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ON A FORMULA OF SPECTRAL DENSITY CHARACTERIZING THE STATIONARY SEA-WAY EFFECT ON OFFSHORE STRUCTURES ELEMENTS

Abstract

Analytic formula is suggested and substantiated for spectral density characterizing stationary process of wave loading of submerged cylindrical element of offshore structures.

It is known that sea waves are irregular (random). Strength analyses of offshore structures become more certain when they are conducted with regard to irregularity of sea ways. Sea-way is characterized by height over the level of the queite surface. It is assumed that the funtion η randomly depends on the coordinate x and time t. It is represented as follows [1]:

$$\eta(x,t) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_\eta(k,\omega)\, e^{i(kx+\omega t)} dk d\omega. \tag{1}$$

Here $G_\eta(k,\omega)$ is a random spectrum; k is wave number; ω is sea-way frequency; $i^2 = -1$. Wave number k, depending on wave, is connected with frequency ω by different relations. For example, for small amplitude waves on the surface of deep fluid $\omega^2 = gk$, where g is free fall acceleration.

On the basis of formula (1) pressure and stress of submerged metallic cylindrical elements may also be represented in relations similar to formula (1). This indicates that sea-way and also pressure and stress connected with wave effect may be characterized by temporary spectral densities.

Let a submerged cylindrical element be subjected to the effect of small amplitude waves appearing on the surface of deep fluid. The ends of the cylindrical element were secured from linear permutations and rotations. The element is not filled with water. External pressure P acting on a cylindrical element consists of a sum of two pressures p_1 and $p_2 : P = p_1 + p_2$, where p_1 is hydrostatic pressure, p_2 is pressure related with wave effect. Fatigue failure of cylindrical elements at the nodes of their connection with basic column occurs under the action of external pressure. Stress state at connection nodes of cylindrical element beyond the elasticity also consists of two constituents s_0 and s_a, where s_0 is determined stress stipulated by hydrostatic stress. Let s_a be random stress arising from sea-way under the action of random cyclic loading. We shall consider that loading process s_a is a stationary process. Usually for the process s_a the normal distribution law is accepted. It s_a and s_a are distributed by normal law and statically independent, then density probability for the random amplitude s_a is subjected to Rayleigh distribution law [2]:

$$p(s_a) = \frac{s_a}{\sigma_s^2} \exp\left(-\frac{s_a^2}{2\sigma_s^2}\right), \quad (s_a \geq 0), \tag{2}$$

where σ_s^2 is dispersion of random process s_a.

Analysis of statistical date countained in the papers [3,4] shows that the following formula

$$\Phi_{s_a}(\omega) = C\gamma^2 g^3 \nu^{-1} \omega^{-6} \exp\left(-\frac{g^2 f^2\left(b, \frac{y}{h}\right)}{\omega^2 \nu^2}\right), \tag{3}$$

where

$$f\left(b, \frac{y}{h}\right) = b\left(1 - \frac{y}{h}\right) + 2\frac{y}{h} \tag{4}$$

may be suggested as an analytic expression of spectral density of stationary random process s_a.

Besides, ω is frequency of stationary random process s_a; h is sea-water depth, y is the distance of cylindrical element of offshore structure from sea botton, γ is specific weight of water; g is free fall acceleration; ν is wind velocity; C and b are constants that are determined from the condition that dispersion and effective period (mean duration of a cycle or loading block) were equal to the given determined quantities-stress and ordinary period, that correspond to some over developed, seldomly met sea-way in the given water-area.

As is known, wind velocity is a random variable. However, at the absence of statistical data, wind velocity ν may be replaced by its mathematical expectation: $\langle \nu \rangle$. Let's use this remark. And omit corner brackets.

To ground formula (3) we use the following non-standard approach. This approach is in comparison of results of prediction of the same cylindrical element of offshore structure by the stochastic method and appropriate deterministic method:

We use the table taken from [3]:

Table [3]. Data on analysis of service life of an element.

H,m	T,s	c,\%	s,MPa
12-18	12	0,01	35
6-12	10	0,03	20
3-6	9	0,10	10
1,5-3	7	1,00	4
0-1,5	3	98,86	2,7

In the table the heights H and appropriate periods of T waves functioning at the place of offshore structure installation are cited. And here the quantities c during which different range waves act are reduced at percents from general time of structure operation. Amplitudes of the stress s that the considered cylindrical structures element has under the action of appropriate wave range [3] when $h = 60\ m$, $y = 20\ m$, $\gamma = 10kH/m^3$. The Palmgren- Miner method [3] was used by defining service life of the cylindrical element:

$$\sum_{i=1}^{m} \frac{c_i}{N_i T_i} = \frac{1}{T}, \tag{5}$$

here T is the service life of the considered element to fatigue failure, $c_i = \dfrac{n_i T_i}{T}$ are the parts of service life under which cyclic change of stresses with amplitudes s_i happens;

n_i and T_i is the number and period of cyclic change of stresses with amplitude s_i, respectively; N_i is the number of cycles to fatigue failure with amplitude of stresses s_i, N_i are determined by the fatigue curve whose approximation may be performed by the formula

$$N\left(s_0, s_a\right) = N_0 e^{\psi_1\left[1-\left(s_a^2+\psi_2 s_0^2\right)/\ r^2\right]}, \quad \left(0 < s_a < \infty\right), \tag{6}$$

where N_0, ψ_1, ψ_2 are the constants that are defined by the forms of fatigue curves. The values $N_0, \psi_1, \psi_2 > 0$; r is the fatigue range of the material. According our calculations, by using fatigue curve for structural steel used in offshore structures $N_0 = 10^8;\ \psi_1 = 2,4;\ \psi_2 = 0,05;\ r = 7MPa.$

