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Simulation of oil flow in conjugated bed-well system

Shusha A. Kerimova

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Abstract. *A model of non-stationary motion of fluid flow in the conjugate bed-well system is constructed. Pressure on the bottomhole is determined. An analytic expression for determining well productivity for arbitrary change of wellhead pressure with regard to dynamical connection of the bed-well system is obtained.*

Keywords. integral model · filtration · differential equation · fluid · original · pressure · Laplace transform.

Mathematics Subject Classification (2010): 76S05 · 76N17

1 Introduction

All the processes occurring in the system a well-oil transportation line are directly reflected on oil filtration in the bed. Therefore, filtration process of oil and its flow in rising pipes should be jointly considered.

The works [14 - 17, 19, 20, 22, 23, 26] were devoted to this problem. In some of these works [14, 20, 22, 26] the filtration process is simplified or the problem is solved by numerical methods. In the other works [15,16,17,19,23] stationary flow of fluid is considered regardless of dynamical system, a bed-well. The works [3, 4, 6, 7, 11, 12, 21, 24, 25] were devoted to studying one- dimensional and two-dimensional models of filtration of gas-fluid mixture and gassy fluid. In these works, the investigations are carried out regardless of filtration process and fluid flow in the conjugated system, a bed-well. In reality, filtration process and motion of fluid in rising pipes occur interconnectedly. Therefore, as was noted at the beginning they should be considered as a unique system and this paper is devoted to this issue.

2 Problem statement.

Assume that the bed has a circular form of radius R_k. A well of radius r_c is located concentrically to the outer boundary of the bed. By its permeability the bed is accepted as ho-

Sh.A. Kerimova
Institute of Mathematics and Mechanics of NAS of Azerbaijan,
B. Vagabzade, 9, AZ1141, Baku, Azerbaijan
E-mail: shusha_az@rambler.ru

mogeneous, the fluid as homogeneous. It is assumed that at initial moment the well was not acting. In the oil recovery process, at all points of the bed and in the wellhead the pressure drops.

Then the equation of fluid flow filtration will be of the form [1, 2, 8, 18]

$$\frac{\partial^2 \Delta P}{\partial r^2} + \frac{1}{r}\frac{\partial \Delta P}{\partial r} = \frac{1}{\chi}\frac{\partial \Delta P}{\partial t} \quad r_c \le r \le R_k; t > 0, \tag{2.1}$$

where $\Delta P = P_k - P \quad \chi = \frac{k}{\mu\beta^*}$; P is pressure at any point of the bed, MPa; P_k - is pressure on the contour of the bed, MPa; R_k is the radius of bed's contour, $m; r$ is a coordinate,$m; r_c$ the well's radius, $m; t$ time, s ; χ is a pressure conductivity factor, m^2/s; μ is fluid's dynamical viscosity factor, $MPa \cdot s$; β^* is a compressibility factor, MPa^{-1}; k is the effective permeability of the bed, m^2.

Initial and boundary conditions

$$\Delta P|_{t=0} = 0 r_c \le r \le R_k. \tag{2.2}$$

$$\Delta P|_{r=r_c} = P_k - P_c(t) t > 0. \tag{2.3}$$

$$\Delta P|_{r=R_k} = 0 t > 0, \tag{2.4}$$

where, $P_c(t)$ is the pressure on the wellhead, MPa.

The solution of equation (2.1) with regard to conditions (2.2), (2.4) and $\Delta P|_{r=r_c} = P_k - P_c = \Delta P_c = const$ gives the law of pressure distribution in the bed under boundary pressure equal to one [1, 2, 24]

$$\begin{aligned} \Delta P_0 = \frac{\ln\left(\frac{R_k}{r}\right)}{\ln\left(\frac{R_k}{r_c}\right)} - \pi \sum_{\nu=1}^{\infty} \frac{J_0\left(x_\nu \frac{R_k}{r_c}\right) J_0(x_\nu)}{J_0^2\left(x_\nu \frac{R_k}{r_c}\right) - J_0^2(x_\nu)} \left[J_0\left(x_\nu \frac{r}{r_c}\right) Y_0\left(x_\nu \frac{R_k}{r_c}\right) - \right. \\ \left. - Y_0\left(x_\nu \frac{r}{r_c}\right) J_0\left(x_\nu \frac{R_k}{r_c}\right)\right] \exp\left(-\frac{x_\nu^2 \chi t}{r_c^2}\right), \end{aligned} \tag{2.5}$$

where x_ν are the roots of the transcendental equation

$$Y_0(x_\nu) J_0\left(x_\nu \frac{R_k}{r_c}\right) - J_0(x_\nu) Y_0\left(x_\nu \frac{R_k}{r_c}\right) = 0. \tag{2.6}$$

Equation (2.1) with variable boundary condition (2.3) can be solved by the Duhamel integral [1, 2, 24]

$$\Delta P(r,t) = f(t) P_0(0) + \int_0^t f(\tau) \Delta \dot{P}_0[r; (t-\tau)] d\tau. \tag{2.7}$$

Then allowing for boundary condition (2.3) and equation (2.5), from equation (2.7) $f(t) = P_k - P_c(t)$ we get

$$\begin{aligned} \Delta P(r,t) = \int_0^t (P_k - P_c(\tau)) (\pi \sum_{\nu=1}^{\infty} \left[A_\nu\left(x_\nu \frac{R_k}{r_c}\right) U\left(x_\nu \frac{r}{r_c}\right) \left(\frac{x_\nu^2 \chi}{r_c^2}\right)\right] \times \\ \times \exp\left(-\frac{x_\nu^2 \chi (t-\tau)}{r_c^2}\right) d\tau \end{aligned} \tag{2.8}$$

$$A_\nu\left(x_\nu \frac{R_k}{r_c}\right) = \frac{J_0\left(x_\nu \frac{R_k}{r_c}\right) J_0(x_\nu)}{J_0^2\left(x_\nu \frac{R_k}{r_c}\right) - J_0^2(x_\nu)}$$

$$U\left(x_\nu \frac{r}{r_c}\right) = \left[J_0\left(x_\nu \frac{r}{r_c}\right) Y_0\left(x_\nu \frac{R_k}{r_c}\right) - Y_0\left(x_\nu \frac{r}{r_c}\right) J_0\left(x_\nu \frac{R_k}{r_c}\right)\right]$$

$P_c(t)$ is the law of pressure change on the bottom hole to be determined.

