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Parametric investigation of the forced vibration of a "plate + compressible viscous fluid + rigid wall" hydroelastic system

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Abstract. *The paper attempts to investigate the forced vibration of a hydro-elastic system consisting of an elastic plate, compressible viscous fluid and rigid wall with the use of the parametrical presentation of the mechanical constants of the plate material through the mechanical constants of the fluid. It is introduced two parameters one of which is equal to the ratio of the density of the plate material to that of the fluid and the other one is equal to the ratio of the shear wave velocity to the sound speed in the fluid. The numerical results on the influence of these parameters on the frequency response of the interface pressure and normal velocity are presented and discussed. In particular, it is established that an increase in the plate material density causes to decrease in the absolute values of the interface pressure.*

Keywords. hydro-elastic system · compressible viscous fluid · elastic plate · frequency response · forced vibration.

Mathematics Subject Classification (2010): 74H55

1 Introduction

The detail review of the investigations related to the forced vibration of the "plate + compressible viscous fluid + rigid wall" hydro-elastic systems has been made in the paper by Akbarov (2018). It follows from this review paper that in all the investigations carried out in this field concrete numerical results are obtained for the concrete selected materials of the constituencies of the mentioned hydro-elastic system. Consequently, according to these results it is impossible to say how the increase or decrease of the ratio of the densities of the fluid and plate materials, as well as how the increase or decrease of the ratio of the sound speeds in the fluid and plate materials, influences on the interface pressure and interface velocity under vibration of this system. Namely these questions are the subject of the present paper and for this purpose after selection of the fluid in the foregoing system it is introduced the parameters which characterize the ratio of the plate material's density to the

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fluid density and the ratio of the sound speed to the shear wave propagation speed in the plate material. Through the change of these two parameters, it is determined the change of the values of the mechanical constants of the plate material and by this way it is studied the mentioned above questions. Under investigations, as in the paper by Akbarov and Ismailov (2017) and other ones listed papers in the review paper by Akbarov, the motion of the plate is described by the exact equations of elastodynamics in the plane strain state and the flow of the fluid is described within the scope of the linearized Navier - Stokes equations. It is assumed that the fluid is barotropic and Newtonian one.

2 Formulation of the problem

As in the paper by Akbarov and Ismailov (2017), we consider a system consisting of the pre-stressed plate-layer, barotropic compressible Newtonian viscous fluid and rigid wall (Fig.1) and associate the Cartesian coordinate system $Ox_1x_2x_3$ with the upper face plane of the plate and the position of the points of the constituents we determine in this coordinate system. In the introducing coordinate system the plate and the fluid occupy the regions $\{|x_1| < \infty, -h < x_2 < 0, |x_3| < \infty\}$ and $\{|x_1| < \infty, -h_{\mathrm{d}} < x_2 < -h, |x_3| < \infty\}$, respectively. The material of the plate we assume as isotropic and homogeneous and within the scope of this and foregoing assumptions, we investigate the forced vibration of the hydro-elastic system which appear as a result of the action lineal-located time-harmonic forces acting on the free face plane of the plate.

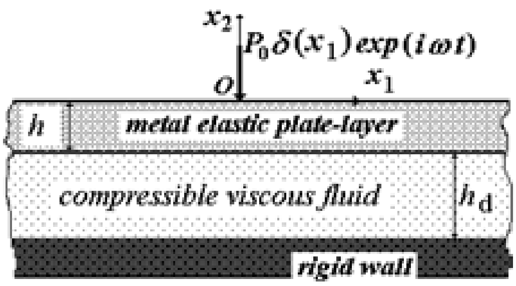

Fig. 1. The sketch of the hydro-elastic system under consideration

For the investigation, we write full system of equations, boundary, compatibility and impermeability conditions, and, according to the geometry and the corresponding geometrical symmetry of the system and external forces acting on this system, we consider plane strain state for the plate and plane-parallel flow for the fluid in the Ox_1x_2 plane.

