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NONLOCAL AXISYMMETRIC FLUID FILTRATION IN VISCOELASTIC STRATA

Abstract

In the paper the axisymmetric weakly-compressible fluid filtration in cased well is considered allowing for interaction of viscoelastic bed with its ambient rocks. The effect of nonlocal bed deformation to pressure distribution is shown. Instant changeover time at working stratum with constant discharge is defined.

In [1-4] the problems of relaxation filtration of elastic fluid are investigated in stratum at given porosity or permeability index in center hole. Nonequilibrium field of pore pressure is defined. The further more precise definitions of loading diagram were conditioned by deflections from linear locking connections that led to construction of nonlinear theory of elastic filtration regime in isolated stratum. Basic difficulty in filtration theory is in consideration of interaction of stratum with mountain range and their viscoelastic properties [5-7].

1. Statement of problem. Let axisymmetric elastic stratum of power h be in mountain range and rigidly associated with rigid foundation. We locate coordinate origin on subface of stratum, and axis z vertically up along the axis of cased well of radius r_c (Fig.1).

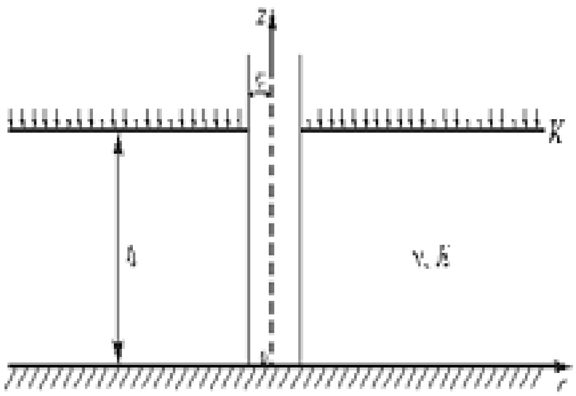

Fig.1

According to [6,7] linearized equations of discontinuity of rigid and fluid phases for isometric process have the form

$$\frac{\partial m}{\partial t}+\beta_1\frac{\partial \sigma^f}{\partial t}-(1-m_0)\,\beta_1\frac{\partial p}{\partial t}-(1-m_0)\,\frac{\partial}{\partial t}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}+\frac{\partial u_z}{\partial z}\right)=0, \qquad (1.1)$$

$$\frac{\partial m}{\partial t}+\beta_2 m_0\frac{\partial p}{\partial t}+m_0\left(\frac{\partial w_r}{\partial r}+\frac{w_r}{r}+\frac{\partial w_z}{\partial z}\right)=0. \qquad (1.2)$$

Consequently, momentum equations of rigid phase in displacements

$$\nabla^2 u_r - \frac{u_r}{r} + \frac{1}{1-2\nu}\frac{\partial e}{\partial r} - 2s\frac{1-\nu}{1-2\nu}\frac{\partial p}{\partial r} = 0,$$
$$\nabla^2 u_z + \frac{1}{1-2\nu}\frac{\partial e}{\partial z} - 2s\frac{1-\nu}{1-2\nu}\frac{\partial p}{\partial z} = 0, \qquad (1.3)$$

and fluid phase

$$\frac{k_0}{\mu_0}\frac{\partial p}{\partial r} = -m_0\left(w_r - \frac{\partial u_r}{\partial r}\right), \quad \frac{k_0}{\mu_0}\frac{\partial p}{\partial z} = -m_0\left(w_z - \frac{\partial u_z}{\partial r}\right). \qquad (1.4)$$

A system of equations (1.1)-(1.4) with respect to the unknown variables u_r, u_z, w_r, w_z, p, m is closed, and σ^f is defined by Hooke's law by displacements

$$\sigma^f_{rr} = 2G\left(\frac{\partial u_r}{\partial r} + \frac{1}{1-2\nu}e\right) + \varepsilon p, \quad \sigma^f_{\theta\theta} = 2G\left(\frac{u_r}{r} + \frac{\nu}{1-2\nu}e\right) + \varepsilon p,$$
$$\sigma^f_{zz} = 2G\left(\frac{\partial u_z}{\partial z} + \frac{1}{1-2\nu}e\right) + \varepsilon p, \quad \sigma^f_{rz} = G\left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}\right). \qquad (1.5)$$

