MECHANICS

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BUCKLING OF NON-UNIFORMLY HEATED DAMAGING ROD

Abstract

On basis of modified relation of one dimensional thermoelasticity for damaging medium, the carrying capacity loss process at unchangeable loading of rod for linear changing by thickness of level of supernormal temperature is investigated. The system of integro-differential equations relative to the amplitude of deflection (for hingly supported rod) and heights of unloading and loading domains of cross-section is obtained. The quantitative analysis is introduced.

The stability and buckling problem of rods as one of the most prevailing structural elements continues to remain in the center of attention of engineering investigations. Incentive to this phenomena is necessity of taking into account new factors and their mutual attention both themselves and previously accounted factors.

In the present paper the simultaneous influence of both damage accumulation process and temperature effects is discovered.

In paper [1] the determining equations for isotropic linear damaging visco-elastic body are given. For elastico-damaging body with regard to temperature stress the determining may be written by analogue with [1] in the form of

$$\begin{cases} э_{ij} = \dfrac{1}{2\mu}\left(1+M^*\right)s_{ij}, \\ \varepsilon = \dfrac{1}{3k}\left(1+L^*\right)\sigma + 3\alpha\left(1+P^*\right)T, \end{cases} \tag{1}$$

where $э_{ij}$ and s_{ij} are deviators, and ε and σ are sphere parts of deformation and stress tensors respectively. Besides M^* is a shear damaging operator, L^* and P^* are operators of solid damaging of mechanical and temperature character. From (1) for uniaxial stress state we obtain

$$\varepsilon = \frac{1}{E}\left(1+M^*+L^*\right)\sigma + \alpha\left(1+P^*\right)T, \tag{2}$$

where

$$M^* = \frac{2}{3}(1+\nu)M^*\ ;\ L^* = \frac{1}{3}(1-2\nu)L^*\ . \tag{3}$$

We accept that the solid damaging is connected mainly with thermostresses, i.e. $L^* = 0$.

Then using the designations

$$\frac{1}{\widetilde{E}} = \frac{1}{E}\left(1+M^*\right);\quad \widetilde{\alpha} = \alpha\left(1+P^*\right). \tag{4}$$

We obtain

$$\varepsilon = \frac{1}{\widetilde{E}}\sigma + \widetilde{\alpha}T\ . \tag{5}$$

Hence for thermoelastic stress subject to damaging we have

$$\sigma = \widetilde{E}\varepsilon - \widetilde{\chi}T\ , \tag{6}$$

where the operator $\tilde{\chi}$ is

$$\tilde{\chi} = \tilde{\alpha}\tilde{E}\ . \tag{7}$$

For simplicity later on we take that $\tilde{\chi} = \chi = const$. Then for stress we obtain

$$\sigma = \tilde{E}\varepsilon - \chi T\ . \tag{8}$$

At monotonically changing stress state, the damaging operator behaves as the ordinary operator of hereditary elasticity

$$M^* x(t) = \int_0^t M(t-\tau)x(\tau)\,d\tau \tag{9}$$

and for cases at determination of stress-deformation state of damaging body the application of Wolterr-Rabotnov's correspondence principle is possible. Besides the inversion

$$\tilde{E} = E\left(1 - N^*\right) = \frac{E}{1 + M^*} \tag{10}$$

holds. The strength criteria according to (1) has the form:

$$\left(1 + M^*\right)|\sigma| = \sigma_0 , \tag{11}$$

where σ_0 is ultimate strength of zero-defect material on compression.

As an example we consider the buckling of non-uniformly heated damaging rod at constant compressive load. The rod on end-walls is hingedly fastened. The temperature plane is assumed constant by length of rod and linear changing by height of section in the direction of buckling

$$T(y) = \frac{T_2 + T_1}{2} - \frac{T_2 - T_1}{2h} y_0\ ,\ \ T_2 > T_1 . \tag{12}$$

Here the axis x_0 coincides with the axis of rod, y_0 and z_0 are principal centroidal axis of section, T_1 and T_2 are temperatures in edge fibers, counted put from some initial temperature T_0; $2h$ is height of section, b is width of cross-section.