Thus, we write the full system of equations of motion of the plate:

$$\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} = \rho \frac{\partial^2 u_1}{\partial t^2}, \frac{\partial \sigma_{12}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} = \rho \frac{\partial^2 u_2}{\partial t^2}.$$

$$\sigma_{11} = (\lambda + 2\mu)\varepsilon_{11} + \lambda\varepsilon_{22}, \sigma_{22} = \lambda\varepsilon_{11} + (\lambda + 2\mu)\varepsilon_{22}, \sigma_{12} = 2\mu\varepsilon_{12},$$

$$\varepsilon_{11} = \frac{\partial u_1}{\partial x_1}, \varepsilon_{22} = \frac{\partial u_2}{\partial x_2}, \varepsilon_{12} = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_2}{\partial x_1}\right). \quad (2.1)$$

In (2.1) the conventional notation is used.

According to Guz (2009), now we write the linearized Navier-Stokes and other field equations for the Newtonian compressible viscous fluid and in these equations the density, viscosity constants and pressure of the fluid we denote by the upper index (2.1).

$$\rho_0^{(1)}\frac{\partial v_i}{\partial t}-\mu^{(1)}\frac{\partial v_i}{\partial x_j\partial x_j}+\frac{\partial p^{(1)}}{\partial x_i}-(\lambda^{(1)}+\mu^{(1)})\frac{\partial^2 v_j}{\partial x_j\partial x_i}=0,\frac{\partial\rho^{(1)}}{\partial t}+\rho_0^{(1)}\frac{\partial v_j}{\partial x_j}=0,$$

$$T_{ij}=\left(-p^{(1)}+\lambda^{(1)}\theta\right)\delta_{ij}+2\mu^{(1)}e_{ij},\theta=\frac{\partial v_1}{\partial x_1}+\frac{\partial v_2}{\partial x_2},$$
$$e_{ij}=\frac{1}{2}\left(\frac{\partial v_i}{\partial x_j}+\frac{\partial v_j}{\partial x_i}\right).a_0^2=\frac{\partial p^{(1)}}{\partial\rho^{(1)}}. \tag{2.2}$$

where $\rho_0^{(2.1)}$ is the fluid density before perturbation. The other notation used in Eq. (2.2) is conventional.

Note that the solution of the system equations in Eq. (2.2) for the 2D plane problems is reduced (see Guz (2009)) to finding the two potentials φ and ψ which are determined from the following equations:

$$\left[\left(1+\frac{\lambda^{(1)}+2\mu^{(1)}}{a_0^2\rho_0^{(1)}}\frac{\partial}{\partial t}\right)\Delta-\frac{1}{a_0^2}\frac{\partial^2}{\partial t}\right]\varphi=0,$$
$$\left(\nu^{(1)}\Delta-\frac{\partial}{\partial t}\right)\psi=0,\Delta=\frac{\partial^2}{\partial x_1^2}+\frac{\partial^2}{\partial x_2^2}, \tag{2.3}$$

where $\nu^{(1)}$ is the kinematic viscosity of the fluid, i.e. $\nu^{(1)}=\mu^{(1)}\big/\rho_0^{(1)}$ and the velocities v_1, v_2 and the pressure $p^{(1)}$ are expressed by the potentials φ and ψ as in the follows.

$$v_1=\frac{\partial\varphi}{\partial x_1}+\frac{\partial\psi}{\partial x_2},v_2=\frac{\partial\varphi}{\partial x_2}-\frac{\partial\psi}{\partial x_1},p^{(1)}=\rho_0^{(1)}\left(\frac{\lambda^{(1)}+2\mu^{(1)}}{\rho_0^{(1)}}\Delta-\frac{\partial}{\partial t}\right)\varphi. \tag{2.4}$$

Assuming that

$$p^{(1)}=-(T_{11}+T_{22}+T_{33})/3, \tag{2.5}$$

we obtain:

$$\lambda^{(1)}=-\frac{2}{3}\mu^{(1)}. \tag{2.6}$$

We recall that $\lambda^{(1)}$ in the equations (2.2) – (2.4) is the second coefficient of the fluid viscosity and namely through this coefficient the compressibility of that, is expressed. Through the assumption (2.5) the second coefficient of the fluid compressibility is expressed the the ordinary (the first) coefficient of the fluid viscosity.

