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PARAMETRIC VIBRATIONS OF NONLINEAR-VISCOELASTIC ROD

Abstract

The rod having the thin-walled structural elements is widely used in techniques. There arise difficulties in analysis of these elements allowing for physical and geometrical nonlinearity and so such problem is solved by the variatioanal method.

In this paper the parametric oscillations of rod allowing for physical and geometrical nonlinearity in elastic medium is investigated by variational method and the characteristic curves are constructed.

The rods having the thin-shelled structural elements are widely used in engineering. There arise difficulties in analysis of these elements allowing for physical and geometrical non-linearity in medium though thin-shelled constructions are simpler by their one-dimensional property. In spite of this, for full description of the carrying capacity of such constructions reliable strength analysis should be performed. The medium is modelled by ground and its influence is considered by the Winkler model.

The solution of such type problems is mathematically difficult, that is extended with regard for dynamical effects that is necessary in problems of constructions of buildings and structures, in vibration problems and etc. In this case we use the variational principle. The application of the variational method is dictated not only by the convinience of numerical calculation, in this case one can learn non-contradictary theory of thin-shelled constructions.

Parametric vibrations of linear, inhomogeneous on thickness bar with regard to physical and geometrical nonlinearity in elastic medium are considered.

The problem is solved by the variational method. Characteristic curves of dependence are constructed.

Let's consider a nonhomogeneous on thickness linear, rectangular in the plan rod of thickness $2h$, length l in the ground. Let a uniformly distributed load of intensity

$$p = p_0 + p_1 \sin \omega_1 t,$$

act on this rod along the upper end wall. Here p_0 is the basic load, p_1 is the load change amplitude, ω_1 is its frequency. It is obvious that the consideration of such a load admits to study a more general case of time dependence of pavement load. Substitute the action of the ground on the rod by the force q_0 distributed along the length of the rod and proportional to the deflection of the bar. It is assumed that the lower end-wall of the rod is rigidly fixed:

$$x=l,\ W=0,\ \frac{\partial W}{\partial x}=0.$$

To study parametric vibrations of bar in the ground we shall use the variational principle.

In conformity to our case we can write the functional in the following form [1]:

$$J=\int_{t_1}^{t_2}\int_{0}^{l}\left\{\dot N\left(\frac{\partial\dot u}{\partial x}+\frac{\partial w}{\partial x}\frac{\partial\dot w}{\partial x}\right)-\dot M\frac{\partial^2\dot W}{\partial x^2}+\frac{1}{2}N\left(\frac{\partial w}{\partial x}\right)^2-\right.$$

$$-\frac{1}{8h^2}\left(\frac{\dot N^2}{E_0}+\frac{6}{h^2}\frac{1}{E_1}\dot N\ \dot M+\frac{9}{h^4}\frac{1}{E_2}\dot M^2\right)-\frac{1}{4h^2}N\times$$

$$\times\left[\int_{t_0}^{t}\left[K_0(t-\tau)N+\frac{3}{h^2}K_1(t-\tau)M\right]d\tau\right]-\frac{3}{4h^4}\dot M\times$$

$$\left.\times\left[\int_{0}^{t}\left[K_1(t-\tau)N+\frac{3}{h^2}K_2(t-\tau)M\right]d\tau\right]-\frac{1}{2}\rho_0\left(\frac{\partial\dot W}{\partial x}\right)^2\right\}dxdt+$$

$$+\int_{0}^{t}\left[\dot u(l)-\dot u(0)\right]2\dot\rho hdt+\int_{t_1}^{t_2}\int_{0}^{l}k\frac{\dot W^2}{2}dtdx,\qquad(1)$$

where $\frac{1}{E_i}=\int\limits_{-h}^{h}\frac{z^i}{E(z)}dz$, ρ_0 is density, t_1 is the beginning of time reference, t_2 is the end of time in the course of which the process is studied, $K_1(t-\tau),\ K_1(t-\tau)$ is the creeping kernal, uW are the permutation vector components of rod's points.

