bar{x}_{t+1})\right) - (0, \partial g(\overline{x}_t, t))$, $t = \overline{T - \Delta, T - 1}$,

5) $-x_T^*(T-1) - x^* \in \partial g(\overline{x}_T, T)$.

Denoting $E_1 = T_{a0(c(-\Delta),c(0))}(\bar{x}_1)$, $grb_t = intT_{gra_t(c(t-\Delta),\cdot)}(\bar{x}_t, \bar{x}_{t+1})$, $t = \overline{1,\Delta}$, $grb_t = intT_{gra_t}(\bar{x}_{t-\Delta}, \bar{x}_t, \bar{x}_{t+1})$, $t = \overline{\Delta + 1, T - 1}$, $E_2 = intT_C(\bar{x}_T)$ consider the inclusion

$$\begin{gathered} z_{t+1} \in b_t(z(t-\Delta), z_t),\ t = \overline{\Delta + 1, T - 1}, \\ z_{t+1} \in b_t(z_t),\ t = \overline{1,\Delta}, \\ z_1 \in E_1,\ z_T \in E_2. \end{gathered} \tag{6}$$

Theorem 2. *Let $a_t : R^{2n} \to 2^{R^n}$, $t = 0, 1, ..., T - 1$, be a multivalued mapping, C and gra_t be closed sets, $g(x,t)$ be functions with respect to x at $t = 1, ..., T$, $x(t) = c(t), (\ t = -\Delta, ..., -1, 0)$ be fixed vectors, there exist a solution of system of inclusions (6), the function $g(x,t)$ satisfy the Lipschithz condition with respect to x at the neighbourhood of $\bar{x}_t$.*

Then in order that the trajectory $\{\bar{x}_t\}_{t=-\Delta}^T$ beginning from $\bar{x}(t) = c(t)$, $t = -\Delta, ..., -1, 0$, and finishing at $\bar{x}_T \in C$ minimize the function $f(\omega) = \sum\limits_{t=1}^{T} g(x_t, t)$

on all admissible trajectories, it is necessary that there would be found such vectors $x_t^*(t-1)$, $t=\overline{1,T}$, $x_s^*(\Delta+s)$, $s=\overline{1,T-1-\Delta}$ and $x^*\in N_C(\bar{x}_T)$ that

1) $x_1^*(0)\in N_{a_0(c(-\Delta),c(0))}(\bar{x}_1)$,

2) $x_t^*(t-1)+x_t^*(t+\Delta)\in a^*_{t(c(t-\Delta))}\left(x_{t+1}^*(t);\ (\bar{x}_t,\bar{x}_{t+1})\right)-\partial g(\bar{x}_t,t)$, $t=\overline{1,\Delta}$,

3) $\left(-x_{t-\Delta}^*(t),\ x_t^*(t-1)+x_t^*(\Delta+t)\right)\in a_t^*\left(x_{t+1}^*(t);(\bar{x}_{t-\Delta},\bar{x}_t,\bar{x}_{t+1})\right)-(0,\partial g(\bar{x}_t,t))$, $t=\overline{\Delta+1,T-1-\Delta}$,

4) $\left(-x_{t-\Delta}^*(t),x_t^*(t-1)\right)\in a_t^*\left(x_{t+1}^*(t);(\bar{x}_{t-\Delta},\bar{x}_t,\bar{x}_{t+1})\right)-(0,\partial g(\bar{x}_t,t))$, $t=\overline{T-\Delta,T-1}$,

5) $-x_T^*(T-1)-x^*\in\partial g(\bar{x}_T,T)$.

The validity of theorem 2 follows from theorem 1.

Theorem 3. *Let $a_t:R^{2n}\to 2^{R^n}$, $t=0,1,...,T-1$, be a multivalued mapping, C and gra_t be closed convex sets, the function $g(x,t)$ satisfy the Lipscithz condition with respect to x at the neighbourhood of $\bar{x}_t$, $t=\overline{1,T}$, where $\{\bar{x}_t\}_{t=-\Delta}^T$, $\bar{x}(t)=c(t)$, $t=-\Delta,...,-1,0$, is a solution of inclusion (1) $\bar{x}_T\in C$.*