Here the variables are deflections from stationary values of quantities, and the following notation are accepted: m is porosity, m_0 is its initial value, k_0 is an absolute permeability index, u_r, u_z are displacement components of solid particle, w_r, w_z are rate components of fluid phase, p is pore pressure, $\sigma^f_{rr}, ..., \sigma^f_{zz}$ are tensor components of effective stress, β_1, β_2 are coefficients of isothermic compressibility of material of rigid and fluid phase, $E, \nu, \mathcal{K}$ are elasticity modulus, Poisson's coefficient and volumetric elasticity modulus, respectively, $\varepsilon = (1-m_0)\,\beta_1\mathcal{K}$, σ^f is mean stress, e is volumetric deformation of stratum skeleton, $G = E\,(1-m_0)\,/2\,(1+\nu)$, $\mathcal{K} = E\,(1-m_0)\,/3\,(1-2\nu)$, $s = (1-2\nu)\,(1-\varepsilon)\,/2G\,(1-\nu)$.

Boundary conditions. The well is cased, therefore

$$u_r = 0, \ \sigma^f_{rz} = 0 \ \text{ at } \ r = r_c, \qquad (1.6)$$

on external boundary we accept impermeability and symmetry conditions

$$\frac{\partial p}{\partial r} = 0, \ u_r = 0, \ \sigma^f_{rz} = 0 \ \text{ at } \ r = R, \qquad (1.7)$$

The displacements are lacking on subface of stratum, i.e. subface is rigid

$$u_r = u_z = 0 \quad \text{at} \quad z = 0. \qquad (1.8)$$

Let the conditions

$$u_r = 0, \quad K_c\,[u_z] = \sigma^f_{zz} - p \ \text{ at } \ z = h. \qquad (1.9)$$

be satisfied on contact between stratum and circumferential range (roofing).

Note that $\sigma^{f}_{zz} - p = \Gamma_{zz}$ is complete vertical stress in poreelastic stratum and must be identified with mountain stress, $[u_z] = u_z(h) - u_z(0)$, K_c is rigidity of lateral deformation of stratum.

Method of solution. Change of pore pressure p by power of stratum along perfect well is significantly less than character change by radius, and therefore $(\partial p/\partial r) >> (\partial p/\partial z)$.

We can obtain solutions of system of differential equations (1.3) under boundary conditions (1.6), (1.7) with the help of Hankel's finite-integral representation [5,6,8]

$$u_r = \frac{\pi^2}{2}\sum_{i=1}^{\infty} u_r^*(i,z,t)\,\Phi_1(\xi_i r),$$
$$u_z = \frac{\pi^2}{2}\sum_{i=1}^{\infty} u_z^*(i,z,t)\,\Phi_0(\xi_i r), \tag{1.10}$$

where

$$u_z^*(\xi_i,z,t) = (A_i + \xi_i z B_i)\,e^{\xi_i z} + (C_i + \xi_i z D_i)\,e^{-\xi_i z} - \frac{\xi_i}{2G} N(\xi_i,z,t),$$

$$u_z^*(\xi_i,z,t) = (-A_i + (3-4\nu - \xi_i z)B_i)\,e^{\xi_i z} + (C_i + (3-4\nu+\xi_i z)D_i)\,e^{-\xi_i z} + \frac{1}{2G}\frac{dN}{dz},$$

$$\Phi_l(\xi_i r) = \frac{\xi_i^2 W_l(r\xi_i)\,J_1(R\xi_i)}{J_1^2(r_c\xi_i) - J_1^2(R\xi_i)}, \quad l = 0,1.$$

$$W_l(\xi_i r) = J_l(r\xi_i)\,Y_1(r_c\xi_i) - Y_l(r\xi_k)\,J_1(r_c\xi_i), \tag{1.11}$$

$$N(\xi_i,z,t) = -2Gs\int_0^h \frac{1}{\xi_i} sh\left[\xi_i(z-\eta)\right] H(z-\eta)\,p^*(\xi_i,\eta,t)\,d\eta,$$

$$u_r^*(\xi_i,z,t) = \int_c^R u_r(r,z,t)\,W_1(r\xi_i)\,r\,dr,$$

$$H(z-\eta) = 0, \quad z-\eta<0, \quad H(z-\eta) = 1, \quad z-\eta>0,$$

$$u_z^*(\xi_i,z,t) = \int_c^R u_z(r,z,t)\,W_0(r\xi_i)\,r\,dr, \qquad p^*(\xi_i,z,t) = \int_c^R p(r,z,t)\,W_0(r\xi_i)\,r\,dr,$$

