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THE SYSTEM OF NEGATIONLESS CALCULUS OF PREDICATES WITH "MEANINGFULNESS" OPERATOR

Abstract

The system of negationless intuitionistic calculus of predicates P_rC in the language, supplemented by the operator]-"meaningfulness" is offered. In such system the inferences, intended for proof of the meaningfulness of formulas and conclusion of the "meaningful" formulae, constructed just from the meaningful formulae are separated. It is shown, that the deductions satisfy to the to the requirement of the negationlessity and that the "meaningfulness" is expansion of the "nonemptiness" concept.

Use of negation in intuitionistic mathematics, and consideration of the "empty" (i.e. false for any mathematical object) predicates always rose a doubt from the point of view of "intuitive clarity" (the basic criterion of the intuitionistic discussions) [1].

The intuitionistic negation is the assumption in the form $A-\rfloor$ where $\perp$ is a contradiction (false proposition).

For this the intuitionistically understandable implication $A \to B$ is interpreted as an explicit presentation of some construction, finishing building the construction A up to construction B. In case of negation there is no mental construction of premise and therefore there cannot be an intuitively clear "finishing building" construction.

It is obvious, that use of empty predicates doesn't satisfy this criterion ("It cannot be intuitively clear the supposition on the existence of squire, which is a circle" [1]). First G.Griss [1,2] paid his attention to these problems. In the works, G.Griss besides methodological substantiations has developed some mathematical theories negation. G.Griss has suggested to use the ratio of distinguishability $\#$ (construction variant of the ratio $\neq$ (is not equal)). Griss's negation $n\ A(t)$ is understood as the implications

$$A(x) \to x \# t$$

Question on deductive possibilities for negationless mathematical theories was considered by Griss and in the works [3], [4]. It turned out, that the deductive possibilities of the negationless arithmetic, analysis and theory of types are sufficiently wide.

However, the formalization of the negationless logic represents significant difficulties connected with a series of collisions, originating thus to which it is necessary to refer the problem of the proof of "nonemptiness" of the formulas, the problem of implications, the problem of dysfunction.

As the history of the question shows, attempts to avoid collisions either did not exclude the remaining ones, or reduced to new, not so simply formulated ones.

The problem of is "nonemptiness" (syntactical criterion of which is a proof of the formulas $\exists x_1 ... \exists x_n A$) has led Griss to the assumption on necessity of "levelness" of the discussions presence of some "pre-mathematics", where the predicates would be checked on nonemptiness while "the math. problem is to draw conclusions" [2].

In present work the variant of calculus of predicates P_rG, in which the logic is separated from rules of construction of "nonempty" predicates is offered. As a matter of fact, here nonemptiness is replaced by of fact, here nonemptiness is replaced by the more wide concept which is called a "meaningfulness".

With this purpose, a symbol of the operator $]$ is added to the language of traditional calculus of predicates.

1. The system P_rG

1.1. The Language of the system includes the variables $(x, y, z, u, v...)$ and the parameters $(a, b, c, d, e, a_1, b_1)$, playing a role of free variables, and also the individual constants $(\xi, \eta, \xi, \xi, \eta)$. The language holds the predicate symbols: $\mathcal{R}_1', \mathcal{R}_2', ...$ and the logical constants $\&, \vee, \rightarrow, \forall, \exists,]$ (meaningfulness).

1.2. Two types of formulae is determined: M-formulae (correspond to ordinary construction of formulae) and G-formulae, formed by the rule: if A is M-formula, then $]\ A^{a_1,...,a_n;}_{x_1,...,x_n;}$ is G-formula ($A^{a_1,...,a_n;}_{x_1,...,x_n;}$ is a result of the substitution to A pairwise various variables $x_1, ..., x_n$ instead of the parameters $a_1, ..., a_n$ (pairwise various)) respectively.

1.3. The structure of a inference of formulae in the system P_rG (where M-conclusions differ and G-construction have a structure of the natural conclusion of Gentzen type)

The deductions are generated as formula trees constructed under the deduction rules. The final formula of a tree appears the deducible formula from the suppositions were at tops of this tree and not eliminated under defined agreements. The deduction of the formula as A from a set of the formulae Γ is written as $\Gamma + A$.

1.4. G-constructions rules.

It is supposed, that there is an effective way of generation of a set of formulae $\{\varphi_i\}_{i\in J}$, where P_i has the form $\mathcal{P}^{n_{k_i}}_{k_i}(t_1, ..., t_n)$ Each formula from this set is called axiom.

If A is an axiom, and B is obtained from A by substituting to it term's instead of parameters, then B we'll call a variant of the axiom A.

Information carrier (i.c.)

a) if $A = \mathcal{P}(\bar{t})$, then a single information carrier is itself.

b) $A = \mathcal{B}\&C$, then any formula of the form $B^c\&C^c$, where B^c and C^c are information carriers of formulae B and C, respectively, is the information carrier of the formula A.

c) if $A = \mathcal{B}\vee C$, then B^c and C^c (B^c and C^c are any information carriers for B and C respectively) are information carriers for A.

d) if $A = \mathcal{B} \rightarrow C$, then the formula $\mathcal{B}^c\&B^c \rightarrow C^c$ is the information carrier for A.

e) $A = \forall x\mathcal{B}$ is the information carrier for A. It and only it .

f) if $A = \exists x\mathcal{B}^a_x$, then any formula of the form $\exists x\,(\mathcal{B}^c)^a_x$ is the information carrier for A.

g) Any formula is its own information carrier. The sense of the notion of information carrier is such that from the "meaningfulness" of any data carrier of the formula A it is possible to bring out the "meaningfulness" of the formula A.

