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REFINED METHODS FOR INVESTIGATION OF DEFLECTED MODE OF ADJOINT STRUCTURAL ELEMENTS OF SHELL

Abstract

Deflected mode of shell structural elements of complicate meridian form is investigated. On an example of spectral segment and circular plate a comparative analysis of other models is given in particular, when the reduction of a shell and application of models close to Winkler-Fouss model is ignored.

Structural elements in many cases consist of one or several parts connected between themselves by different ways. In particular, structural elements of shells may consist of elements different by physical characteristics, connected between themselves by glue interlayers. Besides, by the form these elements in general construction may create complications in geometry and as a result the formulation of strict statement of a problem satisfying the increasing demands of practice is of great difficulty.

In the paper for these objects of investigation the following method is suggested. By means of dissection in those places of construction where there are sharp alternation in geometry, or in physics of the considered object, the latter is substituted by several simple ones both on geometrical and physical characteristics each of which separately sufficiently yields exact investigation.

In view of that each considered element is a shell, then as a final result it is considered for the interaction of thick-shelled structural elements of shells.

Still in forties of the XX century on an example of solution of a contact problem for a plate M.M.Filonenko Borodich insisted on rejection of Kirghoff-Love conjecture [1].

The refined model of S.P.Timoshenko considering both a rotation of a normal to the mean surface after deformation and deformations of the elements normal to the mean surface is on the basis of the paper. Earlier we have considered similar structural elements where as a kinematic model we adopted S.P.Timoshenko type models [2] taking into account the rotation of a normal and ignoring the deformation of a normal itself.

Consideration of deformability of a normal increases the order of solving equations.

By the known way [3] all solving relations are obtained: equilibrium equations of a shell

$$(A_2T_{11})_{,1} - A_{2,1}T_{22} + A_1A_2(k_1T_{13} + X_1) = 0,$$

$$(A_2M_{11})_{,1} - A_{2,1}M_{22} - A_1A_2(T_{13} - M_1) = 0,$$

$$(A_2T_{13})_{,1} - A_1A_2(k_1T_{11} + k_2T_{22} - X_3) = 0,$$

$$(A_2M_{13})_{,1} - A_1A_2(k_1M_{11} + k_2M_{22} + T_{33} - M_3) = 0; \qquad (1)$$

equilibrium equations for each dissected element

$$\left(A_2 M_{11}^k - \delta_k h_k A_2 T_{11}^k\right)_{,1} + \delta_k h_{k,1} A_2 T_{11}^k + \delta_k h_k A_{2,1} T_{22}^k -$$

$$-A_{2,1} M_{22}^k - A_1 A_2 \left(T_{13}^k + \delta_k h_k k_1^k T_{13}^k - M_1^k\right) = 0$$

$$\left(A_2 M_{13}^k - \delta_k h_k A_2 T_{13}^k\right)_{,1} + \delta_k h_{k,1} A_2 T_{13}^k - A_1 A_2 \left[k_1^k \left(M_{11}^k - \delta_k h_k T_{11}^k\right) + \right.$$

$$\left. + k_2^k \left(M_{22}^k - \delta_k h_k T_{22}^k\right) + T_{33}^k - M_3^k\right] = 0, \quad (k = 1, 2) \ ; \tag{2}$$

conjugation conditions for a shell and each dissected element, i.e. continuity condition of components of small permutations vectors by passing the boundaries of the system's elements

$$u + \delta_k h \gamma + 2\delta_k h_k \left(\gamma^k - Y_1^k \gamma_1^k\right) = 0$$

$$\omega + \delta_k h \gamma_1 + 2\delta_k h_k \left(\gamma_1^k + \gamma^k Y_1^k\right) = 0, \quad (k = 1, 2) \tag{3}$$

statistical natural boundary conditions at edges $\alpha^1 = \alpha_a^1 \left(\alpha_b'\right)$ of a shell

$$\begin{array}{ll} T_{11} = R_{11} \quad at \quad \delta u \neq 0 \ , & M_{11} = G_{11} \quad at \quad \delta\gamma \neq 0 \ , \\ T_{13} = R_{13} \quad at \quad \delta\omega \neq 0 \ , & M_{13} = G_{13} \quad at \quad \delta\gamma_1 \neq 0 \ , \end{array} \tag{4}$$

and $\alpha' = \alpha'_{a_k} \left(\alpha'_{b_k}\right)$ of dissected elements

$$\begin{array}{l} M_{11}^k - \delta_k h_k T_{11}^k = 0 \quad at \quad \delta\gamma^k \neq 0 \\ M_{13}^k - \delta_k h_k T_{13}^k = 0 \quad at \quad \delta\gamma_1^k \neq 0 \end{array} \tag{5}$$

