Muhammad KAMALI, Arif A. SALIMOV

HORIZONTAL LIFTS OF AFFINOR FIELDS TO THE TENSOR BUNDLE

Abstract

The purpose of the present paper is to study, using the Vishnevskii operator, the horizontal lifts of affinor fields along a cross-section of the tensor bundle and investigate their transfers.

1. Introduction

Let M_n be a differentiable manifold of class C^∞ and finite dimension n, and let $T_q^p(M_n)$, $p+q>0$ be the bundle over M_n of tensors of type (p,q) : $T_q^p(M_n) = \bigcup\limits_{P\in M_n} T_q^p(P)$, where $T_q^p(P)$ denotes the tensor (vector) spaces of tensors of type (p,q) at $P\in M_n$.

We list below notation used in this paper.

i. $\pi : T_q^p(M_n)$ is the projection $T_q^p(M_n)$ onto M_n.

ii. The indices $i,\ j,...$ run from 1 to n, the indices $\bar{\imath},\ \bar{j},...$ from $n+1$ to $n+n^{p+q} = \dim T_q^p(M_n)$ and the indices $I=(i,\bar{\imath})$, $J=(j,\bar{j}),...$ from 1 to $n+n^{p+q}$. The so-called Einsteins summation convention is used.

iii. $\mathcal{F}(M)$ is the ring of real-valued C^∞ functions on M_n. $\mathcal{I}_q^p(M_n)$ is the module over $\mathcal{F}(M)$ of C^∞ tensor fields of type (p,q).

iv. Vector fields in M_n are denoted by $V,\ W,....$ The covariant derivation with respect to V is denoted by ∇_V. Affinor fields (tensor fields of type (1,1)) are denoted by $\varphi,\ \psi,....$

Denoting by x^j the local coordinates of $P=\pi\left(\tilde{P}\right)$ $\left(\widetilde{P}\in T_q^p(M_n)\right)$ in a neighborhood $U\subset M_n$ and if we make $\left(x^j, t_{j_1...j_q}^{i_1...i_p}\right) = \left(x^j, x^{\bar{j}}\right)$ correspond to the point $\tilde{P}\in\pi^{-1}(U)$, we can introduce a system of local coordinates $\left(x^j, x^{\bar{j}}\right)$ in a neighborhood $\pi^{-1}(U)\subset T_q^p(M_n)$, where $t_{j_1...j_q}^{i_1...i_p} \overset{def}{=} x^{\bar{j}}$ are components of $t\in T_q^p(P)$ with respect to the natural frame ∂_i.

If $\alpha\in\mathcal{I}_p^q(M_n)$, it is regarded, in a natural way (by contraction), as a function in $T_q^p(M_n)$, which we denote by $\iota\alpha$. If α has the local expression $\alpha = \alpha_{i_1...i_p}^{j_1...j_q}\partial_{j_1}\otimes...\otimes \partial_{j_q}\otimes dx^{i_1}\otimes...\otimes dx^{i_p}$ in a coordinate neighborhood $U\left(x^i\right)\subset M_n$, then $\iota\alpha$ has the local expression

$$\iota\alpha = \alpha(t) = \alpha_{i_1...i_p}^{j_1...j_q} t_{j_1...j_q}^{i_1...i_p}$$

with respect to the coordinates $\left(x^j, x^{\bar{j}}\right)$ in $\pi^{-1}(U)$.

Suppose that $A\in\mathcal{I}_q^p(M_n)$. We define the vertical lift ${}^VA\in\mathcal{I}_0^1(T_q^p(M_n))$ of A to $T_q^p(M_n)$ (see [1]) by

$${}^VA(\iota\alpha) = \alpha(A)\circ\pi = {}^V(\alpha(A)),$$

where ${}^V(\alpha(A))$ is the vertical lift of the function $\alpha(A)\in\mathcal{F}(M_n)$. The vertical lift ${}^VA$ of A to $T_q^p(M_n)$ has components

$${}^VA = \begin{pmatrix} {}^VA^j \\ {}^VA^{\bar{j}} \end{pmatrix} = \begin{pmatrix} 0 \\ A_{j_1...j_q}^{i_1...i_p} \end{pmatrix} \tag{1.1}$$

with respect to the coordinates $\left(x^j, x^{\bar{j}}\right)$ in $T_q^p(M_n)$.