1.5. Rules of the G-constructions for G-formulas

$$]\&\frac{]A\ ]B\ \ (A\&B)^{c\ \bar{a}}_{\bar{t}}}{]\,(A\&B)^{\bar{a}}_{\bar{x}}}\qquad]\vee\ \frac{]A\ ]B}{]\ (A\vee B)}$$

$$\rceil \to \frac{\rceil A \ \rceil B \ \ (A \to B)^{c\ \bar{a}}_{\bar{t}}}{\rceil\,(A \to B)^{\bar{a}}_{\bar{x}}}\ \rceil$$

$$\rceil\forall \frac{\forall x A}{\rceil\,(\forall x A)} \qquad \rceil\exists \frac{\rceil A^{\bar{a}\ \ b}_{\bar{x}\ \ t}}{\rceil\ \left(\exists y\,(A^{\bar{a}}_{\bar{x}})^{b}_{t}\right)}$$

$$\rceil_1 \frac{A^{\bar{a}}_{\bar{t}}}{\rceil\,(A^{\bar{a}}_{\bar{x}})} \qquad \rceil_2 \frac{\rceil A^{a}_{t}}{\rceil\ (A^{a}_{x})}$$

If $\rceil A$ is the root of the conclusion formula of the formula tree generated under the G-constructions rules, then we'll say, that $\rceil A$ is G-deducible.

Rules of the M-deduction.

$I - rules$ (*introduction of a operator*) $E - rules$ (*elimination of a operator*)

$$\& \frac{A\ B\ \rceil\ (A\&B)^{\bar{a}}_{\bar{x}}}{A\&B} \qquad \frac{A\&B}{A, B}$$

$$\vee \frac{A\ \ \rceil\ (A \vee B)^{\bar{a}}_{\bar{x}}}{A \vee B} \qquad \frac{A \vee B \quad \begin{matrix}[A]\\ C\end{matrix} \quad \begin{matrix}[B]\\ C\end{matrix}}{C}$$

$$\to \frac{\begin{matrix}[A]\\ \vdots \\ B\end{matrix} \quad \rceil\,(A \to B)^{\bar{a}}_{\bar{x}}}{A \to B} \qquad \frac{A \quad A \to B}{B}$$

$$\forall^{*} \frac{A \ \ \left(\rceil\,(\forall x A)^{\bar{a}}_{\bar{t}}\right)}{\forall x A^{a}_{x}} \qquad \frac{\forall x A^{a}_{x}}{A^{a}_{t}}$$

$$\exists \frac{A^{a}_{t} \quad \left(\rceil\,(\exists x A^{a}_{t})^{\bar{a}}_{\bar{x}}\right)}{\exists x A^{t}_{x}} \qquad (*)\ \frac{\exists x A \quad \begin{matrix}[A^{x}_{t}]\\ \vdots \\ C\end{matrix}}{C}$$

The rules indicated by $*$ are called rules with proper parameters. In the rule $\forall I$ is a proper parameter a isn't contained in supposition, from which the deduction of premises depends, and in the $\rceil E$ rule the eigen parameter B isn't contained in supposition, from which the derivation of $\exists \times A_1$ depends, in $\exists \times A$ and in C.

The G-formulas, in brackets indicate that application of parcel requires their G-constructivety.

The formulas in square brackets are suppositions, eliminated in the process of deduction.

The deduction of the formula A from the set of premises Γ we'll denote $\Gamma \vdash A$.

2. The properties of deductions.

Definition. *G-subformulas of the formula A.*

The concept of G-subformula differs from the concept of the subformula in the ordinary sense (in sense of Kleeng 5), such that subformulas of the formula $\exists x A$ aren't all subformulas of the formulas A^{x}_{t}, but just such, that where t is a parameter not included in $\exists x A$.

The property of "meaningfulness" of the G-subformulas of the M-formula.

Statement 1. *If the formula is "meaningful", then any of it's G-subformula is "meaningful".*

Proof is inductive by the length of the formula.

Statement 2. *If* $\Gamma \vdash A$*, then* A *is "meaningful", i.e.* $\vdash^G]A^{\bar{a}}_{\bar{x}}$

Statement 3. *If* $\Gamma \vdash A$*, then* $\vdash^G]A$*.*

Proof is inductive by the height of the conclusion.

Statement 4. If the conclusion $\Gamma \vdash A$ is given, then for any G-subformula B of the formula, including into the conclusion is true $\vdash^G]B$.

4. Interpretation of "meaningfulness".

Let's denote by $\Gamma \vdash A$ a minimal intuitionistic calculus of the predicates (i.e. intuitionistic calculus without rules, connected with negation) with adding as a base of axioms by an universal closures of the G-axioms.

Supposition 1. *(The property of information carrier).* BM^* *for any information carrier of the formula* A *is true* $A^c \vdash A$ *and* $(A^c)^{\bar{a}}_{\bar{t}} = \left(A^{\bar{a}}_{\bar{t}}\right)^c *$.

Proof is inductive by the length of the formulas.

Theorem 4.1. *If* $\vdash^G A$*, then* $M^* \vdash A$*. If* $\vdash^G]A^{\bar{a}}_{\bar{x}}$*, then* $M^* \vdash \exists x_1 ... \exists x_n \ A^{\bar{a}}_{\bar{x}}$*.*

The proof of the theorem is inductive by the length of the conclusion.

Theorem shows, that the concept "nonemptiness" is a subcept of meaningfulness.

The explained formalization is suitable for the semantic interpretation, that is a subject of the further researches.

References

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