Mehrali O.YUSIFOV

FRICTION EFFECT UPON THE NONLINEAR VIBRATIONS AMPLITUDE OF STRUCTURAL ELEMENTS CONTACTING WITH MEDIUM USING WINKLER'S NONLINEAR MODEL

Abstract

In the paper we study friction effect upon the nonlinear vibration amplitude of structural elements contacting with medium. Under great loads the points of piles perform wide range of permutations, i.e. it is appropriate to calculate the piles within geometrically nonlinear theory. Analysis of loads affecting upon single piles shows that deflection is the prevalent component of permutation vector of pile's point. We use Winkler's nonlinear model to describe soils effect upon the behavior of a pile.

Under great loads the pile's points perform wide range permutations, i.e. it is appropriate to calculate piles within geometrically nonlinear theory. Analysis of loads affecting upon the single piles shows that deflection is the prevalent component of permutation vector of piles point. To describe the soil's effect on behavior of a pile we use Winkler's model [1]. In this case intensity of force of soil upon a pile q is accepted in the form:

$$q = kW(1 - k_1 W)(1 - k_2 W), \tag{1}$$

where k is bed's factor, $k_i\,(i = 1, 2)$ are the factors charactering the soil behavior and moreover at $k_i = 0\,(i = 1, 2)$ we get a linear model [2], W is deflection of bar. The pile is simulated by a linear bar.

Assume that the pile is compressed by the force $N(t)$ and there is adhesion between the pile and soil. Within a linear problem the quantity u longitudinal permutation is found independently of the deflection w. For the nonlinear problem the quantity u -depends on deflection that varies by time. In this case longitudinal permutations must also depend on time. Therefore, there arises necessity of account of inertia forces along the axis X, i.e. unlike linear case, account of transverse vibrations to necessity of account of longitudinal vibrations leads to necessity of account of longitudinal vibrations. Study this problem by the variation method. The functional for calculating the pile in this case will be of the form [1,2]:

$$J = J_1 + \int_0^{\pi}\int_0^{L} \times$$

$$\times \left(\frac{1}{2}k_x u^2 - fukW\left(1 - k_1 W\right)\left(1 - k_2 W\right)W - \frac{1}{2}\rho F\left(\frac{\partial u}{\partial t}\right)^2\right)dxdt,$$

where

$$J_1 = \int_0^T \int_0^L \left\{ N \left[\frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 \right] - M\frac{\partial^2 W}{\partial x^2} - \frac{1}{2E}\left(\frac{N^2}{F} + \frac{M^2}{J}\right) + \right. \tag{1}$$

$$\left. + \frac{1}{2}k\left(1 - k_1 W\right)\left(1 - k_2 W\right)W^2 - \frac{1}{2}\rho F\left(\frac{\partial W}{\partial t}\right)^2 \right\} dxdy +$$

$$+ \int_0^T \left[-\rho F u + N\frac{\partial W}{\partial x}W - M\frac{\partial W}{\partial x} + \frac{\partial M}{\partial x}W \right] dt\,|_{x=0} +$$

$$+ \int_0^T \left[-Nu - N\frac{\partial W}{\partial x}W + M_0 \sin\omega_0 t\frac{\partial W}{\partial x} - \frac{\partial M}{Sdx}W \right] dt\,|_{x=L},$$

where N and M are longitudinal force and moments, respectively, L is the length of the pile, f is a friction factor, k is proportionality factor. u, W, N and M are varying quantities. Having varied the given functional with respect to u and taking into account that $N = -PF$, instead of J_1, we have:

$$J_1 = \int_0^T \int_0^L \left\{ -\rho F\frac{1}{2}\left(\frac{\partial W}{\partial x}\right)^2 - M\frac{\partial^2 W}{\partial x^2} - \frac{1}{2EJ}M^2 + \right.$$

$$\left. + \frac{1}{2}k\left(1 - k_1 W\right)\left(1 - k_2 W\right)W^2 - \frac{1}{2}\rho F\left(\frac{\partial W}{\partial t}\right)^2 \right\} dxdt +$$

$$+ \int_0^T \left[-\rho F\frac{\partial W}{\partial x}W - M\frac{\partial W}{\partial x} + \frac{\partial M}{\partial x}W \right] dt\,|_{x=0} +$$

$$+ \int_0^T \left[PF\frac{\partial W}{\partial x}W + M_0 \sin\omega_0 t\frac{\partial W}{\partial x} - \frac{\partial M}{\partial x}W \right] dt\,|_{x=L} + J_0,$$

where varying quantities are W, M and J_0 is an integral not affecting upon stationary value J_1. Stationary value of the functional J will be found by the Ritsz method. Proceeding from solution of the linear problem we take approximation of varying quantities in the following form:

$$W = W_0 \sin\frac{\pi x}{L}\sin\omega_0 t; M = EJM_0 \sin\omega_0 t\frac{x}{L} + m_0 \sin\frac{\pi x}{L}\sin\omega_0 t;$$

$$\text{and } u = u_0 x + u_1 \sin\omega_0 t \cdot \sin\frac{\pi x}{L} \tag{2}$$

u_0, u_1, ω_0 and M_0 are the desired quantities.