Then for optimality of the trajectory $\{\bar{x}_t\}_{t=-\Delta}^T$ it's necessary that there be found the vectors $x_t^*(t-1)$, $t=\overline{1,T}$, $x_t^*(t+\Delta)$, $t=\overline{1,T-1}$, $x^*\in N_C(\bar{x}_T)$ and the number λ equal to zero or 1 such that

1) $x_1^*(0)\in N_{a_0(c(-\Delta),c(0))}(\bar{x}_1)$,

2) $x_t^*(t-1)+x_t^*(t+\Delta)\in a^*_{t(c(t-\Delta))}\left(x_{t+1}^*(t);\ (\bar{x}_t,\bar{x}_{t+1})\right)-\lambda\partial g(\bar{x}_t,t)$, $t=\overline{1,\Delta}$,

3) $\left(-x_{t-\Delta}^*(t),\ x_t^*(t-1)+x_t^*(\Delta+t)\right)\in a_t^*\left(x_{t+1}^*(t);(\bar{x}_{t-\Delta},\bar{x}_t,\bar{x}_{t+1})\right)-(0,\lambda\partial g(\bar{x}_t,t))$, $t=\overline{\Delta+1,T-1}$,where $x_t^*(t+\Delta)=0$, at $t=\overline{T-\Delta,T-1}$,

4) $-x_T^*(T-1)-x^*\in\lambda\partial g(\bar{x}_T,T)$.

Using corollary 1 and the expression N_{M_t}, $t=\overline{0,T}$, the theorem is proved analogously to theorem 4.2.4 [5].

Condition A. Assume that the condition

$$T_{M_t}(\bar{\omega})-T_{\bigcap\limits_{s=0}^{t-1}M_s}(\bar{\omega})=R^{nT},\quad t=1,2,3,...,T$$

is satisfied.

Theorem 4. *Let $X=R^n$, $a_t:R^{2n}\to 2^{R^n},C\subset R^n$ and gra_t be closed sets, the function $g(x,t)$ satisfy the Lipshcithz condition with respect to x at the neighbourhood of $\bar{x}_t$, $t=\overline{1,T}$, where $\{\bar{x}_t\}_{t=-\Delta}^T$, $\bar{x}(t)=c(t)$, $t=-\Delta,...,-1,0$, is a solution of inclusion (1), $\bar{x}_T\in C$ and condition A be satisfied. Then for the optimality of the trajectory $\{\bar{x}_t\}_{t=-\Delta}^T$ it's necessary that there would be found the vectors $x_t^*(t-1)$, $t=\overline{1,T}$, $x_t^*(t+\Delta)$, $t=\overline{1,T-1}$, $x^*\in N_C(\bar{x}_T)$ such that*

1) $x_1^*(0)\in N_{a_0(c(-\Delta),c(0))}(\bar{x}_1)$,

2) $x_t^*(t-1)+x_t^*(t+\Delta)\in a^*_{t(c(t-\Delta))}\left(x_{t+1}^*(t);\ (\bar{x}_t,\bar{x}_{t+1})\right)-\partial g(\bar{x}_t,t)$, $t=\overline{1,\Delta}$,

3) $\left(-x_{t-\Delta}^*(t),\ x_t^*(t-1)+x_t^*(t+\Delta)\right)\in a_t^*\left(x_{t+1}^*(t);(\bar{x}_{t-\Delta},\bar{x}_t,\bar{x}_{t+1})\right)-(0,\partial g(\bar{x}_t,t))$, $t=\overline{\Delta+1,T-1}$,*where* $x_t^*(t+\Delta)=0$ *at* $t=\overline{T-\Delta,T-1}$,

4) $-x_T^*(T-1)-x^*\in\partial g(\bar{x}_T,T)$.

Proof. If condition A is satisfied, then applying corollary 7.6.5 and theorem 4.1.6 [6] by the induction method we have that $\bigcap\limits_{t=0}^{T}T_{M_t}(\bar{\omega})\subset T_{\bigcap\limits_{t=0}^{T}M_t}(\bar{\omega})$and $N_{\bigcap\limits_{t=0}^{T}M_t}(\bar{\omega})\subset$

$\bigcap_{t=0}^{T} N_{M_t}(\bar{\omega})$. Further, the proof of theorem 4 is similar to the proof of theorem 1. The theorem is proved.

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