Here ξ_i are sequences of positive roots of the equation $W_1(R\xi_i) = 0$, $J_l(r\xi_i)$ and $Y_l(r\xi_i)$ are first and second kind Bessel functions of l -th order, A_i, B_i, C_i, D_i are integration parameters which are defined from boundary conditions (1.8) and (1.9) subject to (1.5) and (1.10)

$$B_i = f_1(\xi_i h)\,sp^*, \quad D_i = f_2(\xi_i h)\,sp^*,$$

$$A_i = \frac{1}{2}\left\{\left[(3-4\nu) f_1(\xi_i h) + f_2(\xi_i h)\right] - 1\right\} sp^*, \tag{1.12}$$

$$C_i = -\frac{1}{2}\left\{\left[(3-4\nu) f_1(\xi_i h) + f_2(\xi_i h)\right] + 1\right\} sp^*,$$

$$f_1(\xi_i h)=\frac{1}{2}\frac{[(3-4\nu)K-2(K+1-\nu)\xi_i h]-(3-4\nu)Ke^{\xi_i h}+[(3-4\nu)(K+\xi_i h)+}{(3-4\nu)[(3-4\nu)Kch(2\xi_i h)-2(1-\nu)\xi_i hsh(2\xi_i h)]-}$$

$$\frac{+(1+2K)\xi_i h]e^{-\xi_i h}-[(3-4\nu)K+2(1-\nu)]e^{\xi_i h}}{-\left\{(3-4\nu)^2K\ +2(\xi_i h)^2[K+2(1-\nu)]\right\}},$$

$$f_2(\xi_i h)=\frac{1}{2}\frac{[(3-4\nu)K-2(1-\nu)\xi_i h]e^{2\xi_i h}-[(3-4\nu)(K-\xi_i h)-}{(3-4\nu)[(3-4\nu)Kch(2\xi_i h)-2(1-\nu)\xi_i hsh(2\xi_i h)]-}$$

$$\frac{-(1+2K)\xi_i h]e^{\xi_i h}+(3-4\nu)Ke^{-\xi_i h}-[(3-4\nu)K+2(K+1-\nu)\xi_i h]}{-\left\{(3-4\nu)^2K\ +2(\xi_i h)^2[K+2(1-\nu)]\right\}},$$

$$N(\xi_i,z,t)=-\left(2Gs/\xi_i^2\right)p^*(\xi_i,z,t),\quad N_z'\approx 0,\ K=K_c/2G$$

The expressions for displacements (1.10) enable to define volumetric deformation

$$e=sp-s(1-2\nu)\pi^2\sum_{i=1}^{\infty}\varphi(\xi_i z)p^*(\xi_i,z,t)\Phi_0(\xi_i r),\qquad \varphi(\xi_i z)=f_1(\xi_i z)e^{\xi_i z}-f_2(\xi_i z)e^{\xi_i z}. \tag{1.13}$$

From continuity equation (1.1), Hooke's law (1.5) and formula (1.13) it succeeds to find analytical connection between porosity and pore pressure

$$m=(1-m_0)(1-\beta_1\mathcal{K})(\beta_1 p+e)=(1-m_0)(1-\beta_1\mathcal{K})\times$$
$$\times\left[(\beta_1+s)p-s(1-2\nu)\pi^2\sum_{i=1}^{\infty}\varphi(\xi_i z)p^*(\xi_i,z,t)\Phi_0(\xi_i r)\right]. \tag{1.14}$$