Let ∇ be a torsion-free affine (linear) connection on M_n. We define the horizontal lift ${}^H V = \bar{\nabla}_V \in \mathcal{I}_0^1(T_q^p(M_n))$ of $V \in \mathcal{I}_0^1(M_n)$ to $T_q^0(M_n)$ [1] by

$$ {}^H V(\iota\alpha) = \iota(\nabla_V \alpha), \quad \alpha \in \mathcal{I}_p^q(M_n). $$

The horizontal lift ${}^H V$ of $V \in \mathcal{I}_0^1(M_n)$ to $T_q^p(M_n)$ has components

$$ {}^H V^j = V^j, \quad {}^H V^{\bar{j}} = V^m \left(\sum_{\mu=1}^{q} \Gamma_{mj_\mu}^s t_{j_1 \ldots s \ldots j_q}^{i_1 \ldots i_p} - \sum_{\lambda=1}^{p} \Gamma_{ms}^{i_\lambda} t_{j_1 \ldots j_q}^{i_1 \ldots s \ldots i_p} \right) \tag{1.2} $$

with respect to the coordinates $\left(x^j, x^{\bar{j}}\right)$ in $T_q^p(M_n)$, where Γ_{ij}^k are local components of ∇ in M_n.

Suppose that there is given a tensor field $\xi \in \mathcal{I}_q^p(M_n)$. Then the correspondence $x \mapsto \xi_x$, ξ_x being the value of ξ at $x \in M_n$, determines a mapping $\sigma_\xi : M_n \mapsto T_q^p(M_n)$, such that $\pi \circ \sigma_\xi = id_{M_n}$, and the n dimensional submanifold $\sigma_\xi(M_n)$ of $T_q^p(M_n)$ is called the cross-section determined by ξ. If the tensor field ξ has the local components $\xi_{k_1 \ldots k_q}^{l_1 \ldots l_p}(x^k)$, the cross-section $\sigma_\xi(M_n)$ is locally expressed by

$$ \begin{cases} x^k = x^k \\ x^{\bar{k}} = \xi_{k_1 \ldots k_q}^{l_1 \ldots l_p}(x^k) \end{cases} \tag{1.3} $$

with respect to the coordinates $\left(x^k, x^{\bar{k}}\right)$in $T_q^p(M_n)$. Differentiating (1.3) by x^j, we see that the n tangent vector fields B_j to $\sigma_\xi(M_n)$ have components

$$ \left(B_j^K\right) = \left(\frac{\partial x^K}{\partial x^j}\right) = \begin{pmatrix} \delta_j^k \\ \partial_j \xi_{k_1 \ldots k_q}^{l_1 \ldots l_p} \end{pmatrix}, \tag{1.4} $$

with respect to the natural frame $\{\partial_k, \partial_{\bar{k}}\}$ in $T_q^p(M_n)$.

On the other hand, the fibre is locally expressed by

$$ \begin{cases} x^k = const\ , \\ t_{k_1 \ldots k_q}^{l_1 \ldots l_p} = t_{k_1 \ldots k_q}^{l_1 \ldots l_p}\ , \end{cases} $$

$t_{k_1 \ldots k_q}^{l_1 \ldots l_p}$ being considered, as parameters. Thus, on differentiating with respect to $x^{\bar{j}} = t_{j_1 \ldots j_q}^{i_1 \ldots i_p}$, we see that the n^{p+q} tangent vector fields $C_{\bar{j}}$ to the fibre have components

$$ \left(C_{\bar{j}}^K\right) = \left(\frac{\partial x^K}{\partial x^{\bar{j}}}\right) = \begin{pmatrix} 0 \\ \delta_{k_1}^{j_1} \ldots \delta_{k_q}^{j_q} \delta_{i_1}^{l_1} \ldots \delta_{i_p}^{l_p} \end{pmatrix} \tag{1.5} $$

with respect to the natural frame $\{\partial_k, \partial_{\bar{k}}\}$ in $T_q^p(M_n)$.

