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STABILITY OF CYLINDRICAL SHELLS, STRENGTHENED BY THE ANNULAR RIBS AND FILLED WITH MEDIUM AT STATIC EXTERNAL PRESSURE

Abstract

The given paper is devoted to research of stability of cylindrical shells, strengthened by the annular ribs and filled with elastic medium at uniform external pressure. Taking the shell as constructively-orthotropic, applying the asymptotic method, the formulae for parameter of the critical stresses are obtained. The analysis of influence of parameters of the external medium of the parameter of the critical stresses is done.

Strengthened cylindrical shells are widely used as a supporting structural elements of constructions of modern technology. At working conditions they are in contact with different media. To design models of strengthened cylindrical shells, filled with medium, for example, different capacities and pipe lines, the constructions of the special setting and etc. are reduced. Therefore development of theory and methods of stability analysis of strengthened cylindrical shells with regard to external influences is an actual problem of great practical value. In literature the description of solution concerns mainly to the strengthened cylindrical shell without medium [1,2]. The stability problems of such constructions with medium practically have not been studied. Note, that the stability of smooth cylindrical shells with filler is sufficiently completely investigated in the papers [3,4 and etc.].

The given paper is devoted to research of stability of cylindrical shells, strengthened by annular ribs and filled with the elastic medium at uniform external pressure. Taking the shell as a constructively-orthotropic, applying the asymptotic method, the formulae for parameter of the critical stresses are obtained. The analysis of influence of parameters of the external medium on parameter of the critical stresses is carried out.

The stability equations of constructively-orthotropic shell, equivalent to the she shell, stregthened by annular ribs and filled with medium has the form [1]:

$$\left(\frac{\partial^2}{\partial\xi^2}+\frac{1-\nu}{2}\frac{\partial^2}{\partial\theta^2}\right)u+\frac{1+\nu}{2}\frac{\partial^2\upsilon}{\partial\xi\partial\theta}-\nu\frac{\partial}{\partial\xi}w=\frac{R^2\left(1-\nu^2\right)}{Eh}q_x$$

$$\frac{1+\nu}{2}\frac{\partial^2 u}{\partial\xi\partial\theta}+\left\{\frac{1-\nu}{2}\left(1+4a^2\right)\frac{\partial^2}{\partial\xi^2}+\right.$$

$$+\left[1+\left(1-\frac{h_s}{R}\right)^2\gamma_s^{(2)}+a^2\right]\frac{\partial^2}{\partial\theta^2}\right\}\upsilon+$$

$$+\left\{-\left[1+\left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]\frac{\partial}{\partial\theta}+(2-\nu)\,a^2\frac{\partial^3}{\partial\xi^2\partial\theta}+\right.$$

$$\left.+\left[a^2-\left(1-\frac{h_s}{R}\right)\delta_s^{(2)}\right]\frac{\partial^3}{\partial\theta^3}\right\}w=\frac{R^2\left(1-\nu^2\right)}{Eh}q_\theta. \tag{1}$$

$$-\nu\frac{\partial u}{\partial\xi}+\left\{-\left[1+\left(1-\frac{h_s}{R}\right)\gamma_s^{(2)}\right]\frac{\partial}{\partial\theta}+(2-\nu)\,a^2\frac{\partial^3}{\partial\xi^2\partial\theta}+\right.$$

$$\left.+\left[a^2-\left(1-\frac{h_s}{R}\right)\delta_s^{(2)}\right]\frac{\partial^3}{\partial\theta^3}\right\}\upsilon+$$

$$+\left[1+\gamma_s^{(2)}+\eta_{s1}^{(2)}+2\left(\delta_s^{(2)}+\eta_{s1}^{(2)}\right)\frac{\partial^2}{\partial\theta^2}+a^2\Delta\Delta+\right.$$

$$\left.+\left(\eta_{s1}^{(2)}+\eta_{s2}^{(2)}\right)\frac{\partial^4}{\partial\theta^4}+\bar{q}\left(\frac{\partial^2}{\partial\theta^2}+1\right)\right]w=$$