Substitute (1.4),(1.14) in (1.2) and after some transformations we obtain a filtration equation of elastic fluid in linear-elastic axisymmetric stratum

$$\frac{\partial}{\partial t}\left[p-\frac{\alpha\pi^2}{2}\sum_{i=1}^{\infty}\varphi(\xi_i z)p^*(\xi_i,z,t)\Phi_0(\xi_i r)\right]=\frac{\chi}{k_0}\nabla\left(k\overline{\nabla}p\right), \tag{1.15}$$

where

$$\alpha=2s(1-\nu)[(1-m_0)(1-\beta_1\mathcal{K})+m_0]/\alpha_1,\quad \chi=k_0/(\alpha_1\mu_0),$$

$$\alpha_1=(1-m_0)(1-\beta_1\mathcal{K})(\beta_1+s)+m_0(1-\beta_1\mathcal{K}),\quad k=k_0\left(1+\frac{m}{m_0}\right)^{\gamma}.$$

Here $\gamma=a_k/a_m$ are permeability and porosity change coefficients, respectively [6,9].

For infinitely extended stratum ($R\to\infty$) disturbances of pressure and displacement must tend to zero by the following form [8]

$$r^{1/2}u_r=0,\ r^{1/2}(\partial u_r/\partial r)=0,\ r^{1/2}u_z=0,\ \ r^{1/2}(\partial u_z/\partial r)=0,\qquad r^{1/2}p=0,\ r^{1/2}(\partial p/\partial r)=0 \tag{1.16}$$

as well as p is a piece-wise continuous function of bounded variation in all finite interval $r_c,\ \rho$, and integral $\int_c^{\rho}|p|\sqrt{r}dr<\infty$. In this case problem (1.1)-(1.9) is solved with the help of Weber-Orr [5,7,8] integral transformation, where the quantities ξ_i are substituted by the continuous quantity ξ. Not carrying out intermediate computations we write the filtration equation

$$\begin{gathered}\frac{\partial}{\partial t}\left[p-\alpha\int\limits_0^\infty\varphi\left(\xi z\right)p^*\left(\xi,z,t\right)\xi W_0\left(r\xi\right)d\xi\right]=\frac{\chi}{k_0}\nabla\left(k\overline{\nabla}p\right),\\ p^*\left(\xi,z,t\right)=\frac{1}{J_1^2\left(r_c\xi\right)+Y_1^2\left(r_c\xi\right)}\int\limits_0^\infty prW_0\left(r\xi\right)d\xi.\end{gathered}\tag{1.17}$$

Thus, the problem is led to defining the solutions of nonlinear differential equations (1.15) or (1.17). In (1.15) the function $\varphi\left(\xi_i z\right)$ characterizes interaction of poreelastic stratum with ambient range. After defining nonstationary distribution of pressure field it is easy to define mode of deformation of stratum from formulae (1.10) and (1.5).

2. Problem on well shutdown. We define pressure distribution in stratum after instantaneous shutting-in of well operating with constant rate Q. Let at $t\le 0$ the pressure in well be $p_0\left(r\right)$. Therefore at $t>0$ we have

$$V\left(r,t\right)=p_0\left(r\right)+p\left(r,t,\right),\tag{2.1}$$

where $p_0\left(r\right)$ is a stationary solution of equation (2.2) such that at $r=r_c,\ t\le 0$

$$r\frac{\partial p_0}{\partial r}=\frac{Q}{2\pi k_0 h}.\tag{2.2}$$

Shutting-in of well at moment $t=0$ means that

$$r\frac{\partial V}{\partial r}=r\frac{\partial p_0}{\partial r}+r\frac{\partial p}{\partial r}=0\quad\text{at}\quad r=r_c$$

or

$$p=0\quad\text{at}\quad t=0,\quad r\frac{\partial p}{\partial r}=-\frac{Q}{2\pi k_0 h}=-q\quad\text{at}\quad r=r_c.\tag{2.3}$$

At $k=k_0$ equation (1.15) is linearized and applying to it Hankel's transformation (and to (1.17) – Weber-Orr) subject to boundary conditions (1.7),(2.3), we have

$$\left[1-\alpha\varphi\left(\xi_i z\right)\right]\frac{dp^*}{dt}+\chi\xi_i^2p^*=-\frac{2\chi}{\pi r_c\xi_i}q.\tag{2.4}$$