Besides all of these, it is assumed the following boundary

$$\sigma_{21}|_{x_2=0}=0,\sigma_{22}|_{x_2=0}=-P_0e^{i\omega t}, \tag{2.7}$$

compatibility

$$\left.\frac{\partial u_1}{\partial t}\right|_{x_2=-h}=v_1|_{x_2=-h},\left.\frac{\partial u_2}{\partial t}\right|_{x_2=-h}=v_2|_{x_2=-h}, \tag{2.8}$$

$$\sigma_{21}|_{x_2=-h}=T_{21}|_{x_2=-h},\sigma_{22}|_{x_2=-h}=T_{22}|_{x_2=-h}, \tag{2.9}$$

and impermeability

$$v_1|_{x_2=-h-h_\mathrm{d}}=0,v_2|_{x_2=-h-h_\mathrm{d}}=0 \tag{2.10}$$

conditions.

This completes the formulation of the problem.

3 Method of solution

We use the same method which is developed and employed in the paper by Akbarov and Ismailov (2017). According to this method, the sought values are presented as $g(x_1, x_2, t) = \bar{g}(x_1, x_2)e^{i\omega t}$ and substituting this expression into the foregoing equations and relations, and replacing the derivatives $\partial(\cdot)/\partial t$ and $\partial^2(\cdot)/\partial t^2$ with constants $i\omega(\bar{\cdot})$ and $-\omega^2(\bar{\cdot})$, respectively, it is obtained the corresponding equations and boundary and contact conditions for the amplitudes of the sought values. For the solution to these equations, it is employed the exponential Fourier transformation with respect to the x_1 coordinate

$$f_F(s, x_2) = \int_{-\infty}^{+\infty} f(x_1, x_2)e^{-isx_1}dx_1 \tag{3.1}$$

to these equations. Taking the problem symmetry into account, with respect to the plane $x_1 = 0$, the originals of the sought values can be represented as follows:

$$\{\sigma_{11}\,; \sigma_{22}; u_2; \varphi; T_{11}; T_{22}; v_2\} =$$

$$= \frac{1}{\pi}\int_0^{\infty} \{\sigma_{11F}\,; \sigma_{22F}; u_{2F}; \varphi_F; T_{11F}; T_{22V}; v_2\}(s, x_2)\cos(sx_1)ds,$$

$$\{\sigma_{12}\,; u_1; \psi; T_{12}; v_1\} = \frac{1}{\pi}\int_0^{\infty} \{\sigma_{12F}\,; u_{1F}; \psi_F; T_{12F}; v_{1F}\}(s, x_2)\sin(sx_1)ds. \tag{3.2}$$

Thus, using the presentions in (3.2) and doing some mathematical manipulations described in the paper by Akbarov and Ismailov (2017) it is obtained the following expressions for the Fourier transforms of the displacements of the plate.

$$u_{2F} = Z_1e^{k_1x_2} + Z_2e^{-k_1x_2} + Z_3e^{k_2x_2} + Z_4e^{-k_2x_2},$$

$$u_{1F} = Z_1a_1e^{k_1x_2} + Z_2a_2e^{-k_1x_2} + Z_3a_3e^{k_2x_2} + Z_4a_4e^{-k_2x_2}, \tag{3.3}$$

where

$$k_1 = \sqrt{-\frac{A_0}{2} + \sqrt{\frac{A_0^2}{4} - B_0}}, k_2 = \sqrt{-\frac{A_0}{2} - \sqrt{\frac{A_0^2}{4} - B_0}},$$

$$A_0 = \frac{AG + B^2 + D}{G}, B_0 = \frac{BD}{G},$$

$$A = X^2 - s^2(\lambda/\mu + 2), B = s(\lambda/\mu + 1), a_1 = \frac{-D - Gk_1^2}{Bk_1^2},$$

$$a_2 = -a_1, a_3 = \frac{-D - Gk_2^2}{Bk_2^2},$$

$$a_4 = -a_3D = X^2 - s^2, G = \lambda/\mu + 2, X^2 = \omega^2h^2/c_2^2, c_2 = \sqrt{\mu/\rho}. \tag{3.4}$$

It is also determined the following expressions for the Fourier transforms of the values related to the fluid motion.