We consider in $\pi^{-1}(U) \subset T_q^p(M_n)$, $n + n^{p+q}$ local vector fields B_j and $C_{\bar{j}}$ along $\sigma_\xi(M_n)$. They form a local family of frames $\{B_j, C_{\bar{j}}\}$ along $\sigma_\xi(M_n)$, which is called the adapted (B, C)- frame of $\sigma_\xi(M_n)$ in $\pi^{-1}(U)$. Taking account of (1.1), (1.2), (1.4) and (1.5) we can easily prove that, the lifts ${}^{V}A$ and ${}^{H}V$ have, respectively, along $\sigma_{\xi}(M_n)$ components of the form

$$ {}^{V}A = \begin{pmatrix} {}^{V}\tilde{A}^{j} \\ {}^{V}\tilde{A}^{\bar{j}} \end{pmatrix} = \begin{pmatrix} 0 \\ A^{i_1 \dots i_p}_{j_1 \dots j_q} \end{pmatrix}, \tag{1.6} $$

$$ {}^{H}V = \begin{pmatrix} {}^{H}\tilde{V}^{j} \\ {}^{H}\tilde{V}^{\bar{j}} \end{pmatrix} = \begin{pmatrix} V^{j} \\ -(\nabla_V \xi)^{i_1 \dots i_p}_{j_1 \dots j_q} \end{pmatrix} \tag{1.7} $$

with respect to the adapted (B,C)- frame, where $(\nabla_V \xi)^{i_1 \dots i_p}_{j_1 \dots j_q}$ are local components of $\nabla_V \xi$ in M_n.

2. Horizontal Lifts of the affinor field to the tensor bundle along a cross-section

In each coordinate neighborhood $U\left(x^{j}\right)$ of M_n, we put

$$ X_{(j)} = \frac{\partial}{\partial x^{j}} \in \mathcal{I}^1_0(M_n), \; A^{(\bar{j})} = \partial_{j_1} \otimes \dots \otimes \partial_{j_p} \otimes dx^{i_1} \otimes \dots \otimes dx^{i_q} \in \mathcal{I}^p_q(M_n), $$

where $\bar{j} = n+1, \dots, n+n^{p+q}$. The local vector fields ${}^{H}X_{(j)}$ and ${}^{V}A^{(\bar{j})}$ span the module of vector fields in $\pi^{-1}(U)$. Hence any tensor field is determined in $\pi^{-1}(U)$ by its action of ${}^{H}X_{(j)}$ and ${}^{V}A^{(\bar{j})}$.

Let $\varphi \in \mathcal{I}^1_1(M_n)$. We define a tensor field ${}^{H}\varphi \in \mathcal{I}^1_1(T^p_q(M_n))$ along the cross-section $\sigma_{\xi}(M_n)$ by

$$ \begin{cases} {}^{H}\varphi\left({}^{H}V\right) = {}^{H}(\varphi(V)), & \forall V \in \mathcal{I}^1_0(M_n), \\ {}^{H}\varphi\left({}^{V}A\right) = {}^{V}(\varphi(A)), & \forall A \in \mathcal{I}^p_q(M_n), \end{cases} \tag{2.1} $$

where $\varphi(A) \in \mathcal{I}^p_q(M_n)$ and call ${}^{H}\varphi$ the horizontal lift of $\varphi \in \mathcal{I}^1_1(M_n)$ to $T^p_q(M_n)$ along $\sigma_{\xi}(M_n)$.