Solution (2.4) under initial condition (2.3) gets the form

$$p^*\left(\xi_i,z,t\right)=-\frac{2q}{\pi r_c\xi_i^3}\left[1-\exp\left(-\frac{\chi\xi_i^2t}{1-\alpha\varphi\left(\xi_i z\right)}\right)\right].\tag{2.5}$$

The Bessel functions including unit compose orthogonal system and therefore, the solution of equation (1.15) is represented in the form

$$p=\frac{\pi^2}{2}\sum_{i=1}^{\infty}\frac{\xi_i^2J_1^2\left(\xi_iR\right)W_0\left(\xi_ir\right)}{\left[J_1^2\left(\xi r_c\right)-J_1^2\left(\xi_iR\right)\right]}p^*\left(\xi_i,z,t\right)+\frac{2q\chi}{R^2-r_c^2}t.\tag{2.6}$$

Problem (1.17) and (2.3) is solved analogously

$$p=-\frac{2q}{\pi r_c}\int_0^{\infty}\left(1-\exp\left(-\frac{\chi\xi^2 t}{1-\alpha\varphi(\xi z)}\right)\right)\frac{W_0(r\xi)\,d\xi}{\xi^2\left[J_1^2(\xi r_c)+Y_1^2(\xi r_c)\right]}. \tag{2.7}$$

It we introduce dimensionless quantities: $x_i=r_c\xi_i$, $R_c=R/r$, $z_c=z/r_c$, $\overline{\chi}=\chi/r_c^2$, $\overline{p}=p/q$, and use the expression $W_0(x_i)=-2/(\pi x_i)$, then the solutions on hole wall (2.6) and (2.7) characterize pressure recovery

$$\overline{p}_c=2\sum_{i=1}^{\infty}\frac{J_1^2(R_c x_i)\left[1-\exp\left(-\frac{x_i^2\overline{\chi}t}{1-\alpha\varphi(z_c x_i)}\right)\right]}{x_i^2\left[J_1^2(x_i)-J_1^2(R_c x_i)\right]}+\frac{2\overline{\chi}}{R_c^2-1}t, \tag{2.8}$$

$$\overline{p}_c=\frac{4}{\pi^2}\int_0^{\infty}\frac{1-\exp\left(-\frac{x^2\overline{\chi}t}{1-\alpha\varphi(z_c x)}\right)}{\xi^2\left[J_1^2(x)+Y_1^2(x)\right]}dx. \tag{2.9}$$

The computation on formulae (2.8) and (2.9) is led by numerical method at the following data:

$$h=30-50m,\ \ r_c=0,1m,\ \ \alpha=0.8;\ \ \nu=0,2;0,3$$
$$m_0=0,16;\ k_0=10^{-12}m^2,\ \beta_1=2.94\cdot10^{-11}Pa^{-1},$$
$$\beta_2=8\cdot10^{-10}Pa^{-1},\ E=2.35\cdot10^{10},\ R=\left(10^2-10^3\right)m.$$

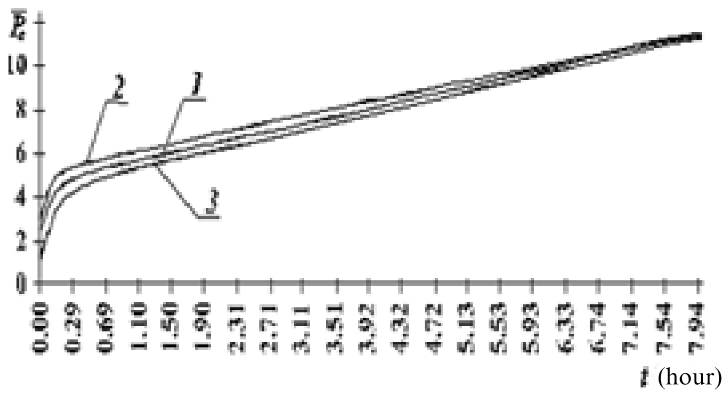

Fig.2

It follows from computations that the well deformation with small rigidity ($K << 1$) of roofing always leads to decrease of values of pressure conductivity factor and initial on-cycle of well, pressure recovery is late (fig.2, fig.3) as compared with isolated well, i.e., $\varphi(\xi z)=0$ (curve 1). However at large $K >> 1$ pressure change near roofing and bottom is more than in interior domain of stratum (curve 3). Such non-uniformity of pressure distribution by stratum power often is reason of premature flooding of well and appearance of horizontal cracks as a result of concentrations of strength on roofing. On hole wall the transient time depending on parameters of stratum from nonlocal elastic regime to local-elastic is approximately estimated for rigid bottom ($K = 10$) $t\tilde{}10^4$ sec , for soft one ($K = 0,1$) $t\tilde{}10^5$ sec .