$$=\frac{R^2\left(1-\nu^2\right)}{Eh}q_z.$$

Here $\bar{\gamma}_s^{(2)}=\frac{F_s}{L_{1h}}\left(1+k_1\right),\ \xi=\frac{x}{R},\ \ \theta=\frac{y}{R},\ \ \delta_s^{(2)}=\frac{h_s}{R}\bar{\gamma}_s^2,$
$\eta_{s2}^{(2)}=\dfrac{E_s\left(1-\nu^2\right)}{E}\bar{\eta}_s^{(2)},\ \ \eta_{s1}^{(2)}=\dfrac{E_sJ_{xs}\left(1-\nu^2\right)\left(1+k_1\right)}{EL_1R^2h},$
$\bar{\eta}_s^{(2)}=\left(\frac{h_s}{R}\right)^2\bar{\gamma}_s^{(2)},\ a^2=\dfrac{h^2}{12R^2},\ \Delta=\dfrac{\partial^2}{\partial\xi^2}+\dfrac{\partial^2}{\partial\theta^2},$
$\gamma_s^{(2)}=\dfrac{E_s\left(1-\nu^2\right)}{E}\bar{\gamma}_s^{(2)},\ \bar{q}=\dfrac{\sigma_y\left(1-\nu^2\right)}{E},$ F_s is an area of cross-section of the ring, L_1 is a length of the shell, R is a radius of the shell cover, h is a thickness of the shell, k_1 is quantity of rings, E_s, E are modules of elasticity of ring and shell, respectively, ν is the Poisson coefficient, σ_y is a stress, J_{xs} is a moment of inertia of cross-section of the ring with respect to axis $ox,, u, v, w$ are components of vector of points displacements of surface of the shell, q_x, q_θ, q_z are components of the pressure vector from medium to the shell.

The balance equations of medium in the vector form has the form [5]:

$$a_e^2qraddiv\vec{s}-a_t^2rotrot\vec{s}=0 \tag{2}$$

Here $a_t\sqrt{\dfrac{\lambda+2\mu}{\rho}},\quad a_e=\sqrt{\dfrac{\mu}{\rho}}$ are wave velocities of longitudinal and transverse waves in the medium respectively; $\vec{s}\left(s_xs,\theta\right),s_z$ is a displacement vector, λ,μ are Lame coefficients. The systems of balance equations of shell (1) of medium (2) are complemented by contact conditions. It is supposed, that the contact between the shell and medium is sliding, i.e. at $r=R$

$$w = s_z \tag{3}$$

$$q_x = -\sigma_{rx} = 0,\ q_\theta = -\sigma_{r\theta} = 0,\ q_z = -\sigma_{rr} \tag{4}$$

Let's mark, that condition (3) is the equality of displacement vectors, and conditions (4) is the equality of stress vectors of the shell and medium.

Components of the stress vector $\sigma_{rx}, \sigma_{r\theta}, \sigma_{rr}$ by [5], are defined in the following way:

$$\begin{gathered}
\sigma_{rx} = G_s \left(\frac{\partial S_x}{\partial r} + \frac{\partial S_r}{\partial x} \right); \quad \sigma_{r\theta} = G_s \left(\frac{\partial S_x}{r\partial \theta} + \frac{\partial S_\theta}{\partial x} \right) \\
\sigma_{rr} = 2G_s \left(\frac{\partial S_r}{\partial r} + \frac{\nu_s \Delta_1}{\partial x 1 - 2\nu_s} \right) \\
\Delta_1 = \frac{\partial S_x}{\partial r} + \frac{\partial S_\theta}{r\partial \theta} + \frac{\partial S_r}{\partial r} + \frac{S_r}{r}; \quad G_s = \frac{E_s}{2\left(1+\nu_s\right)}
\end{gathered} \tag{5}$$

Here ν_s is a Poisson coefficient, $E_{\bar{s}}$ is an elasticity module of medium material. Complementing by contact conditions (3), (4) the balance equation of cover (1) of medium (2) we arrive at the contact problem on stability of the shell, reinforced by the annular ribs and filled with medium. In other words, the problem on stability of strengthened shell with the medium, is reduced to the combined integration of equations of the shells theory of medium at fulfillment of indicated conditions of the surface of their contact.