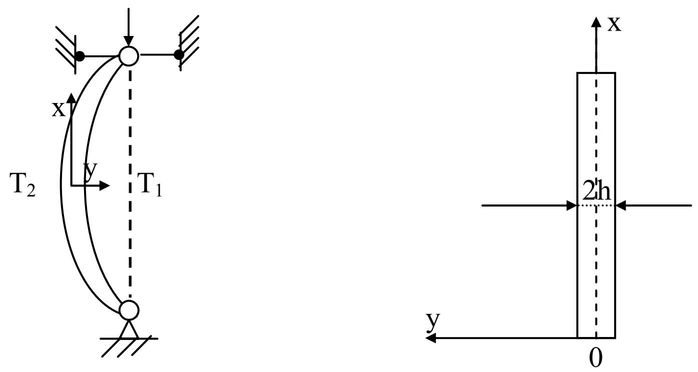


Fig. 1

Assume that at some value of centroidal posed axial compressed force, the buckling occurs at which the load remains constant. Then the fibers lying on concave side will be in continued load state, lying on convex side will be in unload state testing the lengthening. In this case cross-section of rod will consist of two parts: one of that the convex side is elastically deformed, on the other concave side the damage accumulation process described by the homogeneous equation (2) or (4) occurs. In fig. 2 Ω_1 is a loading domain with the current height $h_1(t)$; Ω_2 is a unloading domain with the current height $h_1(t)$. The axis oz composes current neutral axis. Assuming that the cross sections of rod in bending is plane and the buckling occurs in the direction of opposite y for complementary deformations we have

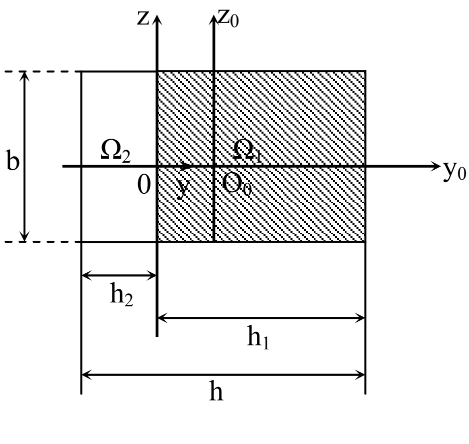


Fig. 2

$$\Delta\varepsilon = -\frac{y}{\rho}. \qquad (13)$$

Here $\rho = -\frac{\partial^2 \vartheta}{\partial x^2}$, where ϑ is deflection, ρ is radius of curvature.

Here the distance y is counted from neutral axis. The stresses in loading and unloading bands will be

$$\begin{cases} \Delta\sigma_2 = E_2\Delta\varepsilon_2 - \chi_2 T; & \Delta\varepsilon_2 > 0, \\ \Delta\sigma_1 = E_1\left(1-N^*\right)\Delta\varepsilon_2 - \chi_1 T; & \Delta\varepsilon_1 < 0. \end{cases} \qquad (14)$$

Since the considered buckling is at unchangeable load, then resultant additional effects must be equal to zero

$$\int_{\Omega_1} \Delta\sigma_1 d\omega + \int_{\Omega_2} \Delta\sigma_1 d\omega = 0$$

or subject to the formulas (13), (14):

$$E_2 s_2 \frac{\partial^2 \vartheta}{\partial x^2} + E_1\left(1-N^*\right)s_1 \frac{\partial^2 \vartheta}{\partial x^2} - \chi_1\left(\gamma\Omega_1 - \frac{\beta}{h}s_1\right) - \chi_2\left(\gamma\Omega_2 - \frac{\beta}{h}s_2\right) = 0, \qquad (15)$$