Let ${}^{H}\tilde{\varphi}^{K}_{L}$ be components of ${}^{H}\varphi$ with respect to the adapted (B,C)- frame the croos-section $\sigma_{\xi}(M_n)$. Then, from (2.1) we have

$$ \begin{cases} {}^{H}\tilde{\varphi}^{K}_{L} \; {}^{H}\tilde{V}^{L} = {}^{H}\left(\widetilde{\varphi(V)}\right)^{K}, & (i) \\ {}^{H}\tilde{\varphi}^{K}_{L} \; {}^{H}\tilde{A}^{L} = {}^{V}\left(\widetilde{\varphi(A)}\right)^{K}, & (ii), \end{cases} \tag{2.2} $$

where $\left({}^{V}\left(\widetilde{\varphi(A)}\right)^{K}\right) = \begin{pmatrix} 0 \\ {}^{V}\left(\widetilde{\varphi(A)}\right)^{\bar{k}} \end{pmatrix} = \begin{pmatrix} 0 \\ \varphi^{l_1}_{m} A^{m l_2 \dots l_p}_{k_1 \dots k_q} \end{pmatrix}, \quad p > 0.$

First, consider the case where $K = k$. In this case, *(i)* of (2.2) reduces to

$$ {}^{H}\tilde{\varphi}^{k}_{l} \; {}^{H}\tilde{V}^{l} + {}^{H}\tilde{\varphi}^{k}_{\bar{l}} \; {}^{H}\tilde{V}^{\bar{l}} = {}^{H}\left(\widetilde{\varphi(V)}\right)^{k} = (\varphi(V))^{k} = \varphi^{k}_{l} V^{l}. \tag{2.3} $$

Since the right-hand side of (2.3) are functions depending only on the base coordinates x^{i}, the left-hand side of (2.3) are too. Then, since ${}^{H}\tilde{V}^{\bar{l}}$ depend on fibre coordinates, from (2.3) we obtain

$$ {}^{H}\tilde{\varphi}^{k}_{\bar{l}} = 0, \tag{2.4} $$

which implies

$$^{H}\tilde{\varphi}_{l}^{k}=\varphi_{l}^{k}. \tag{2.5}$$

When $K=k$, *(ii)* of (2.2) can be rewritten, by virtue of (1.6), (2.4) and (2.5), as $0=0$.

When $K=\bar{k}$, *(ii)* reduces to

$$^{H}\tilde{\varphi}_{l}^{\bar{k}}\ {}^{V}\tilde{A}^{l}+{}^{H}\tilde{\varphi}_{\bar{l}}^{\bar{k}}\ {}^{V}\tilde{A}^{\bar{l}}={}^{V}\left(\varphi\tilde{(A)}\right)^{\bar{k}}$$

or

$$^{H}\tilde{\varphi}_{\bar{l}}^{\bar{k}}A_{r_1\ldots r_q}^{s_1\ldots s_p}=\varphi_{m}^{l_1}A_{k_1\ldots k_q}^{ml_2\ldots l_p}=\varphi_{s_1}^{l_1}\delta_{s_2}^{l_2}\ldots\delta_{s_p}^{l_p}\delta_{k_1}^{r_1}\ldots\delta_{k_q}^{r_q}A_{r_1\ldots r_q}^{s_1\ldots s_p},$$

for all $A\in\mathcal{I}_1^1(M_n)$, which implies

$$^{H}\tilde{\varphi}_{\bar{l}}^{\bar{k}}=\varphi_{s_1}^{l_1}\delta_{s_2}^{l_2}\ldots\delta_{s_p}^{l_p}\delta_{k_1}^{r_1}\ldots\delta_{k_q}^{r_q},\quad\left(x^{\bar{l}}=t_{r_1\ldots r_q}^{s_1\ldots s_p},x^{\bar{k}}=t_{k_1\ldots k_q}^{l_1\ldots l_p}\right) \tag{2.6}$$

where δ_k^r is the Kronecker symbol.