Since we shall investigate parametric vibrations, i.e. we'll consider periodic vibrations, then on this basis of the variational principle, we'll accept

$$t_2=t_1+T,$$

where T is the period of parametric vibrations. For simplicity we accept $t_1=0$. Proceeding from the variational principle we should set iitial conditions at the moment $t=t_1$ and $t=t_2$. In view of periodicity of the desired solution there is no necessity in this. Note that a periodic solution exists if the creeping kernel is damping. In this connection periodicity will appear at large values of time. At differentiations with respect to parameter, it is necessary to understand the parameter characterizing the loading. In the considered case the loading depends on two parameters P_0 and P_1. In general case they are connected with one another. But if there was a problem by solving of which it was necessary to give only specific values of P_0 and P_1 then one could determine their final values one by the another and to take for example

P_0 for a differentiation parameter. For the present problem it will be necessary to construct dependences in which one of parameters will change. Then calculation is complicated. In this case, it is appropriate to introduce two differentiation parameters. Initially we accept $P_0 = 0$, for differentiation parameter we take P_0. The system of equations, is solved up to specific value of P_0. Then we take P_1 as a differentiation operator. The initial conditions in this case have already been solved at the previous stage. Thereby the problem may be solved in two stages.

In differential (1), $\dot{W}, \dot{M}, \dot{N}$ are the varying quantities. Let's find its stationary value. Initially let's vary it with respect to $\dot{u}$. With regard to above said $\dot{u}$ is excluded from consideration. As a result functional (1) has the form:

$$J = \int\limits_{t_1}^{t_2}\int\limits_0^l \left\{ -2h\dot{\rho}\frac{\partial w}{\partial x}\frac{\partial \dot{w}}{\partial x} + M + h\rho\left(\frac{\partial w}{\partial x}\right)^2 - \right.$$

$$-\frac{1}{8h^2}\left(\frac{4h^2\dot{\rho}}{E_0} + \frac{12\dot{\rho}}{E_1 h}\dot{M} + \frac{9}{h^4}\frac{1}{E_2}\dot{M}^2\right) + \frac{1}{2h}\dot{\rho}^2\times$$

$$\times\left[\left[\int\limits_0^t K_0(t-\tau)(-2\rho h) + \frac{3}{h^2}K_1(t-\tau)M\right]d\tau\right] - \frac{3}{4h^4}\dot{M}\times$$

$$\left.\times\left[\int\limits_0^t\left[K_1(t-\tau)(-2\rho h) + \frac{3}{h^2}K_2(t-\tau)M\right]d\tau\right] - \frac{1}{2}\rho_0\left(\frac{\partial w}{\partial x}\right)^2\right\} dxdt +$$

$$+\int\limits_{t_1}^{t_2}\int\limits_0^l k\frac{\dot{W}}{2}dtdx. \qquad (2)$$

It is obvious that the stationary value of the cited functional (2) as the stationary value of functional (1) is determined by solving the stated problem. In functional (2) the number of varying quantities are $\dot{M}$ and $\dot{W}$. We find them by using the Rietz method. Proceeding from the expected behaviour of the rod we take the approximation for the desired quantities as follows:

$$W = \sin\frac{\pi x}{l}\left(W_0\cos\omega t + W_1\sin\omega t\right),$$

$$M = \sin\frac{\pi x}{l}(M_0\cos\omega t + M_{1\,\sin}\omega t), \qquad (3)$$

where W is the vibration frequency. In a general case the frequency of stationary vibrations is the unknown quantity which is to be found from the stationary state condition of the functional. But at present case we are interested in the beginning of parametric resonance depending on external parameters.

Based on the Reitz method we put approximation (3) into the expression of functional (2) and integrate it with respect to variables x and t. As a result, we get

$$J=\int_{t_1}^{t_2}\int_{0}^{l}-2h\dot{\rho}\frac{\pi}{2}\left(W_0\dot{W}_0+W_1\dot{W}_1\right)\left(\frac{\pi}{l}\right)^2\frac{\pi}{2}-2h\rho_1\times$$

$$\times\left(\dot{W}_0W_1+W_0\dot{W}_1\right)\frac{\pi^2}{8}\left(\frac{\pi}{l}\right)^2+\left(\frac{\pi}{l}\right)^2\frac{\pi^2}{4}\left(\dot{M}_0\dot{W}_0+\dot{M}_1\dot{W}_1\right)-$$

$$-h\left(\frac{\pi}{l}\right)^2\frac{\pi}{4}\left\{\rho_0\left(\dot{W}^2+\dot{W}_1^2\right)+\frac{\rho_1}{2}\dot{W}_0\dot{W}_1\right\}-\frac{1}{8h^2}\times$$

$$\times\left\{4h^2\frac{\pi}{E_0}\left(\pi\;\dot{\rho}_0^2+\frac{\pi}{2}\dot{\rho}_1^2\right)-\frac{24}{E_1h}\left(2\dot{\rho}_0\dot{M}+\frac{4}{3}\dot{\rho}_1\dot{M}_0\right)+\right.$$