where $\gamma = \frac{(T_1+T_2)}{2}$; $\beta = \frac{T_2 - T_1}{2}$, $s_k = \int_{\Omega_k} y d\omega$ are static inertia moments with respect to neutral axis of section

$$s_1 = \frac{bh_1^2}{2}; \quad s_2 = -\frac{bh_2^2}{2}. \qquad (16)$$

Equating the sum of interior forces with respect to neutral axis to exterior time:

$$\int_{\Omega_1} \Delta\sigma_1 \cdot y d\omega + \int_{\Omega_2} \Delta\sigma_2 \cdot y d\omega = M$$

or subject to the relations (14)

$$E_1\left(1-N^*\right)J_1 \frac{\partial^2 \vartheta}{\partial x^2} + E_2 J_2 \frac{\partial^2 \vartheta}{\partial x^2} - \chi_1\left(\gamma s_1 - \frac{\beta}{h}J_1\right) - \chi_2\left(\gamma s_2 - \frac{\beta}{h}J_2\right) = M, \qquad (17)$$

where J_k are inertia moments of the square Ω_k with respect to neutral axis

$$J_1 = \frac{bh_1^3}{3}; \quad J_2 = \frac{bh_2^3}{3}. \qquad (18)$$

Moment to exterior forces- is the moment of compressive forces

$$M = -P \cdot \vartheta(x,t). \tag{19}$$

Thus we have the following system of differential integral equations with respect to three functions $h_1(t), h_2(t), \vartheta(x,t)$:

$$\begin{cases} \{E_1 h_1^2(t) - E_2 h_2^2(t)\}\dfrac{\partial^2 \vartheta(x,t)}{\partial x^2} - 2\gamma\{\chi_1 h_1(t) + \chi_2 h_2(t)\} - \beta\{\chi_1 h_1^2(t) - \chi_2 h_2^2(t)\} - \\ - E_1 \int\limits_0^t N(t-\tau) h_1^2(\tau) \dfrac{\partial^2 \vartheta(x,\tau)}{\partial x^2} d\tau = 0 , \\ \{E_1 h_1^3(t) + E_2 h_2^3(t)\}\dfrac{\partial^2 \vartheta(x,t)}{\partial x^2} - 1{,}5\gamma\{\chi_1 h_1^2(t) - \chi_2 h_2^2(t)\} + \beta\{\chi_1 h_1^3(t) + \chi_2 h_2^3(t)\} - \\ - E_1 \int\limits_0^t N(t-\tau) h_1^3(\tau) \dfrac{\partial^2 \vartheta(x,\tau)}{\partial x^2} d\tau + \dfrac{3}{b} P \cdot \vartheta(x,t) = 0 , \\ h_1(t) + h_2(t) = h . \end{cases} \tag{20}$$

Let the rod by end-walls be hingedly supported, then we can represent deflection in the form of

$$\vartheta(x,t) = f(t)\sin\frac{\pi x}{l}. \tag{21}$$

Moving for the representation (21) in a system of the equations (2) we obtain

$$\begin{cases} \{E_1 h_1^2(t) - E_2 h_2^2(t)\} f(t) + \dfrac{2\gamma l^2}{\pi^2}\{\chi_1 h_1(t) + \chi_2 h_2(t)\} - \dfrac{\beta l^2}{\pi^2}\{\chi_1 h_1^2(t) - \chi_2 h_2^2(t)\} - \\ - E_1 \int\limits_0^t N(t-\tau) h_1^2(\tau) f(\tau) d\tau = 0 , \\ \{E_1 h_1^3(t) - E_2 h_2^3(t)\} f(t) + \dfrac{1{,}5\gamma l^2}{\pi^2}\{\chi_1 h_1^2(t) - \chi_2 h_2^2(t)\} - \dfrac{\beta l^2}{\pi^2}\{\chi_1 h_1^3(t) - \chi_2 h_2^3(t)\} - \\ - E_1 \int\limits_0^t N(t-\tau) h_1^3(\tau) f(\tau) d\tau + \dfrac{3 l^2 P}{\pi^2 b} f(t) = 0 , \\ h_1(t) + h_2(t) = h . \end{cases} \tag{22}$$