When $K=\bar{k}$, (i) of (2.2) reduces to

$$^{H}\tilde{\varphi}_{l}^{\bar{k}}\ {}^{H}\tilde{V}^{l}+{}^{H}\tilde{\varphi}_{\bar{l}}^{\bar{k}}\ {}^{H}\tilde{V}^{\bar{l}}={}^{H}\left(\varphi\tilde{(V)}\right)^{\bar{k}}. \tag{2.7}$$

Making use of the Vishnevskii operator we shall investigate components $^{H}\tilde{\varphi}_{l}^{\bar{k}}$. Now we consider the Vishnevskii operator on the module $\mathcal{I}_q^p(M_n)$ [2, p.184]:

$$(\Phi_\varphi\xi)_{kj_1\ldots j_q}^{i_1\ldots i_p}=\varphi_k^m\nabla_m\xi_{j_1\ldots j_q}^{i_1\ldots i_p}-\begin{cases}\varphi_m^{j_1}\nabla_k\xi_{j_1\ldots j_q}^{mi_2\ldots i_p}, & p>0\\ \varphi_{j_1}^{m}\nabla_k\xi_{mj_2\ldots j_q}^{i_1\ldots i_p}, & q>0.\end{cases} \tag{2.8}$$

Remark. When $\Phi_\varphi\xi=0$ for a pure tensor field ξ and for φ-connection ($\nabla\varphi=0$) in M_n with complex structure φ,ξ is said to be analytic.

Adopt the case where $p>0$, for example. After some calculations, from (2.8) we have

$$V^k(\Phi_\varphi\xi)_{kj_1\ldots j_q}^{i_1\ldots i_p}+\varphi_m^{i_1}\nabla_V\xi_{j_1\ldots j_q}^{mi_2\ldots i_p}=\nabla_{\varphi\ V}\xi_{j_1\ldots j_q}^{i_1\ldots i_p}, \tag{2.9}$$

for any $V\in\mathcal{I}_0^1(M_n)$ with local components V^k.

Using (1.7) from (2.9) we have

$$V^l(\Phi_\varphi\xi)_{lk_1\ldots k_q}^{l_1\ldots l_p}+\varphi_l^{l_1}\nabla_V\xi_{k_1\ldots k_q}^{ll_2\ldots l_p}=$$

$$=V^l(\Phi_\varphi\xi)_{lk_1..k_q}^{l_1\ldots l_p}+\varphi_{s_1}^{l_1}\delta_{s}^{l_2}\ldots\delta_{s_p}^{l_p}\delta_{k_1}^{r_1}\ldots\delta_{k_q}^{r_q}\nabla_V\xi_{r_1\ldots r_q}^{s_1\ldots s_p}=$$

$$={}^{H}V^l(\Phi_\varphi\xi)_{lk_1\ldots k_q}^{l_1\ldots l_p}-\varphi_{s_1}^{l_1}\delta_{s}^{l_2}\ldots\delta_{s_p}^{l_p}\delta_{k_1}^{r_1}\ldots\delta_{k_q}^{r_q}\ {}^{H}V^{\bar{l}}=-{}^{H}\left(\varphi\tilde{(V)}\right)^{\bar{k}}$$

or

$$^{H}V^l(\Phi_\varphi\xi)_{lk_1\ldots k_q}^{l_1\ldots l_p}-\varphi_{s_1}^{l_1}\delta_{s}^{l_2}\ldots\delta_{s_p}^{l_p}\delta_{k_1}^{r_1}\ldots\delta_{k_q}^{r_q}\ {}^{H}V^{\bar{l}}=-{}^{H}\left(\varphi\tilde{(V)}\right)^{\bar{k}}. \tag{2.10}$$

From (2.6), (2.7) and (2.10) we write

$$\left({}^{H}\tilde{\varphi}_{l}^{\bar{k}}+(\Phi_\varphi\xi)_{lk_1\ldots k_q}^{l_1\ldots l_p}\right)V^l=0$$

or

$$^{H}\tilde{\varphi}_{l}^{\bar{k}}=-\left(\Phi_{\varphi}\xi\right)_{lk_1\ldots k_q}^{l_1\ldots l_p}.$$