In close stratum at fixed time the pressure growths with decrease of radius of external boundary R. Besides, terminal R causes unbounded growth of pressure in time. Thus, the solution of elastic problem at large t and constant discharge in closed stratum is not always justified. This contradicts primitive conjectures with smallness of divergence of parameters of stratum from datum values.

3. The process of unsteady filtration of fluid in deep-seated seams with hereditary character under the given functional dependence of parameters of stratum (porosity and permeability) is investigated in [1-4]. However, the definition of these parameters during solving problems [5-7] is of interest. By solving these problems Volterra principle should be used and for this in Laplace mappings the solutions of problem of linear filtration theory of elastic regime elastic constants must be replaced by corresponding rheological operators. Note that Volterra principle is reasonable in those cases if boundary conditions don't change in deformation process as well as Poisson coefficients and rigidity are constant.

Applying Laplace transformation to equation (1.15) ($k = k_0$) under initial condition (2.3) and using Volterra principle we obtain

$$\lambda\left[p_\lambda - \frac{\pi^2\alpha_\lambda(\lambda)}{2}\sum_{i=1}^{\infty}\varphi(\xi_i z)\, p^*_\lambda(\xi_i, z, t)\,\Phi_0(\xi_i r)\right] =$$

$$= \chi_\lambda(\lambda)\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p_\lambda}{\partial r}\right) + \frac{\partial^2 p_\lambda}{\partial z^2}\right], \tag{3.1}$$

where

$$p_\lambda = \int_0^\infty p(z,r,t)\,e^{-\lambda t}dt, \qquad H_\lambda = \int_0^\infty H(t)\,e^{-\lambda t}dt,$$

$$\alpha_\lambda = \frac{2(1+\nu)(1-2\nu)^2(1-\varepsilon)(1+H_\lambda(\lambda))}{a + b(1+H_\lambda(\lambda))}, \quad \chi_\lambda = \frac{\chi_0}{a + b(1+H_\lambda(\lambda))}, \tag{3.2}$$

$$\chi_0 = k_0(1-m_0)(1-\nu)E/\mu_0, \; b(\lambda) = \left(1-\varepsilon^2\right)(1+\nu)(1-2\nu)(1+H_\lambda(\lambda)),$$

$$a = (1-m_0)(1-\nu)\left[(1-m_0)(1-\beta_1\mathcal{K})\beta_1 + m_0\beta_2\right]E.$$

We pass from (3.1) to original

$$\frac{\partial}{\partial t}\left\{p - \frac{\pi^2}{2}\sum_{i=1}^{\infty}\left[\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} a_\lambda(\lambda)\,\lambda\varphi(\xi_i z)\,p^*_\lambda(\xi_i,z,t)\,e^{\lambda t}dt\right]\Phi_0(\xi_i r)\right\} =$$

$$= \chi_0\Delta\left(\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\frac{P_\lambda(r,z,\lambda)\,e^{\lambda t}}{a+b(1+H_\lambda(\lambda))}d\lambda\right). \tag{3.3}$$

Obtained equation (3.3) is basic filter equation of loosely compressible fluid to center hole in linear hereditary closed stratum.

