Aydin I. MEKHTIYEV

BULGING-IN OF A RIGID TITLE BLOCK IN ANISOTROPIC HALF-PLANE WITH A RIGID ELLIPTICAL CORE FOR WANT OF FRICTION UNDER A TITLE BLOCK

Abstract

On the basis of the unknown S.C.Lekhnitskiy and L.A.Galin the algorithm of the solution is designed and these outcomes for a problem about bulging-in of a rigid title block without friction in an anisotropic half-plane keeping a rigid elliptical core are adduced. The solution is obtained in a manifestative analytic form to within a series of unknowns of factors, the definition which one is reduced to the solution of finite systems of linear algebraic equations.

On the basis of the known solutions of S.C.Lekhnitskiy and L.A.Calin the algorithm of the solution is designed and these outcomes for a problem about indentation of a rigid die without friction in an anisotropic half-plane, containing a rigid elliptical core are adduced.

The problem about action of the rigid die with plane base on the linear boundary of the anisotropic half-plane at absence of friction under a die, when half-plane contains an arbitrary situated rigid elliptical core is considered, fig.1

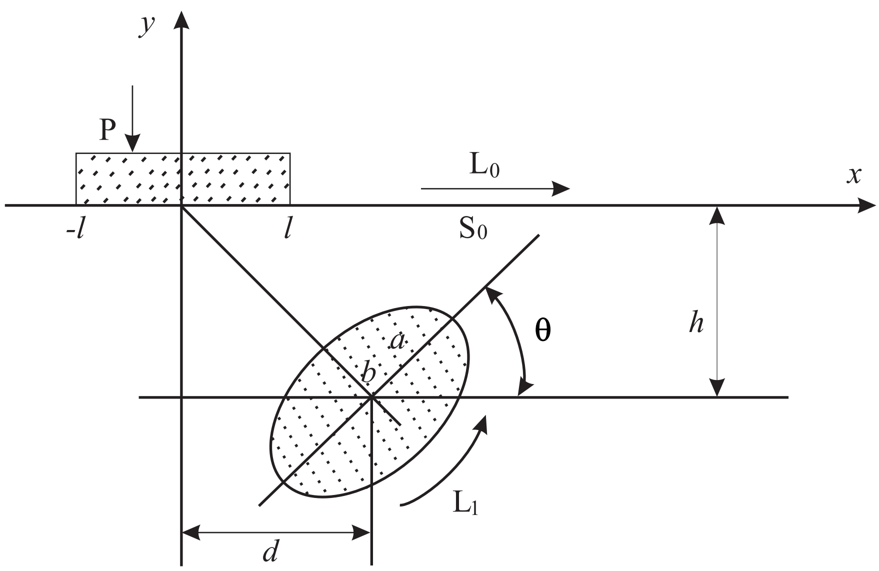

Fig. 1.

The real axis *ox* we'll direct along linear boundary of the half-plane, the axis *oy* is perpendicular to it aside an external normal. The width of the die's base is $2l$

and it is symmetric with respect to axis oy. The die is pressed in with the force P. The center of the burned elliptical core has the coordinates $(d; -h)$. The lengths of big and small semi-axis are equal to a and b, respectively, moreover θ is an angle of slope of big semi-axis to the horizontal coordinate Ox.

The physical boundary conditions will be the followings:

$$\begin{cases} \sigma_{yy} = 0; \ \sigma_{xy} = 0; \ y = 0; \ |x| > 0, \\ \\ \dfrac{\partial \nu}{\partial x} = 0; \ \sigma_{xy} = 0; \ y = 0; \ |x| < l, \end{cases} \tag{1}$$

$$u_x = 0; \ u_y = 0; \ (x, y) \in L_1. \tag{2}$$

As is known [1] a solution of the plane problem of anisotropic elasticity theory is reduced to finding a pair of complex potentials $\Phi_j(z_j)$ of the variables $z_j = x + \mu_j y$, where μ_j are complex roots of the corresponding characteristic equation, given by elastic modules of the half-plane material, for the orthotropic material they only imaginary $\mu_j = i\beta_j$. The functions $\Phi_j(z_j)$ are analytical in the complex planes z_j, corresponding to the domain s_0 in the physical plane z, where s_0 is a domain of the half-plane $y < 0$ except for the domain s_1 of the elliptical core with the contour L_1 (fig.1).