Thus the complete lift $^{c}\varphi$ of φ has along the pure cross-section $\sigma_{\xi}^{\varphi}\left(M_n\right)$ components

$$\begin{cases} ^{H}\varphi_{l}^{k}=\varphi_{l}^{k}, & ^{H}\varphi_{\bar{l}}^{k}=0, \\ ^{H}\varphi_{l}^{\bar{k}}=-\left(\Phi_{\varphi}\xi\right)_{lk_1\ldots k_q}^{l_1\ldots l_p}, & ^{H}\varphi_{\bar{l}}^{\bar{k}}=\begin{cases} \varphi_{s_1}^{l_1}\delta_{s_2}^{l_2}\ldots\delta_{s_p}^{l_p}\delta_{k_1}^{r_1}\ldots\delta_{k_q}^{r_q}, & p>0 \\ \delta_{s_1}^{l_1}\ldots\delta_{s_p}^{l_p}\varphi_{k_1}^{r_1}\delta_{k_2}^{r_2}\ldots\delta_{k_q}^{r_q}, & q>0 \end{cases} \end{cases} \tag{2.11}$$

with respect to the adapted (B,C)- frame of $\sigma_{\xi}^{\varphi}\left(M_n\right)$.

3. Transfer of the Horizontal lift

Let M_n be a paracompact manifold with a Riemannian metric. We shall mean by the Riemannian metric a symmetric covariant tensor field g of degree 2 which is nondegenerate. If g is a pure tensor field $\left(g\left(\varphi X,Y\right)=g\left(X,\varphi Y\right),\forall X,Y\in\mathcal{I}_0^1\left(M_n\right)\right)$, then a manifold M_n with an affinor φ-structure is called an almost B-manifold [2, p.31] and this will be denoted by V_n.

Suppose that $T_1^1\left(V_n\right)$ and $T_2^0\left(V_n\right)$ are the tensor bundle of type (1,1) and (0,2) over V_n, respectively. Clearly that $\dim T_1^1\left(V_n\right)=\dim T_2^0\left(V_n\right)=n+n^2$. Let the diffeomorphism $f:T_1^1\left(V_n\right)\to T_2^0\left(V_n\right),\ y^I=y^I\left(x^J\right),\ I,\ J=1,\ldots,n+n^2$, be defined by a local expression such that

$$\left.\begin{array}{l} y^{i}=x^{i}\ , \\ y^{\bar{i}}=t_{ij_1}=g_{im}t_{j_1}^{m}\ . \end{array}\right\}$$

Since

$$x^{\bar{k}}=t_{k_1}^{l_1},$$

$$\frac{\partial y^{\bar{i}}}{\partial x^{\bar{k}}}=\frac{\partial}{\partial x^{\bar{k}}}\left(t_{ij_1}\right)=\frac{\partial}{\partial x^{\bar{k}}}\left(g_{im}t_{j_1}^{m}\right)=\frac{\partial}{\partial x^{\bar{k}}}\left(g_{il_1}t_{k_1}^{l_1}\delta_{j_i}^{k_1}\right)=g_{il_1}\delta_{j_1}^{k_1},$$

$$0=\frac{\partial y^{\bar{i}}}{\partial x^{k}}=\frac{\partial t_{ij_1}}{\partial x^{k}}=\frac{\partial}{\partial x^{k}}\left(g_{im}t_{j_1}^{m}\right)=\left(\partial_k g_{im}\right)t_{j_1}^{m},$$

we have

$$A=\left(\frac{\partial y^I}{\partial x^K}\right)=\begin{pmatrix} \frac{\partial y^i}{\partial x^k} & \frac{\partial y^i}{\partial x^{\bar{k}}} \\ \frac{\partial y^{\bar{i}}}{\partial x^k} & \frac{\partial y^{\bar{i}}}{\partial x^{\bar{k}}} \end{pmatrix}=\begin{pmatrix} \delta_k^i & 0 \\ 0 & \delta_{j_1}^{k_1}g_{il_1} \end{pmatrix}.$$