$$\left.+\frac{9}{h^4}\frac{\pi^2}{4E_2}\left(\dot{M}_0^2+\dot{M}_1^2\right)\right\}+\frac{\dot{\rho}_0}{2h}J_1+\frac{\dot{\rho}_1}{2h}J_2-\frac{3}{4h^4}\dot{M}_0\times$$

$$\times\left\{-2hp_1\Phi\Phi_{1c}^{(2)}\frac{8}{3}+\frac{3}{h^2}\frac{\pi^2}{4}\left(M_0\Phi_{2c}-M_1\Phi_{2s}\right)\right\}^2-$$

$$-\frac{3}{4h^4}\dot{M}_1\left\{-8h\rho_0\Phi_{10}-\frac{8}{3}h\rho_1\Phi_{1s}^{(2)}+\frac{3}{h^2}\frac{\pi^2}{4}\times\right.$$

$$\left.\times\left(M_0\Phi_{2s}-M_1\Phi_{2c}\right)\right\}-\frac{1}{2}\rho_0\frac{\pi^2}{4}\left(\dot{W}_0^2+\dot{W}_1^2\right)W^2+k\frac{\pi^2}{4}\dot{W}^2, \qquad (4)$$

where J_i are some functions from M_i and ρ_i not affecting on stationary values of the function J, Φ_{ic} and Φ_{is} are Fourier transform of its cosine and sine from the kernel $K_i(t-\tau)$, respectively.

Thus, finding the stationary value of functional (2) was led to finding the stationary value of function (2). This value is determined from the following system:

$$\frac{\partial J}{\partial\dot{W}_0}=0;\;\frac{\partial J}{\partial\dot{W}_1}=0;\;\frac{\partial J}{\partial\dot{M}_5}=0;\;\frac{\partial J}{\partial\dot{M}_8}=6. \qquad (5)$$

Based on the variational principle for the solution of the obtained system we take the following initial conditions: in the absence of load there is no stress and deflection in rods. System (5) is a quasilinear system of differential equations and calculated numerically.

The results of calculations are in Fig.1

The creeping kernel is accepted in tue form:

$$K(z,t)=\frac{a}{E(z)}e^{-\beta t},$$

where a is a mechanical parameter, β is an index of exponent. In this case Fourier transform is of the form:

$$\Phi_0=a\int_0^{\infty}e^{-\beta y}dy=\frac{a}{\beta};$$

$$\Phi_0 = a\int_0^\infty e^{-\beta y}\sin\omega y dy = a\frac{\omega}{\beta^2+\omega^2};$$

$$\Phi_c = a\int_0^\infty e^{-\beta y}\cos\omega y dy = a\frac{\beta}{\beta^2+\omega^2}.$$

For the convenience of calculations along with reduced above pure parameters we introduce the following quantities

$$W_0 = C_0 h;\ W_1 = C_1 h;\ \omega = \omega_0\beta;\ \rho_0 = \frac{\rho E_x h^3}{\rho^2};$$

$$\rho_i = \tau_i E_x h^3;\ \frac{1}{E_i = h^{i+1}}\frac{1}{E_x}\frac{1}{e_i}.$$

In fig. 1. the dash dot lines shows vibrations in ground. Calculations show that discount of ground influence leads to the critical load increase.

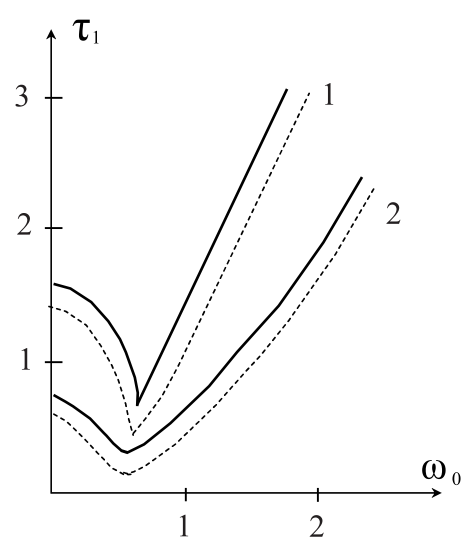

Fig. 1.

Fig. 1. The dependence of τ_1 from ω_0 for the following values of parameters

$$1:\alpha = -0,5;\ \beta_0 = 1;\ \tau_0 = 0,03;\ \rho_2 = 0,3;\ 2:\alpha = 0;\ k = 2.4\cdot 10^4 n/m^3.$$

References

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