In terms of the functions $\Phi_j(z_j)$ boundary conditions (1), (2) will be written:

$$\begin{cases} \Phi_1'(t_0) + \overline{\Phi_1'(t_0)} + \Phi_2'(t_0) + \overline{\Phi_2'(t_0)} = 0, \\ \\ \mu_1 \Phi_1'(t_0) + \overline{\mu}_1 \overline{\Phi_1'(t_0)} + \mu_2 \Phi_2'(t_0) + \overline{\mu}_2 \overline{\Phi_2'(t_0)} = 0, \end{cases} , \tag{3}$$

$$t_0 = x; \ |t_0| > l.$$

$$\begin{cases} q_1 \Phi_1'(t_0) + \bar{q}_1 \overline{\Phi_1'(t_0)} + q_2 \Phi_2'(t_0) + \overline{q}_2 \overline{\Phi_2'(t_0)} = 0, \\ \\ \mu_1 \Phi_1'(t_0) + \overline{\mu}_1 \overline{\Phi_1'(t_0)} + \mu_2 \Phi_2'(t_0) + \overline{\mu}_2 \overline{\Phi_2'(t_0)} = 0, \end{cases} \tag{4}$$

$$t_0 = x; \ t_0 < l.$$

$$\begin{cases} p_1 \Phi_1(t_1) + \overline{p}_1 \overline{\Phi_1(t_1)} + p_2 \Phi_2(t_2) + \overline{p}_2 \overline{\Phi_2(t_2)} = 0, \\ \\ q_1 \Phi_1(t_1) + \overline{q}_1 \overline{\Phi_1(t_1)} + q_2 \Phi_2(t_2) + \overline{q}_2 \overline{\Phi_2(t_2)} = 0, \end{cases} \tag{5}$$

$$t_k = x + \mu_k y; \ (x, y) \in L_1,$$

where p_k and q_k, $k = 1; 2$ are coefficients, expressed by the known way [1] of elastic modules of the half-plane material. Acting as in [4] let us introduce on L_1 two unknown auxiliary functions $\omega_j(z_j)$; $j = 1, 2$ and using the theorem on analytical extensor of functions through contour, we'll obtain the following representation of the functions $\Phi_j(z_j)$ by functions $s_0 \cup s_1$ analytical in the all continuous half-plane $\Phi_{0,j}(z_j)$:

$$\Phi_j(z_j) = \Phi_{0,j}(z_j) - J_j(z_j). \tag{6}$$

Here

$$J_j(z_j) = \frac{1}{2\pi i} \int\limits_{L_{1,j}} \frac{\omega_j(t_j)}{t_j - z_j} dt_j, \tag{7}$$

where $L_{1,j}$ are contours, corresponding to the contour z_j in the complex planes L_1.

Taking into account representations (6) in boundary conditions (3), (4) we'll obtain the following auxiliary mixed boundary problem for entire lower half-plane:

$$\begin{cases} \Phi'_{0,1}(t_0) + \overline{\Phi'_{0,1}(t_0)} + \Phi'_{0,2}(t_0) + \overline{\Phi'_{0,2}(t_0)} = R(t_0); \quad |t_0| > l \\ \mu_1 \Phi'_{0,1}(t_0) + \overline{\mu}_1 \overline{\Phi'_{0,1}(t_0)} + \mu_2 \Phi'_{0,2}(t_0) + \overline{\mu}_2 \overline{\Phi'_{0,2}(t_0)} = Q(x); \qquad t_0 \in L_0 \\ q_1 \Phi'_{0,1}(t_0) + \overline{q}_1 \overline{\Phi'_{0,1}(t_0)} + q_2 \Phi'_{0,2}(t_0) + \overline{q}_2 \overline{\Phi'_{0,2}(t_0)} = M(x); \qquad |t_0| < l \end{cases} \tag{8}$$

where the following notation are taken:

$$\begin{cases} R(t_0) = J'_1(t_0) + \overline{J'_1(t_0)} + J'_2(t_0) + \overline{J'_2(t_0)}; \\ Q(t_0) = \mu_1 J'_1(t_0) + \overline{\mu}_1 \overline{J'_1(t_0)} + \mu_2 J'_2(t_0) + \overline{\mu}_2 \overline{J'_2(t_0)}; \\ M(t_0) = q_1 J'_1(t_0) + \overline{q}_1 \overline{J'_1(t_0)} + q_2 J'_2(t_0) + \overline{q}_2 \overline{J'_2(t_0)}. \end{cases} \tag{9}$$