In relations mentioned above A_1, A_2 are Liame parameters; k_1, k_2 are principal curvatures; k_1^k, k_2^k are the curvatures of coordinate lines on mean surfaces of dissected elements; $T_{11},\ T_{22},\ T_{13},\ M_{11},\ M_{22},\ M_{13},\ T_{11}^k,\ T_{22}^k,\ T_{13}^k,\ T_{33}^k,\ M_{11}^k,\ M_{22}^k,\ M_{13}^k$ are intrinsic efforts and moments for a shell and each dissected element, respectively; $u, \omega, \gamma, \gamma_1, \gamma^k, \gamma_1^k$ are the components of vectors of small permutations of a shell and each dissected element, respectively; $Y_1^k = H_{k,1}/A_1,\ H_k = h + h_k$ is a distance between mean surface and corresponding dissected element; $R_{11}, R_{13}, G_{11}, G_{13}$ are the components of vectors of contour effects $\bar{R}$ and moments $\bar{G}$ acting on the edges of a shell; the sign "," in indices denotes a derivative in corresponding direction.

In view of the fact that the investigated object by means of faces interacts with other structural elements, the vectors of small permutations at faces of stacks are assumed to be given, in particular, for simplicity they may equal zero.

In consequence of this condition tangential components of small permutations vector of peeled elements are expressed by angular permutation components.

For this reason system (2) for each dissected element consists of not four equations in axially symmetric state, but of free equations.

Relations (1), (2), (3), (4), (5) including physical and kinematic relations compose a complete system of solving formulae to determine deflected mode of the considered class of shell structural elements in refined statement.

Equations (1), (2)-(3) are integrated in bounds $[\alpha'_a, \alpha'_b]$, $\left[\alpha'_{a_k}, \alpha'_{b_k}\right]$, respectively. The integration is carried out by a numerical way. A finite sums method with using integrating matrices device [4] is chosen as a numerical method according to which the first derivatives of small permutations components are chosen as unknown variables.

It is based on the fact that in strength analysis namely the first derivatives of small permutations are defined. Here the permutations are approximated by the following way $u = C_u + \int\limits_{\alpha'_a}^{\alpha'_b} (...)\, d\alpha'$, where C_u is defined according to boundary conditions.

Performing the described procedure and allowing for physical and kinematic relations for equation (1) we get

$$\frac{d}{d\alpha'}\left([A]\{T\}\right) + [B]\{T\} = \{L\}$$

where $\{T\} = \{T_{11}, T_{22}, T_{33}, T_{13}, M_{11}, M_{22}, M_{13}\}^T$; $[A]$, $[B]$ are the matrices of order 4×7, $\{L\}$ is a column of fourth order

$$\{T\} = [D]\{\varepsilon\}, \ \{\varepsilon\} = \{\varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}, \varepsilon_{13}, æ_{11}, æ_{22}, æ_{13}\}^T,$$

$$\{\varepsilon\} = [E_1]\{U_1\} + [E_2]\{U_2\},$$

$$\{U_1\} = \{u, \omega, \gamma, \gamma_1\}^T, \quad \{U_2\} = \left\{u_{,1}, \omega_{,1}, \gamma_{,1}, \gamma_{1,1}\right\}^T.$$

Here $u_{,1}, \omega_{,1}, \gamma_{,1}, \gamma_{1,1}$ denote a derivative with respect to α^1

$$\{U_1\} = [J_1]\{U_2\} + \{C_u\},$$

$$\{U_1\}_{\alpha'=\alpha'_b} = [J_3]\{U_2\} + \{C_u\}, \qquad \{C_u\} = \{U_1\}_{\alpha'=\alpha'_a},$$

where $[J_1]$ is an integrating matrix that denotes the integration from α'_a to $\alpha^1 \le \alpha'_b$, $[J_2]$ is an integrating matrix denoting the integration from $\alpha^1 \ge \alpha'_a$ to α'_b, $[J_3]$ is an integrating matrix denoting the integration from α'_a to α'_b