Rod loading capacity of the bar will be determined prevailing from the parallel operating buckling and damaging processes. The question here is very important" what happens earlier: stability loss or distraction because of accumulation of critical volume of damages? We analyze the case when carrying capacity loss in consequence of catastrophic increasing defection will be prevalent.. For this from the system of equation (22) it follows to determine the amplitude of the deflection $f(z)$. For qualitative analysis we use the approximate estimate

$$\begin{cases} N^* f(t) = (N^* \cdot 1) f(t) = A(t) f(t), \\ A(t) = N^* \cdot 1 = \int\limits_0^t N(\tau) d\tau. \end{cases} \tag{23}$$

Then the system of the equation (22) has the form

$$\begin{cases} \left\{E_1\left(A(t)-1\right)h_1^2(t)+E_2h_2^2(t)\right\}f(t)-\dfrac{2\gamma l^2}{\pi^2}\left\{\chi_1h_1(t)+\chi_2h_2(t)\right\}+ \\ +\dfrac{\beta l^2}{\pi^2}\left\{\chi_1h_1^2(t)+\chi_2h_2^2(t)\right\}=0 \\ \left\{E_1\left(A(t)-1\right)h_1^3(t)-E_2h_2^3(t)+\dfrac{3l^2P}{\pi^2 b}\right\}f(t)-\dfrac{1{,}5\gamma l^2}{\pi^2}\left\{\chi_1h_1^2(t)-\chi_2h_2^2(t)\right\}+ \\ +\dfrac{\beta l^2}{\pi^2}\left\{\chi_1h_1^3(t)+\chi_2h_2^3(t)\right\}=0\ , \\ h_1(t)+h_2(t)=h\ . \end{cases} \tag{24}$$

We can find stability loss moment without determination of the function $f(t)$. For this it's necessary to give stability loss condition as inversion condition of amplitude of the deflection $f(t)$ at infinity. Tending to zero the coefficients at the function $f(t)$ at the first two equation of the system (24):

$$\begin{cases} E_1\left(A(t)-1\right)h_1^2+E_2h_2^2=0 \\ E_1\left(A(t)-1\right)h_1^3-E_2h_2^3+\dfrac{3l^2P}{\pi^2 b}=0\ , \\ h_1+h_2=h\ . \end{cases} \tag{25}$$

corresponds to this.

Solving this system we find the value of critical force for arbitrary moment of time t

$$P_{kp}=\frac{\pi^2 bE_2h^3}{3b^2}\cdot\frac{\eta^2(t)}{\left(1+\eta(t)\right)^2}\ , \tag{26}$$

when the height h_1, h_2 of loading and unloading domains will be

$$h_1=\frac{h}{1+\eta(t)}\ ;\quad h_2=\frac{\eta(t)}{1+\eta(t)}h\ , \tag{27}$$

where

$$\eta(t)=\sqrt{\frac{E_1}{E_2}\left(1-A(t)\right)}\ . \tag{28}$$

At the given unchangeable compassion force P the relation (26) allows to determine the corresponding time of carrying capacity loss

$$\int_0^{t_{kp}}N(\tau)d\tau=1-\frac{E_2}{E_1}\left\{\frac{\pi h}{l}\sqrt{\frac{bhE_2}{3P}}-1\right\}^{-2}\ . \tag{29}$$