The solution of boundary problem (8) we'll search as a sum of two functions:

$$\Phi'_{0,j}(z_j) = \varphi_j(z_j) + \psi_j(z_j).) \tag{10}$$

For the function $\varphi_j(z_j)$ we'll choose the boundary conditions in the following way:

$$\begin{cases} \varphi_1(t_0) + \overline{\varphi_1(t_0)} + \varphi_2(t_0) + \overline{\varphi_2(t_0)} = 0; \quad |\, t_0| > l \\ \mu_1 \varphi_1(t_0) + \overline{\mu}_1 \overline{\varphi_1(t_0)} + \mu_2 \varphi_2(t_0) + \overline{\mu}_2 \overline{\varphi_2(t_0)} = 0; \quad t_0 \in L_0 \\ q_1 \varphi_1(t_0) + \bar{q}_1 \overline{\varphi_1(t_0)} + q_2 \varphi_2(t_0) + \bar{q}_2 \overline{\varphi_2(t_0)} = V'(t_0); \quad |\, t_0| < l\ , \end{cases} \tag{11}$$

where

$$V'(t_0) = M(t_0) - \left[q_1 \psi_1(t_0) + \overline{q}_1 \overline{\psi_1(t_0)} + q_2 \psi_2(t_0) + \overline{q}_2 \overline{\psi_2(t_0)} \right]. \tag{12}$$

Then from (8) and (11) for the functions $\psi_j(z_j)$ we'll unambiguously obtain the following boundary conditions:

$$\begin{cases} \psi_1(t_0) + \overline{\psi_1(t_0)} + \psi_2(t_0) + \overline{\psi_2(t_0)} = R(t_0); \\ \mu_1 \psi_1(t_0) + \overline{\mu}_1 \overline{\psi_1(t_0)} + \mu_2 \psi_2(t_0) + \overline{\mu}_2 \overline{\psi_2(t_0)} = Q(t_0); \quad t_0 \in L_0. \end{cases} \tag{13}$$

Conditions (13) define first basic boundary problem for continuous lower half-plane and by [1] it's solution has the form:

$$\begin{cases} \psi_1(z_1) = \dfrac{1}{\mu_1-\mu_2}\dfrac{1}{2\pi i}\int\limits_{L_0}\dfrac{R(t_0)\mu_2-Q(t_0)}{t_0-z_1}dt_0; \\ \psi_2(z_2) = \dfrac{1}{\mu_2-\mu_1}\dfrac{1}{2\pi i}\int\limits_{L_0}\dfrac{R(t_0)\mu_1-Q(t_0)}{t_0-z_1}dt_0. \end{cases} \tag{14}$$

In its turn boundary conditions (11) correspond to the problem on indentation without half-plane, when a profile of the die's base is given by the function $y=V(x)$. Following the technique given in [2] it's solution will be:

$$\begin{cases} \varphi_1(z_1) = \dfrac{\mu_2}{2\pi(\mu_1-\mu_2)}\left[\dfrac{1}{s\chi(z_1)}\int\limits_{-l}^{l}\dfrac{\chi^+(t_0)V'(t_0)}{t_0-z_1}dt_0 - \dfrac{iP}{\chi(z_1)}\right]; \\ \varphi_2(z_2) = \dfrac{\mu_1}{2\pi(\mu_2-\mu_1)}\left[\dfrac{1}{s\chi(z_2)}\int\limits_{-l}^{l}\dfrac{\chi^+(t_0)V'(t_0)}{t_0-z_2}dt_0 - \dfrac{iP}{\chi(z_2)}\right], \end{cases} \tag{15}$$

where

$$\chi(z_j) = \sqrt{z_j^2-l^2}. \tag{16}$$

At that in (16)such branch for radical is chosen, that $\lim\limits_{z\to\infty}\chi(z)/z=1$, moreover s is a complex parameter, expressed by elastic constants of half-plane material as in [2].

So, formulas (6), (10), (14) and (15) completely define the required functions $\Phi_j(z_j)$ and thereby the solution of the given problem. However these functions are defined to within auxiliary functions $\omega_j(t_j)$ on the contours $L_{1,j}$. For their definition let's represent these functions as expansions into Fourier series on unit circle, on which the contours $L_{1,j} (j=1;2)$ are mapped by the following functions:

$$\begin{cases} z_j = \bar{z}_{0,j} + \dfrac{c_j e^{i\theta_j}}{2}\left(\rho_j\xi_j + \dfrac{1}{\rho_j\xi_j}\right); \\ t_j = \bar{z}_{0,j} + \dfrac{c_j e^{i\theta_j}}{2}\left(\rho_j\sigma + \dfrac{1}{\rho_j\sigma}\right), \end{cases} \tag{17}$$

where $\bar{z}_{0,j}=d-\mu_j h$; ξ_j are auxiliary complex planes, $\sigma=e^{i\alpha}$ is an affix of contour of the unit circle γ_j in them.