Similar transformations are carried out for relations (2)-(5). As a result, for the stated problem the vector of unknowns looks like

$$\{U\} = \left\{\{u_{,1}, \omega_{,1}, \gamma_{,1}, \gamma_{1,1}\}, \ \left\{\gamma^k_{,1}, \gamma^k_{1,1}\right\}_1, ..., \left\{\gamma^k_{,1}, \gamma^k_{1,1}\right\}_n, \{C_u\}, \{C\}, \left\{C^k_u\right\}_1, \left\{C^k\right\}_1,\right.$$
$$\left. ..., \left\{C^k_u\right\}_n, \left\{C^k\right\}_n, \left\{Q^k\right\}_1, ..., \left\{Q^k\right\}_n\right\}^T$$

where $\{C\} = (\{T\})_{\alpha'=\alpha'_b}$, $\left\{Q^k\right\}_i = \left\{q^k_1, q^k_3\right\}$ are reactive efforts acting in stratification domains.

On the basis of above mentioned method the spherical segment strength analysis representing an element of real construction at axially symmetric deformation is calculated.

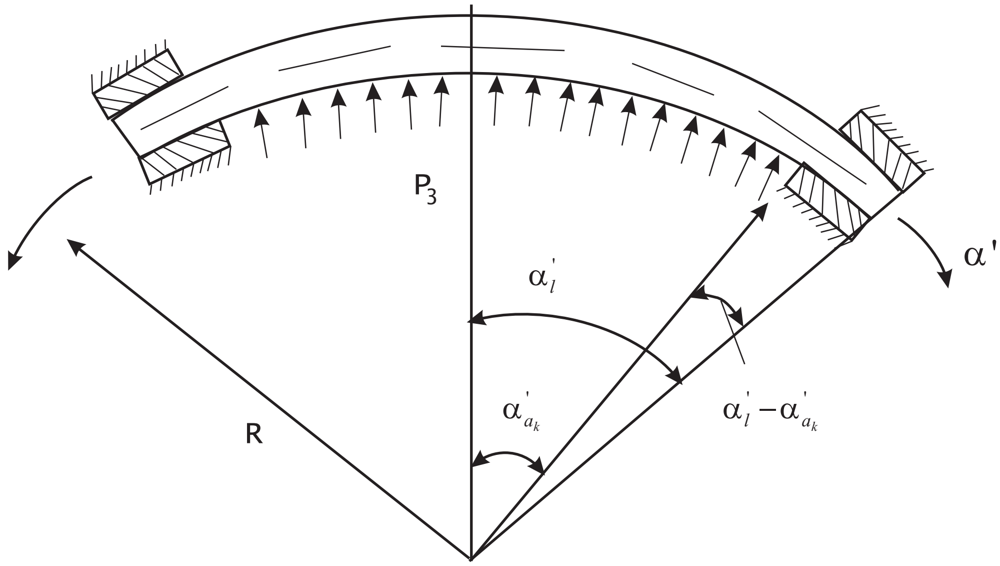

Fig.1

The construction is a spherical segment framed from the both faces by a narrow band made of different material. Excess pressure $P_3 = 0,1$ MPa acts on internal face of the shell. Geometrical and physico-mechanical characteristics of a shell have the following sense: radius of mean surface of the shell $R = 535$ mm, thickness of the shell $2h = 10$ mm; Poisson coefficient $\nu = 0,38$, elasticity modulus of the shell's material $E = 3090$ MPa (oriented glass CT-1 at 20^0C), a half of a solid angle of the shell $\alpha'_b = 0,424$ rad., the angle occupied by a frame equals $\alpha'_{b_k} - \alpha'_{a_k} = 0.0424$ rad., the thickness of a frame $2h_k = 2,5$ mm, Poisson coefficient of frame's material (rubber) $\nu^k = 0,485$, elasticity modulus of frame's material $E^k = 3,0$ MPa.