By determining service life of the considered cylindrical element for accounting concentration of stresses or dynamic character of interaction process that is characteristic for some periods of waves, nominal stress amplitudes were multiplied by the coefficient $k_\sigma = 2$. According to calculations conducted by formula (5) allowing for (6) at the above-mentioned numerical data the quantity T became equal $6,78 \cdot 10^8 s$ or $21,5$ years.

Now let's perform analytic calculations by using static method [2]. To determine the expected service life of the considered cylindrical element let's use the formula [2]:

$$T = T_e\left[\int\limits_0^\infty \frac{p\left(s_a\right) ds_a}{N\left(s_0, s_a\right)}\right]^{-1}, \tag{7}$$

where $p\left(s_a\right)$ is the distribution density of the process s_a; T_e is the effective period of the process s_a; $N = N\left(s_0, s_a\right)$ is the function by which the fatigue curve approximation is performed.

By using (2) and (6) in (7) we'll have:

$$T = T_e N_0 e^{\psi_1\left(1-\psi_2\frac{s_0^2}{r^2}\right)}\left[\frac{\frac{1}{2}\left(\frac{r}{\sigma_s}\right)^2 - \psi_1}{\left(\frac{r}{\sigma_s}\right)^2}\right]. \tag{8}$$

By deriving (8) it was assumed that it holds the inequality $\sigma_s < r\sqrt{2\psi_1}$.

Now let's cite concrete calculation. Let's use the above-cited numerical data. In this case $p_1 = 0,6MPa$. At this the value of stress intensity s_0 for Poisson coefficient $0,3$ was: $s_0 = 20,8\ MPa$. In this case we have $\left(\frac{s_0}{r}\right)^2 = 8,83$. Let's determine the dispersion σ_s^2 and effective period T_e. These quantities are defined by the by the following formulae [2]:

$$\sigma_s^2 = \int\limits_0^\infty \Phi_{s_a}\left(\omega\right) d\omega; \quad T_e = 2\pi\frac{\sigma_s}{\sigma_{\dot{s}}}; \quad \sigma_{\dot{s}}^2 = \int\limits_0^\infty \Phi_{s_a}\left(\omega\right)\omega^2 d\omega. \tag{9}$$

Take into account (3) in relations (9). We'll have

$$\sigma_s^2 = \frac{3\sqrt{\pi}}{8} \frac{C\gamma^2\nu^4}{g^2 f^5\left(b, \frac{y}{h}\right)}; \quad T_e = \frac{\sqrt{6}\pi\nu}{gf\left(b, \dfrac{y}{h}\right)}. \tag{10}$$

Let's determine the quantities C and b contained in formulae (10). According to the above-mentioned table maximal stress of the element holds under the action of the wave of height $12-18$ m and of period 12 c. Accept $\sigma_{s_a}^{\max} = 14,2$ MPa; $T_{\max} = 12$ c corresponds to the above-mentioned height and period $\nu_{\max} = 90\dfrac{m}{c}$. By taking into account these data in (10) we'll have: $b = 7,655$; $f\left(b, \dfrac{1}{3}\right) = 5,77$; $C = 2,96 \cdot 10^4$. Using the obtained data and accepting $\nu = 30$ m/c, we determine σ_s and T_e : $\sigma_s \approx 1,58$ MPa; $T_s = 4c$ by formula (10). And on the base of formula (8) we get: $T = 6,25 \cdot 10^8 s$ or 19,8 years. Different of values of service lives obtained by deterministic and stochastic methods is $7,7\%$. Now according to the table mentioned above we can see that stress distribution by wave ranges may be approximately represented by Rayleigh distribution (2) for $\sigma_s = 1,58$ MPa. The correspondence of the values of the quantity T obtained on the basis of stochastic and deterministic approaches indicates on acceptability of the suggested formula (3) for the spectral density of the random process s_a.

References

[1]. Bolotin V.V. *Application of the methods of probability theory and reliability theory in design of structures.* M.: "Stroyizdat", 1971, 255 p.

[2]. Bolotin V.V. *Statistical methods in structural mechanics.* M.: "Stroyizdat", 1965, 279 p.

[3]. Dawson T.N. *Offshore structural engineering.* New Jersey: "Englewood Cliffs", 1983, 288 p.

[4]. Maddox N.R., Wildenstein A.N. *A spectral fatigue analysis for offshore structures.* Proc. VII Annual Offshore Technology Conference, 1975, pp.185-193.

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