The inverse of the mapping f is written as

$$x^{l}=y^{l},\quad x^{\bar{l}}=t_{r_1}^{s_1}=g^{s_1m}t_{mr_1}.$$

Suppose that $y^{\bar{j}}=t_{ll_1}$, we have

$$A^{-1}=\left(\frac{\partial x^L}{\partial y^J}\right)=\begin{pmatrix} \delta_j^l & 0 \\ 0 & g^{s_1l}\delta_{r_1}^{l_1} \end{pmatrix}.$$

which is the Jacobian matrix of inverse mapping f^{-1}.

Theorem. *Suppose that* $^{H}\underset{1}{\varphi}$ *and* $^{H}\underset{2}{\varphi}$ *denote the horizontal lifts of the affinor field* φ *to* $T_1^1\left(V_n\right)$ *and* $T_2^0\left(V_n\right)$ *along the cross-section* $\xi_j^i\left(x\right)$ *and* $\xi_{ij}\left(y\right)$*, respectively. If* $\Phi_{\varphi}g=0$*, then* $^{H}\underset{2}{\varphi}$ *is transferred from* $^{H}\underset{1}{\varphi}$ *by means of the diffeomorphism* f*.*

Proof. Let $\Phi_\varphi g = 0$. Taking account of $g_{mj}\varphi_i^m = g_{im}\varphi_j^m$ and (2.11), we obtain

$$\overset{H}{\underset{2}{\varphi}} = \left(\overset{H}{\underset{2}{\varphi}}{}^I_J\right) =$$

$$= \begin{pmatrix} \varphi_j^i & 0 \\ -\left(\Phi_\varphi \xi\right)_{j_i j_1} & \varphi_i^l \delta_{j_1}^{l_1} \end{pmatrix} =$$

$$= \begin{pmatrix} \varphi_j^i & 0 \\ -g_{im}\left(\Phi_\varphi \xi\right)_{jj_1}^m - \left(\Phi_\varphi g\right)_{jim} \xi_{j_1}^m & \varphi_i^l \delta_{j_1}^{l_1} \end{pmatrix} =$$

$$= \begin{pmatrix} \varphi_j^i & 0 \\ -g_{im}\left(\Phi_\varphi \xi\right)_{jj_1}^m & \varphi_i^l \delta_{j_1}^{l_1} \end{pmatrix} =$$

$$= \begin{pmatrix} \delta_k^i & 0 \\ 0 & \delta_{j_1}^{k_1} g_{il_1} \end{pmatrix} \begin{pmatrix} \varphi_l^k & 0 \\ -\left(\Phi_\varphi \xi\right)_{lk_1}^{l_1} & \delta_{k_1}^{r_1} \varphi_{s_1}^{l_1} \end{pmatrix} \begin{pmatrix} \delta_j^l & 0 \\ 0 & g^{s_1 l} \delta_{r_1}^{l_1} \end{pmatrix},$$

where $y^{\bar{i}} = t_{ij_1}$ and $y^{\bar{j}} = t_{ll_1}$.

Corollary. *If ∇ is a Riemannian connection, then $\overset{H}{\underset{2}{\varphi}}$ is transferred from $\overset{H}{\underset{1}{\varphi}}$ by means of the diffeomorphism f.*

References

[1]. Ledger A., Yano K. *Almost complex structures on tensor bundles.* J.Diff.Geom., 1(1967), No4, pp.355-366.

[2]. Vishnevskii V.V., Shirokov A.P., Shurygin V.V. *Spaces over algebras.* Kazan Gos. Univ., Kazan, 1985, 285 p. (Russian)

Muhamed Kamali, Arif A. Salimov
Ataturk University,
Faculty of Arts and Sciences. Department of Mathematics.
25240, Erzurum, Turkey.

Received September 03, 2001; Revised May 19, 2003.
Translated by authors.