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DURABILITY CRITERION OF FIBROUS COMPOSITES

Abstract

The durability criterion of fibrous composites is suggested, taking into account structural contribution each of component of composite in their interaction. It is shown, that it is generalization of earlier known durability criterion. The comparison with experiment is given.

The necessity of the adequate description of carrying capacity and longevity of composite materials and designs on their basis has resulted in creation of numerous strength criteria. Each of them has the working area and naturally can not claim for any universality. It wold be possible conditionally to divide them into two groups: empirical criterion, for example [3,4] and criterion obtained on the basis of certain general mechanico-mathematical principles, for example [1,2,5-9]. It reflects the selected way of research-from experiment to the theory or visa-verse. The most favorable case is the covering of criterion from mentioned above different groups, that is absolute confirmation of the correctness and validity of both criterions and allows to define their capabilities. In the given paper the noted is shown on an example of two durability criterions: empirical and phenomenological. In [1] the phenomenological criterion of long-term durability founded on the suppositions is offered that on each of eigenstate of body occur a personal processes of destractions, they are autonomous and moment of carrying capacity loss is determined by a summary level of damage-destruction on all eigenstates:

$$\sum_{\alpha=1}^{r} C_\alpha \left[\left(\sigma_{(\alpha)ij} + M_\alpha^* \sigma_{(\alpha)ij}\right)\left(\sigma_{(\alpha)ij} + M_\alpha^* \sigma_{(\alpha)ij}\right)\right]^{1/2} = 1. \quad (1)$$

Here $\sigma_{(\alpha)ij}$ are tensors of stresses, M_α^* are damage operators of eigenstates, C_α are empirical constants, $r \leq 6$ is a number of eigen bodies.

The criterion determining instantneous durability, on the base of (1) will take the from:

$$\sum_{\alpha=1}^{r} C_\alpha \sigma_\alpha = 1, \quad (2)$$

where $\sigma_\alpha = \left(\sigma_{(\alpha)ij} \cdot \sigma_{(\alpha)ij}\right)^{1/2}$ are norms of tensors of stresses of eigenstates.

In [3] empirical criterion of durability of composites unidirectionally reinferced by fibers is given. It is supposed that destruction of similar composites are determined by shift efforts in the binding matrix τ_m and efforts of breaking of fiber from the matrix σ_m:

$$\sigma_m + C\tau_m = a \quad (3)$$

Show, that the durability criterion (2) for the indicated class of composites involves the criterion of the form (3), basing its correctness and what is more makes possible to interpret widely this criterion, revealing the ignored factors and marking their role with the characteristics of the criterion (2).

As is known [1] for unidirectional reinforced composites the number of eigenstates of body is equal to four. At this the components of stress tensors of the eigenstates $\sigma_{(\alpha)ij}$ have the form:

$$\begin{cases} \sigma_{(1)11} = \frac{\sqrt{3}}{2} f\left(p, \sigma_k\ \varphi\right) \cdot \sin\varphi \\ \sigma_{(1)22} = \frac{\sqrt{3}}{2} f\left(p, \sigma_k\ \varphi\right) \cdot \left(\sin\varphi + \sqrt{3}\sin\left(\varphi_0 - \varphi\right) \cdot \sin^2\theta\right) \\ \sigma_{(1)33} = \frac{\sqrt{3}}{2} f\left(p, \sigma_k\ \varphi\right) \cdot \left(\sin\varphi + \sqrt{3}\sin\left(\varphi_0 - \varphi\right) \cdot \cos^2\theta\right) \\ \sigma_{(1)23} = \frac{3}{4} f\left(p, \sigma_k\ \varphi\right) \cdot \sin\left(\varphi_0 - \varphi\right) \cdot \sin 2\theta \\ \sigma_{(1)12} = \sigma_{(1)13} = 0 \end{cases}$$

$$\begin{cases} \sigma_{(2)11} = \frac{\sqrt{3}}{2} g\left(p, \sigma_k\ \varphi\right) \cdot \cos\varphi \\ \sigma_{(2)22} = \frac{\sqrt{3}}{2} g\left(p, \sigma_k\ \varphi\right) \cdot \left(\cos\varphi - \sqrt{3}\sin\left(\varphi_0 - \varphi\right) \cdot \sin^2\theta\right) \\ \sigma_{(2)33} = \frac{\sqrt{3}}{2} g\left(p, \sigma_k\ \varphi\right) \cdot \left(\cos\varphi - \sqrt{3}\sin\left(\varphi_0 - \varphi\right) \cdot \cos^2\theta\right) \\ \sigma_{(2)23} = \frac{3}{4} g\left(p, \sigma_k\ \varphi\right) \cdot \cos\left(\varphi_0 - \varphi\right) \cdot \sin 2\theta \\ \sigma_{(2)12} = \sigma_{(2)13} = 0 \end{cases}$$

$$\begin{cases} \sigma_{(3)11} = \sigma_{(3)12} = \sigma_{(3)13} = 0 \\ \sigma_{(3)22} = \tau_{m,k} \cdot \sin 2\theta \\ \sigma_{(3)23} = \tau_{m,k} \cdot \cos^2\theta \\ \sigma_{(3)23} = \frac{3}{4} g\left(p, \sigma_k\ \varphi\right) \cdot \cos\left(\varphi_0 - \varphi\right) \cdot \sin 2\theta \\ \sigma_{(3)12} = -\tau_{m,k} \cdot \sin 2\theta \end{cases}$$