Consider approximation (2) in functional (1), having adopted $T = \dfrac{\pi}{\omega_0}$ we get:

$$J = \frac{L}{\omega_0 \pi}\left\{ \frac{1}{2}PF\frac{\pi^4}{L^2}W_0^2 + \left(EJM_0 + \frac{\pi}{\partial x 2}m_0\right)\frac{\pi^3}{L}W_0 - \frac{1}{2EJ} \times \right.$$

$$\times\left(E^2J^2M_0^2\frac{\pi^2}{6}+\pi EJM_0m_0+\frac{\pi^2}{4}m_0^2\right)+\frac{\pi^2kW_0^2}{8}-\frac{\pi^2}{8}\rho F\omega_0^2W_0^2\Bigg\}-$$

$$-\frac{EJ}{\omega_0}\frac{\pi}{2L}M_0W_0+\frac{L}{\omega_0\pi}\left\{\frac{1}{6}k_xu_0^2L^2\pi^2-\frac{1}{4\pi^2}k_xu_1^2-\frac{1}{4\pi^2}kfu_1W_0-\right.$$

$$\left.-\left(k_1+k_2\right)u_1W_0-\frac{1}{8\pi^2}\rho Fu_1^2\omega_0^2-2k_xLu_0u_1-2kfLu_0W_0\right\}.$$

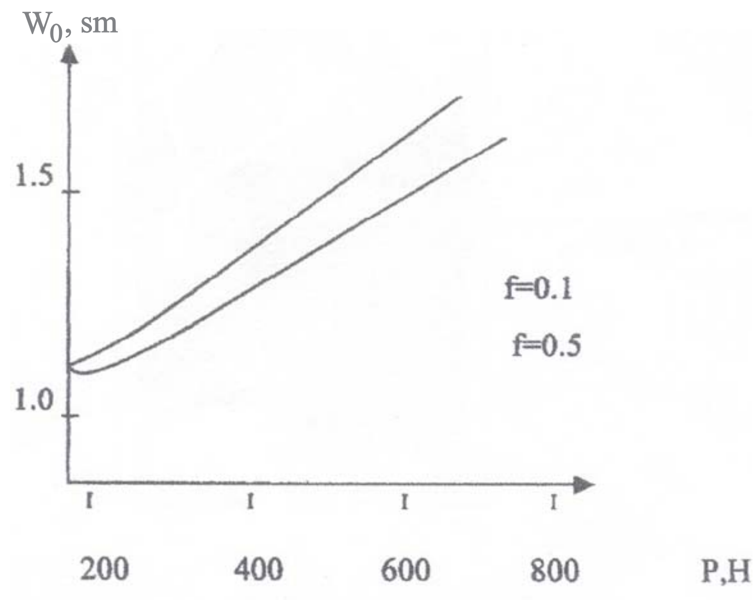

Fig.1. Dependence of deflection of the pile on longitudinal load.

Stationary value of the obtained function is determined from the following system:

$$\frac{\partial J}{\partial m_0}=\frac{\pi^3}{2L}W_0-\frac{1}{2EJ}\left(\pi EJM_0+\frac{\pi^2}{2}m_0\right)=0$$

$$\frac{\partial J}{\partial W_0}=-PF\frac{\pi^4}{L^2}W_0+\left(EJM_0+m_0\frac{\pi}{2}\right)\cdot\frac{\pi^3}{L}+\frac{\pi^2}{4}kW_0-$$

$$-\frac{EJ\pi^3}{2L^2}M_0-\frac{1}{4\pi^2}kfu_1-2kfLu_0=0. \qquad (3)$$

$$\frac{\partial J}{\partial u_0}=\frac{1}{3}k_xu_0L^2\pi^2-2k_xLu_1-2kfLW_0=0,$$

$$\frac{\partial J}{\partial u_1}=-\frac{1}{2\pi^2}k_xu_1-\frac{1}{4\pi^2}kfW-\frac{1}{4\pi^2}\rho Fu_1\omega_0^2-2k_xLu_0=0.$$

From this system we determine the dependence of deflection from the parameters of the problem. Within Winker's linear model the characteristic dependence is of the form:

$$W_0 = \frac{\frac{\pi^3}{2L^2}M_0}{-PF\frac{\pi^2}{L^2} + EJ\frac{\pi^4}{L^4} - k - \rho F\omega_0^2 + kfQ}.$$

Notice that the expression for Q is easily obtained from the last two equations of system (3).

Dependence of W_a on effective force P for different values of friction factor f is represented in fig.1. For the other parameters the followings were adopted:

$$k_x = 2\cdot 10^7\frac{H}{m^2};\quad k = 6\cdot 10^7\frac{H}{m^2};\ E = 2\cdot 10^{11}\frac{H}{m^2};\ \upsilon = 0,3;$$

$$\omega_0 = 200\ c^{-1};\ M_0 = 400\ Hm;\ L = 3m;\ F = 270sm^2,$$

$$k_1 = 0,01, k_2 = 003.$$

It is seen from fig. 1 that deflection of the pile decreases due to increase of friction factor.

References

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[2]. Panovko Ya.G. *Bases of applied theory of vibrations and shock.* Politekhnika, 1990, 272 p. (Russian)

Mehrali O.Yusifov
Azerbaijan State Pedagogical University
34, U.Hajibeyov str., AZ1001, Baku, Azerbaijan
Tel.: (99412) 564 19 87 (apt.)

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