Consider a problem on pressure recovery at instant shutting-in of well working with the given discharge

$$r\frac{\partial p}{\partial r} = -q\,(t) \quad \text{at} \quad r = r_c, \;\; \frac{\partial p}{\partial r} = 0, \;\; r = R. \tag{3.4}$$

Consequently, applying Laplace and Hankel's transformation to problem (3.3) and (3.4) we obtain

$$p^*_\lambda\,(\xi_i, z, t) = -\frac{2q_\lambda\,(\lambda)}{\pi r_c \xi_i \lambda}\frac{\chi_0}{[a + b\,(1 + H_\lambda\,(\lambda))]\,[1 - a_\lambda\,(\lambda)\,\varphi\,(\xi_i z)] + \chi_0 \xi_i^2}.$$

From here we pass to original

$$p\,(r, z, t) = \frac{\pi^2}{2}\sum_{i=1}^{\infty}\frac{\xi_i^2 J_1^2\,(R\xi_i)\,W_0\,(r\xi_i)}{\left[J_1^2\,(r_c\xi) - J_1^2\,(R\xi_i)\right]}p^*\,(\xi_i, z, t) +$$
$$+\frac{2\chi_0}{R^2 - r_c^2}\frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty}\frac{q_\lambda\,(\lambda)\,e^{\lambda t}d\lambda}{\lambda\,[a + b\,(1 + H_\lambda\,(\lambda))]\,[1 - a_\lambda\,(\lambda)\,\varphi\,(\xi_i z)]}, \tag{3.5}$$

where

$$p^*_\lambda\,(\xi_i, z, t) = -\frac{2\chi_0}{\pi r_c \xi_i}\frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty}\frac{q_\lambda\,(\lambda)\,e^{\lambda t}d\lambda}{\lambda\,[a + b\,(1 + H_\lambda\,(\lambda))]\,[1 - a_\lambda\,(\lambda)\,\varphi\,(\xi_i z)] + \chi_0\xi_i^2}.$$

If we take a kernel of integro-differential equation (3.3) the damped exponential function $H\,(t) = \theta_* e^{-\theta' t}$ which is equivalent to using the model of viscoelastic Kelvin stratum [5] and $q\,(t) = const$, $\overline{q}\,(\lambda) = q/\lambda$, $\alpha \approx 2\,(1 - \nu) = const$, then from (3.5) we find p in the explicit form

$$\overline{p}_c = 2\sum_{i=1}^{\infty}\frac{J_1^2\,(R_c x_i)\,\Phi\,(x_i, z_c, t)}{x_i^2\left[J_1^2\,(x_i) - J_1^2\,(R_c x_i)\right]} + \frac{2\overline{\chi_0}}{(R_c^2 - 1)\,(a + b)\,\theta' + b\theta_*}\times$$
$$\times\left[\theta' t + \frac{b\theta_*}{(a + b)\,\theta' + b\theta_*}\left(1 - e^{\frac{(a+b)\theta' + b\theta_*}{a + b}t}\right)\right], \tag{3.6}$$

where

$$\Phi\,(x_i, z_c, t) = 1 + \frac{\overline{\chi_0} x^2}{(a + b)\,(\lambda_1 - \lambda_2)}\left(\frac{\lambda_1 + \theta'}{\lambda_1}e^{\lambda_1 t} - \frac{\lambda_2 + \theta'}{\lambda_2}e^{\lambda_2 t}\right), \tag{3.7}$$

$$\lambda_{1,2} = -\frac{(a + b)\,\theta' + b\theta_* + \overline{\chi}_0 x_i^2}{2\,(a + b)} \pm \sqrt{\left(\frac{(a + b)\,\theta' + b\theta_* + \overline{\chi}_0 x_i^2}{2\,(a + b)}\right)^2 - \frac{\overline{\chi}_0\theta' x_i^2}{a + b}},$$

$$\overline{\chi}_0 = \chi_0 / r_c^2\,(1 - \alpha\varphi\,(x_i, z_c)).$$

The problem for infinite extended stratum ($R \to \infty$) is solved with the help of Weber-Orr and Laplace integral transformations

$$\overline{p}_c = \frac{4}{\pi^2} \int_0^\infty \frac{\Phi(x,z,t)\,dx}{x^3 \left[J_1^2(x) + Y_1^2(x)\right]}. \tag{3.8}$$

If we consider the fluid filtration in viscoelastic maxwellian stratum, then it follows from (3.7) and (3.8)

$$p_c = \frac{4\overline{\chi}_0}{\pi^2} \int_0^\infty \frac{\left[1 - \exp\left(\dfrac{b\theta_* + \overline{\chi}_0 x_i^2}{a+b} t\right)\right] dx}{x\left(b\theta_* + \overline{\chi}_0 x^2\right)\left[J_1^2(x) + Y_1^2(x)\right]}. \tag{3.9}$$

At $\theta_* = 0$ solution (3.9) passes to solution of nonlocal-elastic regime of filtration (2.9).