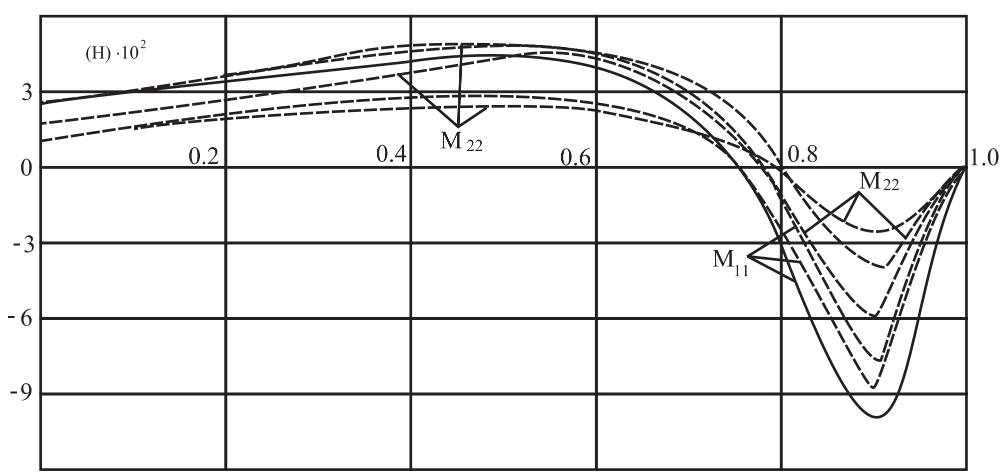

Fig.2

In fig.2 the results of conducted calculations are cited. Whence it is seen how much the results of calculations obtained by other methods are refined.

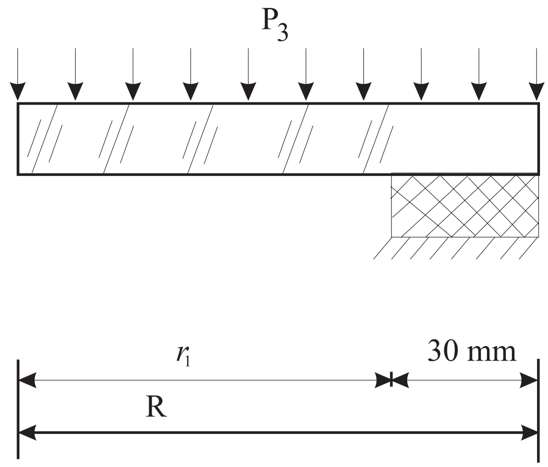

Fig.3

In frames of the discussed method it was carried out investigations of deflected mode of a circular plate of radius $R = 220$ mm rested on elastic frame rigidly fixed by it in the form of ring of internal radius $r_1 = 190$ mm and width 30 mm. Uniformly distributed load $P_3 = -0,1$ MPa uniformly acts on a circular plate (fig.3). Physico-mechanical characteristics of a circular plate and frame are the followings: plate's thickness $2h = 10$ mm, elasticity modulus $E = 3150$ MPa; Poisson coefficient $\nu = 0,38$, frame's thickness $2h_2 = 2,5$ mm; elasticity modulus $E_2 = 3,0$ MPa, Poisson coefficient $\nu_2 = 0,485$.

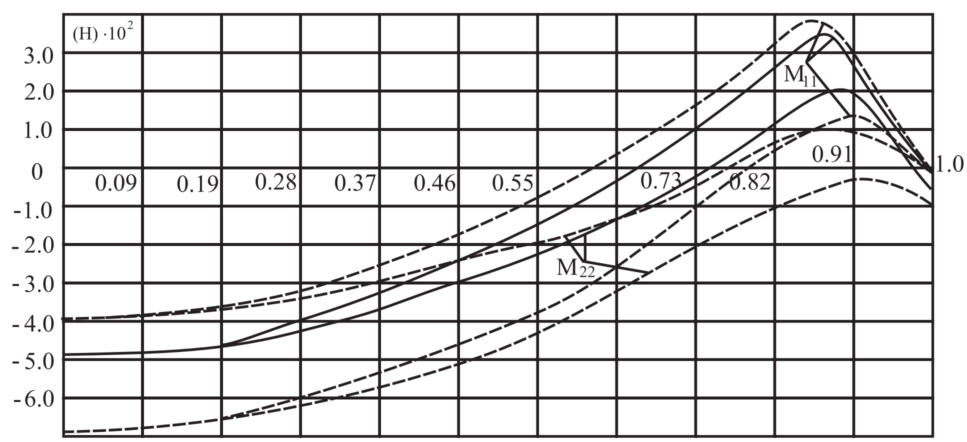

Fig.4

The results of calculations are in fig.4. The full line indicates the results (fig.4) of the present paper, dash-dot lines indicates the results of the method when the deformation of cross element of the plate is ignored, the dotted lines give the results of the method when the frames are deformed like Winkler-Fouss model. Here it is obviously seen how much the neglect of cross reduction of a shell and in particular application of a model close to Winkler-Fouss model in frames affect on determination of deflected mode of the considered structural element.

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