$$\varphi_F = \omega h^2\tilde{\varphi}_F, \psi_F = \omega h^2\tilde{\psi}_F, \tilde{\varphi}_F = Z_5e^{\delta_1x_2} + Z_7e^{-\delta_1x_2}, \tilde{\psi}_F = Z_6e^{\gamma_1x_2} + Z_8e^{-\gamma_1x_2},$$

$$v_{1F} = \omega \,\_h[-Z_5se^{\delta_1x_2} - Z_7se^{-\delta_1x_2} + Z_6e^{\gamma_1x_2} + Z_8e^{-\gamma_1x_2}],$$

$$v_{2F} = \omega \,\_h[Z_5\delta_1e^{\delta_1x_2} - Z_7\delta_1e^{-\delta_1x_2} - Z_6se^{\gamma_1x_2} - Z_8se^{-\gamma_1x_2}], \tag{3.5}$$

where

$$\delta_1 = \sqrt{s^2 - \frac{\Omega_1^2}{1 + i4\Omega_1^2/(3N_w^2)}}, \gamma_1 = \sqrt{s^2 + iN_w^2},\ \Omega_1 = \frac{\omega h}{a_0}, N_w^2 = \frac{\omega h^2}{\nu^{(1)}}. \quad (3.6)$$

Thus, using the expressions in (3.3) and (3.5), and constitutive relations in (2.1) and (2.2) we determine also the expressions of the Fourier transforms of the stresses acting in the plate and in the fluid. The unknown constant $Z_1, \ldots, Z_8$ which enter into these expressions are determined from the conditions (2.7) – (2.10) and finally, the originals of the sought values are determined from the expressions in (3.2). However, this determination is made numerically by employing the corresponding algorithm and PC programs composed by the author the validity of which is confirmed with the comparison of the corresponding results obtained in the paper by Akbarov and Ismailov (2017).

4 Numerical results and discussions

First of all, we note that under calculation the infinite intervals $[0, \infty]$ in the improper integrals in (3.2) is replaced with the finite intervals $[0, S_1^*]$ which is divided into the N number shorter intervals. In each of these shorter intervals the integrals are calculated by the useof the Gauss integration algorithmwith ten nodes. The values of the S_1^* and N are determined from the convergence requirement. In the present investigation it is established that for obtaining the numerical results with the 10^{-5} accuracy it is sufficient to take $S_1^* = 9$ and $N = 2000$. Consequently, the numerical result which will be discussed below are obtained within the scope of the foregoing assumptions.

In the numerical investigation we assume that the material of the fluid is Glycerin with viscosity coefficient $\mu^{(1)} = 1,393kg/(m \cdot s)$, density $\rho_0^{(1)} = 1260kg/m^3$ and sound speed $a_0 = 1927m/s$ (Guz (2009)). We also introduce the notation $c_2 = \sqrt{\mu/\rho}$ which is the shear wave propagation velocity in the layer material. After selection of the fluid's material constants we determine the plate material constants as follows:

$$\rho \Big/ \rho_0^{(1)} = k_1, c_2/a_0 = k_2, \mu = (c_2)^2\rho \quad (4.1)$$

So that, selecting the values for the k_1 and k_2 it is determined the mechanical constant of the plate material through the mechanical constants of thefluid material. In this cases an increase (a decrease) in the values of the k_1 means an increase (a decrease) in the density of the plate material with respect to the density of the fluid. As well as, an increase (a decrease) in the values of the k_2 means an increase (a decrease) shear wave propagation velocity of the plate material with respect to the sound speed in the fluid. The aim of the present numerical investigation is the determination how the change of the k_1 and k_2 effect on the values of the interface pressure (i.e. at $x_2 = -h$) T_{22} and velocity v_2.

Thus, we consider numerical results which are obtained in the case where $h_d/h = 2$. These results are given in Figs. 2, 3 and 4.

Thus, it follows from these figures that an increase in the values of the plate material density (i.e. an increase in the values of k_1 (Figs. 2)) causes to decrease of the absolute values of the interface pressure T_{22}, however the mentioned increase causes (Fig. 3) causes to increase the absolute values of the interface normal velocity. At the same time, it follow from Fig. 4 that an increase in the shear modulus of the plate material (i.e. an increase in the values of the k_2) also causes to decrease of the values T_{22}, however the values of the dimensionless velocity $v_2\mu h/(\mathrm{P}_0 c_2)$ do not depend on the parameter k_2. Nevertheless, it follows from the expressions in (4.1) that the dimensional values of the this velocity v_2 increase with the k_2.