$$\begin{cases} \sigma_{(4)51} = -\frac{1}{9}\sigma_m; \quad \sigma_{(8)22} = \frac{1}{2}\sigma_m \cos^2\theta \\ \sigma_{(4)35} = \frac{1}{2}\sigma_m \sin^2\theta; \ \sigma_{(4)23} = -\frac{1}{4}\sigma_m \sin 2\theta \\ \sigma_{(4)12} = \sigma_{(4)13} = 0 \end{cases}$$

where

$$f\left(p, \sigma_k, \varphi\right) = \sqrt{3}p\sin\varphi + \sigma_k \sin\left(\varphi - \varphi_0\right);$$

$$g\left(p, \sigma_k, \varphi\right) = \sqrt{3}p\cos\varphi + \sigma_k \cos\left(\varphi - \varphi_0\right).$$

Here the direction of the vector k determines an axis of reinforcement, and m is a vector perpendicular to reinforcing fibers, θ is an angle between axes oq a reinforcement and extension (fig. 1), φ-is a material constant, describing the degree of difference of a material from volumetric-isotrozic for which $\varphi = \varphi_0$, where $tg\varphi_0 = \sqrt{2}$.

Besides, σ_k-ii an effort in a fiber, σ_m-is an eafort normal to a fiber, $\tau_{m,k}$-is shift effort in a binding matrix, $p = 1/3\sigma_{ii}$-is mean stress or hydrostatic pressure.

Intensities or norms of stress tensors of eigenstates will be:

$$\begin{cases} \sigma_1 = \sqrt{\frac{3}{2}} f\left(p, \sigma_k, \varphi\right); \ \sigma_0 = \sqrt{\frac{3}{2}} g\left(p, \sigma_k, \varphi\right) \\ \sigma_3 = \sqrt{2}\tau_{m,k}; \quad \sigma_4 = \frac{\sqrt{2}}{2}\sigma_m \end{cases} \tag{4}$$

Then the durability criterion (2) will take the following expression:

$$C_1 f\left(p, \sigma_k, \varphi\right) + C_2 g\left(p, \sigma_k, \varphi\right) + C_3\tau_{m,k} + C_4\sigma_m = 1 \tag{5}$$

The sense of last two members of the left hand side of criterion is clear, it is a contribution to definition of durability rf composite of shift effort in a binding matrix and normal effort of breaking of a matrix from a fiber, i.e. that is dexermined by the criterion (3). Then the difference of the criterion (5) from (3) is presence of tjo first addends in (5). As it follows from their expressions they take into account influencing of mean hydrostatic pressure, efforts in a fiber and parameter distinguishing a material of eomposite from volumetric-isocropic, i.e. proceeding form general principles, these characteristics should be considered on a level with those two characteristics of the criterion (3) deterhining according to this criterion durability of unidirectional reinforced composite at stretching. For clearness of the sense of expressions we'll write out them for a case of isotropy, when $f(p,\sigma_k,\varphi)$, $g(p,\sigma_k,\varphi)$

$$\sin\varphi_0=\sqrt{2/3};\quad \cos\varphi_0=1/\sqrt{3};$$

$$f(p,\sigma_k,\varphi)=\sqrt{2}p;\; g(p,\sigma_k,\varphi)=p-\sigma_k, \tag{6}$$

i.e. the first addend in (5) would be possible basically to connect allowing for mean hydrostatic pressure and second addend constraint of material of composite in direction orthogonal to a fiber.

Fig. 1.

It should be supposed that the durability criterion (3) is a particular case of criterion (5) and justifieh itself in those cases when influencing of hydrostatic pressure effort in a fiben and degree of difference of composite from volumetric-isotropic on durability of composite is unessential, i.e. when the durability of composite is deterjined basicly by the characteristics of a binding matrix and adhesion between a fiber and matrix or simply speaking the destruction of composite occur on a matrix or on boundary of a matrix with fiber. Thus durability criterion (5) is universal with respect to durability criterion (3) and it is capable to forecasn carrying capacity for wiaer class of unidirectional reinforced composites.

An experimental approbation of durability criterion (5) is given on an example of carbon fiber-reinforced plastic KMY-3L. For the values of angle $\theta = 0^0,\ 13^0,\ 22,5^0$, 45^0 and 90^0 on the fig. 2. The calculational curves of relations of norms of eigenstates from the angle θ are constructed, where σ_p-is a tensile stress and the number of curves corresponds to the number of eigenstates.

These curves ajlows to receive idea about contributions of the norms of eigenstates for different angles between the directions of stretching and reinforcement of composite. In fig.3 the curves of dependence of durability limit of samples are reduced depending on the rngle θ is received on the base of experiment (hatcied line) and calculated by the durability criterion (5) (firm line).

At this as the value of parameter describing degree of difference of material of composite from valumetric-isotropic is taken $\varphi = 0,854$. A good agreement of comparison of design and experimental curves is noted in fig. 2. It shows their sufeicient agreement.

Fig. 2. Fig. 3.

At paper [1] for the consmdered composite the parameters of deflective operators in the criterion (8) were determined. Using them for the destruction time $t_p = 50$ the following values of tensile stress for the angle $\theta = 45^0$; $\sigma_p = 2,5\ kg/mm^2$ (experiment), $\sigma_p = 2,9\ kg/mm^2$ (calculation) were obtained.

The sufficient proximity of design and experimental data testifies the correctness and validity of given durability criterion and distinguished by clear physical sense reflecting the structure of a composite.

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