If as kernel we take the exponential $N(t)=\lambda\exp(-\mu t)$, then we obtain

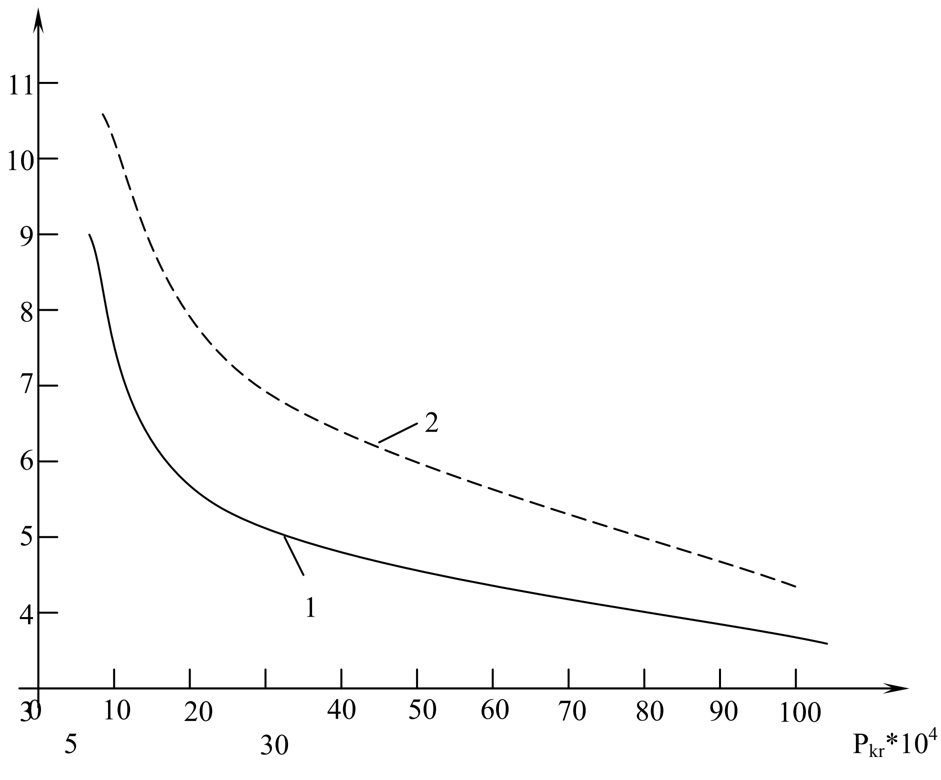
$$t_{kp}=\frac{1}{\mu}\ln\left\{1-\frac{\mu}{\lambda}\left[1-\frac{E_2}{E_1}\left(\frac{\pi h}{l}\sqrt{\frac{bhE_2}{3P}}-1\right)^{-2}\right]\right\}^{-1}\ . \tag{30}$$

From the positivity condition of logarithm we obtain the constrains to the quantity of compression force for which the stability loss occurs after the lapse of some time $t = t_{kp}$:

$$\left(1 + V\frac{E_2}{E_1}\cdot\frac{\mu}{\mu-\lambda}\right) < P < \left(1+\sqrt{\frac{E_2}{E_1}}\right)^{-2}; \quad \mu > \lambda . \qquad (31)$$

Thus framed of the given statement of problem the existence of non-homogeneous temperature field doesn't influence to critical parameters, but only for the given compressed loading it influences to the deflection critical quantity, that unlike failing the homogeneous temperature field it maybe concretely determined.

On fig. 3 the graphs of independence the critical time of compressed loading for values of the parameters $\frac{h}{l} = 0{,}1$; $\frac{\mu}{\lambda} = 1$ are introduced, where curve 1 corresponds the value $\frac{E_2}{E_1} = 0{,}1$, and the curve 2 – significantly $\frac{E_2}{E_1} = 0{,}01$.



1-$\frac{E_2}{E_1} = 0{,}1$, 2- $\frac{E_2}{E_1} = 0{,}01$

Fig. 3

References

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[2]. Vollir A.S. *Stability of deformed systems.* M., 1967, 984p. (in Russian)

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