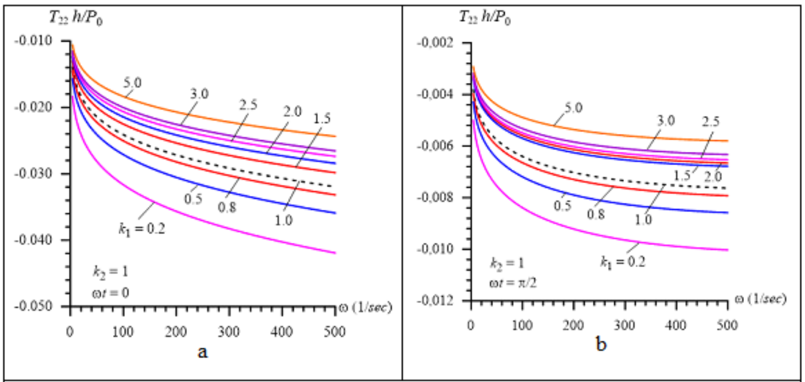

Fig. 2. The influence of the change of the k_1 on the values of the interface pressure in the cases where $\omega t = 0$ (a) and $\omega t = \pi / 2$ (b)

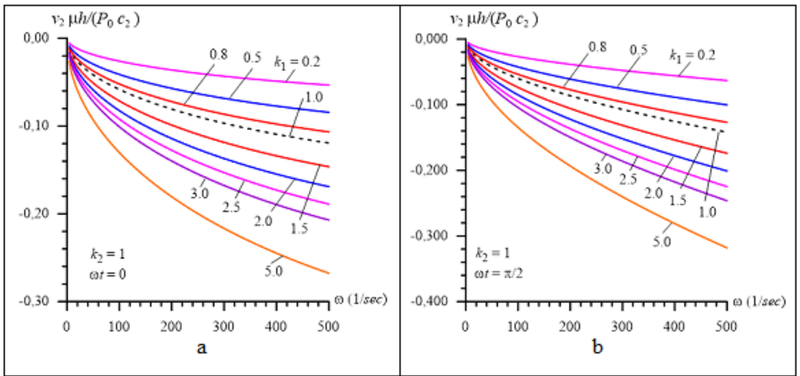

Fig. 3. The influence of the change of the k_1 on the values of the interface normal velocity in the cases where $\omega t = 0$ (a) and $\omega t = \pi / 2$ (b)

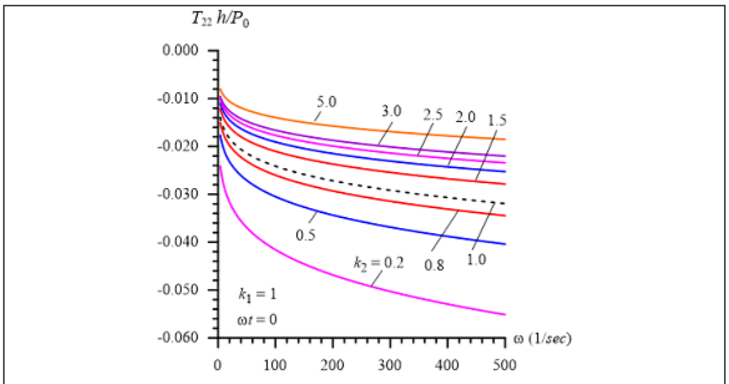

Fig. 4. The influence of the change of the k_2 on the values of the interface pressure in the cases where $\omega t = 0$

5 Conclusions

Thus, in the present paper it is attempt to make a parametrical study the forced vibration of the hydro-elastic system consisting of elastic plate, compressible viscous fluid and rigid wall through introducing the parameters which estimate the ratio of the densities of the fluid and plate materials and the ratio of the sound speeds in the fluid and plate materials.

The investigations are made within the scope of the exact equations of elastodynamics in the plane – strain state for the plate and within the scope of the linearized Navier-Stokes equations for the barotropic compressible viscous fluid. Numerical results on the frequency response of the interface pressure and of the normal velocity are presented for various values of these parameters. According to these results, it is formulated the following concrete conclusions:

- An increase in the values of the plate material density with respect to the fluid density the absolute values of the interface pressure decrease, however the absolute values of the normal velocity increase;

- The foregoing conclusion occurs also with respect of the increase in the values of the values of the shear wave propagation velocity with respect to the sound speed in the fluid.

According to the foregoing results, it can be also concluded that for obtaining the foregoing type results gives certain orient for selection of the materials of the considered type hydro-elastic system with respect to decrease of the interface pressure or interface normal velocity. Therefore, in the author's view, in future, it is necessary to develop investigations started in the present paper.

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