We'll represent the solution of system (1) in the form

$$u = u_0 \cos kx \cos n\theta,\ \bar{v} = \bar{v}_0 \sin kx \sin n\theta,$$

$$w = w_0 \sin kx \cos n\theta, \quad (n = 0, 1, 2, ...) \tag{6}$$

The solution of system (2) has the form [5]:

$$\begin{gathered}
S_x = \left[\left(-kr \frac{\partial I_n(kr)}{\partial r} - 4\left(1-\nu_s\right) k I_n(kr) \right) A_s + k I_n(kr) B_s \right] \cos n\theta \cos kx \\
S_\theta = \left[-\frac{n}{r} I_n(kr) B_s - \frac{\partial I_n(kr)}{\partial r} C_s \right] \sin n\theta \sin kx \\
S_r = \left[-k^2 r I_n(kr) B_s - \frac{\partial I_n(kr)}{\partial r} B_s + \frac{n}{r} I_n(kr) C_s \right] \cos n\theta \cos kx\ .
\end{gathered} \tag{7}$$

Here I_n is a modified n-th order Bessel function of the first kind.

After substitution (6) in (1) the solution of problem is reduced to the homogeneous system of linear algebraic equations. Equating the determinant of this system to zero, we obtain the following equation with respect to the parameter $\chi = kR$

$$\alpha_{1n}\chi^8-\alpha_{2n}\chi^6+\alpha_{3n}\chi^4-\alpha_{4n}\chi^2+\alpha_{5n}=0 \tag{8}$$

where

$$\alpha_{1n}=a^2;\ \alpha_{2n}=\frac{2a^2}{1-\nu}\left(1+\gamma_s^{(2)}-\nu\right)+2a^2n^2;$$

$$\alpha_{3n}=1+\gamma_s^{(2)}-\nu^2+a^2n^4\left(6+\frac{5-\nu}{1-\nu}\gamma_s^{(2)}\right)-2\delta_s^{(2)}n^2+$$

$$+\eta_{s1}^{(2)}\left(n^2-1\right)^2\eta_{s2}^{(2)}n^4-2a^2n^4\left(4-\nu^2+\frac{2\left(2-\nu\right)}{1-\nu}\gamma_s^{(2)}\right)-\bar{q}\left(n^2-1\right)-q_z^{(0)};$$

$$\alpha_{4n}=2n^2\left(n^2-1\right)^2\left\{a^2\left(2+\frac{\gamma_s^{(2)}}{1-\nu}\right)+\frac{\eta_{s1}^{(2)}}{1-\nu}\left(1+\gamma_s^{(2)}-\nu\right)+\frac{1}{1-\nu}\eta_{s2}^{(2)}\right\}+$$

$$+2n^2\left(n^2-1\right)^2\left[\nu\delta_s^{(2)}+a^2\gamma_s^{(2)}\left(n^2-1+\nu\right)\right]-$$

$$-\frac{2\bar{q}}{1-\nu}n^2\left(n^2-1\right)^2\left(1+\gamma_s^{(2)}-\nu\right)+\frac{2q_z^{(0)}n^2}{1-\nu}$$

$$\alpha_{5n}=n^4\left(n^2-1\right)^2\left[\eta_{s2}^{(2)}+\left(1+\gamma_s^{(2)}\right)\left(a^2+\eta_{s1}^{(2)}\right)\right]-$$

$$-\bar{q}n^4\left(n^2-1\right)\left(1+\gamma_s^{(2)}\right)+q_z^{(0)}n^4\left(1+\gamma_s^{(2)}\right)$$

$q_z^{(0)}$ in α_{in} is an amplitude of the stresses vector component q_z:

$$q_z=\tilde{k}w=q_z^{(0)}\cos n\theta\sin kx \tag{9}$$

For finding the expression for $q_z^{(0)}$ we'll use the asymptotic formulae for logarithmic derivative of the Bessel function I_n $(\chi<<n;\ n>>1)$:

$$\frac{I_n'\left(\chi\right)}{I_n\left(\chi\right)}\approx-\frac{n}{\chi}+\frac{\chi}{2n} \tag{10}$$