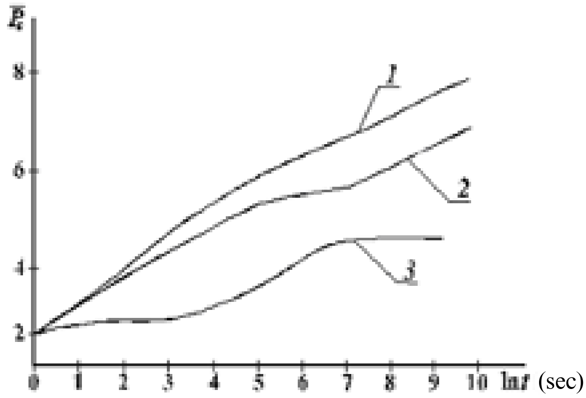

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Fig.3

Computations are carried out by abovementioned data subject to time of relaxation: $\theta_* = \left(10^{-2} - 10^{-3}\right)\sec^{-1}$, $\theta' = \left(10^{-1} - 10^{-2}\right)\sec^{-1}$. Firm lines 1 in fig. 3 corresponds to the solution of the problem in linear-elastic stratum, 2 – in viscoelastic Kelvin stratum, 3 – in viscoelstic maxwell stratum. It is obvious from Fig.3 that changeover time of pressure change is defined by relaxation parameters of stratum and equals approximately $10^2 \le t \le 10^4\,\mathrm{sec}$. Then the process passes to elastic regime of filtration at which instant elasticity coefficient decreases $\left[E\theta' / \left(\theta' + \theta_k\right)\right] < E$. In viscoelastic maxwellian fluid saturated medium nonuniform process quickly arrives at steady-state regime which doesn't agree with practical workers.

It turned out with decline of viscosity of matrix of stratum, pressure change in it with time decreases and choice of optimal conditions is related with relaxation parameters of stratum.

References

[1]. Alishayev M.G., Mirzadjanzade A.Kh. *On consideration of delay phenomena in filtration theory.* Isvestiya vuzov SSSR, neft i gaz, 1975, No6, pp.71-74. (Russian)

[2]. Abasov M.T., Jalilov K.N., Kerimov Z.A., Mirzoyeva D.R. *On filtration of fluid in relaxation-compressible and creeping well at rate and pressure relaxation.* Izvestiya AN Azerb., ser. nauk. o Zemle, 2000, No2, pp.25-38. (Russian)

[3]. Ametov I.M., Basniev K.S. *Filtration of fluid and gas in creeping medium.* Isvestiya AN SSSR, Mechanika zhidkosti i gaza, 1981, No2, pp.150-153. (Russian)

[4]. Molokovich Yu.M., Osipov P.P. *Bases of theory of relaxation filtration.* KSU, Kazan, 1987, 113p. (Russian)

[5]. Ramazanov T.K. *Fluid filtration in linear hereditary stratum.* All Union research institute of matural gas. Coll. Of papers Pecularities of completion of Caspian basin. M.,1986, pp.18-27. (Russian)

[6]. Nikolaevskii V.N. *Geomechanics and fluiddynamics.* M.:"Nedra", 1996, 447p. (Russian)

[7]. Nikolaevskii V.N., Ramazanov T.K. *Mode of deformation of stratum and pressure recovery in well.* Mechanics of deformable body. Strength and viscoelastioplasticity, AN SSSR, IPM, M.: "Nauka", 1986, pp.94-105. (Russian)

[8]. Ditkin V.A., Prudnikov A.P. *Integral transformations and operational calculus.* M.: "Nauka", 1971, 542p. (Russian)

[9]. Faat J. *Compressibility of sanod stresses at low to Moderate Pressures.* Bulleten of the American Association of Petroleum Geologists, 1958, v.42, No8, pp.1924-1957.

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