The inverse to (17) function will be:

$$\xi_j = \lambda_j(z_j) = \frac{z_j-\bar{z}_{0,j}+\sqrt{(z_j-\bar{z}_{0,j})^2-c_j^2e^{2i\theta_j}}}{\rho_j c_j e^{i\theta_j}}. \tag{18}$$

At that the function $\lambda_j(z_j)$ is analytically out of the contours $L_{1,j}$ in the complex planes z_j. In the radicals in (18) such branch is chosen, that $\lim\limits_{z_j\to\infty}\lambda_j(z_j)/z_j=1$.

On the contours $L_{1,j}$ it holds $\xi_j\Big|_{\gamma_j}=\lambda_j(z_j)\Big|_{L_{1,j}}=\sigma=e^{i\alpha}$.

The parameters c_j, ρ_j, θ_j included into (17) are expressed by the parameters a, b, d, h and μ_j as in [3].

So, expanding the functions $\omega_j(t_j)$ into Fourier series on the contours γ_j:

$$\omega_j(t_j(\sigma)) = \sum_{n=-\infty}^{\infty} \alpha'_{j,n}\sigma^n. \tag{19}$$

Taking into account them in representations (7) and using the properties of the Cauchy integrals, we'll obtain:

$$J_j(t_j) = \sum_{n=1}^{\infty} \alpha_{j,n}\lambda_j^{-n}(z_j), \tag{20}$$

where

$$\alpha_{j,n} = a'_{j,n}\rho_j^{-2n} - \alpha'_{-j,n}.$$

According to the properties of the functions $\lambda_j(z_j)$ the functions $J_j(z_j)$ (20) are analytical functions of the variable z_j out of the ellipses $L_{1,j}$ with the center at the point $\bar{z}_{0,j}$, including infinite point. The functions:

$$\bar{J}_j(z_j) = \sum_{n=1}^{\infty} \bar{\alpha}_{j,n}\bar{\lambda}_j^{-n}(z_j), \tag{21}$$

$$\bar{\lambda}_j(z_j) = \frac{z_j - z_{0,j} + \sqrt{(z_j - z_{0,j})^2 - c_j^2 e^{-2i\theta_j}}}{\rho_j c_j e^{i\theta_j}},$$

will be analytical functions of the variable z_j out of the ellipses $L_{2,j}$ with the center at the $z_{0,j}$, symmetrical to the ellipses $L_{1,j}$ with respect to linear boundary of half-plane.

Then the functions $J_j(z_j)$ and $\bar{J}_j(z_j)$ will be regular functions in the upper and lower half-planes, respectively. Using this from (14) on the base of the Cauchy integral's properties, we'll obtain

$$\begin{cases} \psi_1(z_1) = \dfrac{1}{\mu_1 - \mu_2}\left[(\bar{\mu}_1 - \mu_2)\,\bar{J}'_1(z_1) + (\bar{\mu}_2 - \mu_2)\,\bar{J}'_2(z_1)\right]; \\ \psi_2(z_2) = \dfrac{1}{\mu_2 - \mu_1}\left[(\bar{\mu}_2 - \mu_2)\,\bar{J}'_1(z_2) + (\bar{\mu}_2 - \mu_1)\,\bar{J}'_2(z_2)\right]. \end{cases} \tag{22}$$

So, representation of solution of the problem is obtained to unknown series of the constant coefficients $\alpha_{j,n}$ of series (20).

For their definition substituting the representation of solution (6), (10), (22) and (15) to boundary conditions (5) on the mapped unit contour γ_j, multiplying the obtained expression by $\dfrac{1}{2\pi i}\sigma^m d\sigma$ and integrating over contour of the unit circle, we'll have an infinite system of the real linear algebraic equations with respect to real and imagine parts of the unknown coefficients $\alpha_{j,n}$. After definition, by the known formulae [1] on the basis of the same representations of solution (6), (10), (22) and (15) it is possible to find steess at any point of half-plane.

References

[1]. Lekhnitsky S.C. *The theory of elasticity of an anisotropic body.* M.: 1977, p.416. (Russian)

[2]. Galin L.A. *The contact problems of the theory of elasticity and viscosity.* M.: 1980, p.304. (Russian)

[3]. Mekhtiyev A.I. T*o solution of the problem on indentation of a rigid die to anisotropic half-plane, weakened by the free elliptical hole, for absence of friction under a die.* "Nauch. Trudy" AzTu, 2002, No,3, pp.-19-28. (Russian)

[4]. Mekhtiyev A.I. *First boundary problem for anisotropic half-plane, weakened by elliptical hole, with built in edge.* Mechanics, machinovedeniye, 2002, No2, pp.15-18. (Russian)

Aydin I. Mekhtiyev
Azerbaijan Technical University.
25, H.Javid av., AZ1073, Baku, Azerbaijan.
Tel.:(99412) 439 14 56 (off.)

Received January 19, 2004; Revised May 24, 2004.
Translated by Mamedova Sh.N.