Using formulae (7), (5), (10) and contact conditions (3), (4) for $q_z^{(0)}$ we find

$$q_z^{(0)}=\tilde{\chi}\left(1-\nu\right)nE_s^*=\tilde{q}_z^{(0)}n;\quad \tilde{\chi}=\frac{1-\nu^2}{2\left(1+\nu_s\right)};$$

$$E_s^*=\frac{E_s}{Eh_*},\quad h_*=\frac{h}{R},\quad E_s/E<<1.$$

For facilitation of analysis of the roots of characteristic equation (8) we'll simplify its coefficients having preserved only their quantities that define their order. Research shows, that parameters $\alpha_{in}\,(i=1,2,...,5)$ essentially influence on order of the coefficients $\eta_{s2}^{(2)},a^2,n,\delta_s^{(2)}$. It is obvious, that by studying stability of shells the characteristic equation is of interest into which the parametric members enter with maximum exponent of variability.

The analysis shows, that the formulae can be essentially simplified only in case, when at loss of stability of shell long waves are formed. Such forms of wave formation at loss of stability are realized in case when shells are loaded by the external pressure, or jointly by external pressure and axial contracting forces. In case, when the inequality is fulfilled

$$\eta_s^{(2)} n^4 << 1, \tag{11}$$

we'll obtain the approximate characteristic equation

$$\begin{gathered}\left(1+\gamma_s^{(2)}-\nu\right)\chi^4-2\nu n^2\delta_s^{(2)}\left(n^2-1\right)\chi^2+ \\ +\left[\eta_{s2}^{(2)}+\left(a^2+\eta_{s1}^{(2)}\right)\left(1+\gamma_s^{(2)}\right)\right]n^4\left(n^2-1\right)^2- \\ -n^4\left(n^2-1\right)^2\left(1+\gamma_s^{(2)}\right)\bar{q}+n^s\left(1+\gamma_s^{(2)}\right)\tilde{q}_z^{(0)}=0\end{gathered} \tag{12}$$

From (12) we'll obtain

$$\chi^2=l_{11}^{-1}n^2\left(n^2-1\right)\times$$

$$\times\left[\nu\delta_s^{(2)}\pm\sqrt{\nu^2\left(\delta_s^{(2)}\right)^2-l_{11}\left(l_{12}-\frac{\bar{q}\left(1+\gamma_s^{(2)}\right)}{n^2-1}+\frac{n\left(1+\gamma_s^{(2)}\right)\tilde{q}_z^{(0)}}{\left(n^2-1\right)^2}\right)}\right] \tag{13}$$

where

$$l_{11}=1+\gamma_s^{(2)}-\nu^2;\quad l_{12}=\eta_{s2}^{(2)}+\left(a^2+\eta_{s1}^{(2)}\right)\left(1+\gamma_s^{(2)}\right)$$

As the problem is reduced to the charateristic equation of the fourth degree, on the each edge of the shell can be satisfied only two boundary conditions.

The addends of equation (12), which don't contain χ^2, describe the state of the shell, independent of this coordinate. Hence, critical pressure in infinitely long shell, filled with medium $\bar{q}_0$ must be defined by the formula

$$\bar{q}_0=\frac{n^2-1}{\left(1+\gamma_s^{(2)}\right)}l_{12}+\frac{\tilde{q}_z^{(0)}}{n^2-1}$$

As the critical pressure of any way strengthened shell of the finite length must be greater of equal to $\bar{q}_0$, radicand in (3) is greater than zero. Hence, χ^2 can take two values B_1^2 and $-B_2^2$, where B_1^2 and B_2^2 are real positive numbers.

So, in the considered case equation (8) has such roots: $iB_2, -iB_2, B_1, -B_1$. Using (13) we can express B_1 by B_2, that allows to solve the stability problems of the considered constructions for different boundary conditions

$$B_1^2 = B_2^2 + \frac{2\nu\delta_s^{(2)} n^2 \left(n^2 - 1\right)}{1 + \gamma_s^{(2)} - 1}.$$

Let's cite one of boundary conditions for finding B_2:

$$at \quad \xi = 0 \quad and \;\; at \quad \xi = \xi_1 \quad N_x = 0, \quad \nu = 0, \tag{14}$$

where N_x are normal efforts in the cross sections of constructively-orthotropic shell, defined by the formula

$$N_x = \frac{Eh}{(1-\nu^2)} \left[\frac{\partial u}{\partial \xi} + \nu \left(\frac{\partial \nu}{\partial \theta} - w \right) \right]$$

In case (14), the equations for $\bar{B}_2 = B_2 \xi_1$ have the roots $m\pi \;\; (m = 1, 2, 3)$. In general case for calculation of parameter of the critical stresses $\bar{q}$, we'll obtain:

$$\bar{q} = \frac{1}{n^2 \left(n^2 - 1\right) \left(1 + \gamma_s^{(2)}\right)} \left[l_{11} B_2^4 + 2\nu\delta_s^{(2)} n^2 \left(n^2 - 1\right) B_2^2 + \right.$$

$$\left. + l_{12} \left(n^2 - 1\right)^2 n^4 + \left(1 + \gamma_s^{(2)}\right) n^5 \tilde{q}_z^{(0)} \right] \tag{15}$$

For getting $\bar{q}_{kp}$ it is necessary to substitute the values B_2 and n to this formula providing minimum of $\bar{q}$. In this case calculation formula (15) will take the form

$$\bar{q} = \frac{1}{n^2 \left(n^2 - 1\right) \left(1 + \gamma_s^{(2)}\right)} \left[l_{11} \left(\tfrac{\pi}{\xi_1}\right)^4 + 2\nu\delta_s^{(2)} n^2 \left(n^2 - 1\right) \left(\tfrac{\pi}{\xi_1}\right)^2 + \right.$$

$$\left. + l_{12} \left(n^2 - 1\right)^2 n^4 + \left(1 + \gamma_s^{(2)}\right) n^5 \tilde{q}_z^{(0)} \right].$$

If we suppose, that the value B_2, providing minimum of $\bar{q}$, is not equal to zero and weakly depends on n and take $n >> 1$, then minimizing $\bar{q}$ by n, we obtain the formulae for calculation $\bar{q}_{kp}$ and n:

$$\bar{q}_{kp} = \frac{1}{\tilde{n}^2 \left(\tilde{n}^2 - 1\right) \left(1 + \gamma_s^{(2)}\right)} \left[l_{11} \left(\tfrac{\pi}{\xi_1}\right)^4 + 2\nu\delta_s^{(2)} \tilde{n}^2 \left(\tilde{n}^2 - 1\right) \left(\tfrac{\pi}{\xi_1}\right)^2 + \right.$$

$$\left. + l_{12} \left(\tilde{n}^2 - 1\right)^2 \tilde{n}^4 + \left(1 + \gamma_s^{(2)}\right) \tilde{n}^5 \tilde{q}_z^{(0)} \right] \tag{16}$$

where $\tilde{n}$ is defined from the approximate equation of the fourth degree

$$n^4 + Pn + Q = 0 \tag{17}$$

where

$$P = -\frac{\left(1+\gamma_s^{(2)}\right)\tilde{q}_z^{(0)}}{2l_{12}}; \quad Q = -\frac{18l_{12}+4\nu\delta_s^{(2)}B_2^2}{2l_{12}}$$

It is known, that the solution of equation (17) of the fourth degree is reduced to the solution of cubic equation

$$A^3 - 4QA - P^2 = 0 \tag{18}$$

and two quadratic equations

$$\begin{array}{l} X^2 - \sqrt{A}X + \frac{1}{2}\left(A + \sqrt{A^2-4Q}\right) = 0 \\ X^2 + \sqrt{A}X + \frac{1}{2}\left(A - \sqrt{A^2-4Q}\right) = 0 \end{array} \tag{19}$$

Using (18) and (19) for $n = \tilde{n}$, satisfying the requirements of the problem, we'll obtain

$$\tilde{n} = \frac{\sqrt{2\sqrt{A^2-4Q} - A - \sqrt{A}}}{2} \tag{20}$$

where

$$A = \sqrt[3]{\frac{27}{2}p^2 + \frac{3}{2}\sqrt{-3D}} + \sqrt[3]{\frac{27}{2}p^2 - \frac{3}{2}\sqrt{-3D}}; \quad D = 256Q^3 + 27p^2.$$

Substituting (20) in formulae (15), we can obtain the value of the critical force $\bar{q}_{kp}$. Formulae for $\bar{q}_{kp}$ show, that the account of influence of filler and increasing the number of transverse ribs the value of the critical force is increased.

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