Influx of fluid from the bed to the well per unit of time is defined by the formula

$$Q|_{r=r_c} = -2\pi r_c b \frac{k}{\mu} \left.\frac{\partial \Delta P}{\partial r}\right|_{r=r_c}. \tag{2.9}$$

Then allowing for formula (2.8) from expression (2.9) we get

$$\begin{aligned} Q_c(t) = -2\pi r_c b \tfrac{k}{\mu} \Bigg[\int_0^t (P_k - P_c(\tau)) (\pi \sum_{\nu=1}^{\infty} \left[A_\nu \left(x_\nu \tfrac{R_k}{r_c}\right) \left(\tfrac{x_\nu^2 \chi}{r_c^2}\right)\right] \times \\ \times \left[-J_1(x_\nu) Y_0\left(x_\nu \tfrac{R_k}{r_c}\right) + Y_1(x_\nu) J_0\left(x_\nu \tfrac{R_k}{r_c}\right)\right] \exp\left(-\tfrac{x_\nu^2 \chi (t-\tau)}{r_c^2}\right) d\tau\Bigg]. \end{aligned} \tag{2.10}$$

Now let us consider fluid motion in the pipe. Accepting the fluid as dropping compressible homogeneous, for the fluid equation in the pipe and continuity equation we will have [9,13]

$$\begin{aligned} -\frac{\partial P}{\partial x} &= \frac{\partial Q}{\partial t} + 2aQ, \\ -\frac{1}{c^2}\frac{\partial P}{\partial t} &= \frac{\partial Q}{\partial x}, \end{aligned} \tag{2.11}$$

where $Q = \rho u,\ \rho$ is fluid's density, u is the fluid flow rate averaged in cross section of the pipe, $c^2 = \frac{\partial P}{\partial \rho}$ is sound propagation velocity in fluid, Q is mass flow rate, a is resistance coefficient per unit area.

Having differentiated the first equation of expression (2.11) in time t, the second coordinate in x and subtracted them from each other, we get

$$\frac{\partial^2 Q}{\partial t^2} = c^2 \frac{\partial^2 Q}{\partial x^2} - 2a\frac{\partial Q}{\partial t}. \tag{2.12}$$

We represent the velocity of cross-section of the fluid's column as a sum of two velocities

$$u = u_r + u_e, \tag{2.13}$$

where u_e is the velocity of motion of fluid's column as a solid (transfer velocity), u_r is the velocity of cross sections of the fluid's column under its compressibility (relative velocity) [21].

Substituting expression (2.13) in formula (2.12) we get

$$\rho\frac{\partial^2 u_e}{\partial t^2} + \rho\frac{\partial^2 u_r}{\partial t^2} = \rho c^2 \frac{\partial^2 u_r}{\partial x^2} - 2a\rho\left(\frac{\partial u_e}{\partial t} + \frac{\partial u_r}{\partial t}\right). \tag{2.14}$$

As equation (2.14) is linear, it is separated into two equations

$$\frac{\partial^2 u_e}{\partial t^2} + 2a\frac{\partial u_e}{\partial t} = \frac{\dot{P}_c - \dot{P}_y}{\rho l}, \tag{2.15}$$

where $P_y(t)$ is the pressure at the wellhead

$$\frac{\partial^2 u_r}{\partial t^2} = c^2\frac{\partial^2 u_r}{\partial x^2} - 2a\frac{\partial u_r}{\partial t} + \frac{\dot{P}_y - \dot{P}_c}{\rho l}. \tag{2.16}$$

Having located the origin in the coordinate axis in the lower section of the pipe and directing it upwards for initial and boundary conditions we get

$$u_e|_{t=0} = 0 \tag{2.17}$$

$$\left.\frac{du_e}{dt}\right|_{t=0} = 0 \tag{2.18}$$

$$u_r|_{t=0} = 0, \qquad \left.\frac{\partial u_r}{\partial t}\right|_{x=0} = 0 \tag{2.19}$$

$$\left.\frac{\partial u_r}{\partial x}\right|_{x=l} = 0 \tag{2.20}$$

$$u_r|_{x=0} = 0 \tag{2.21}$$

$$f u_e|_{x=0} = -\frac{2\pi k}{\mu} r_c h \left.\frac{\partial \Delta P}{\partial r}\right|_{r=r_c}. \tag{2.22}$$

Applying the Laplace transform and taking into account the convolution theorem [10] from the expression (2.16) with regard to initial conditions (2.17) and (2.18), we get

$$\begin{aligned} u_e = {} & \tfrac{1}{\rho l}\int\limits_0^t P_c(\tau)\exp\left[-2a(t-\tau)\right]d\tau - \tfrac{1}{\rho l}\int\limits_0^t P_{wh}(\tau)\exp\left[-2a(t-\tau)\right]d\tau - \\ & + \tfrac{1}{2al}\exp(-2at)\left[P_c(0) - P_{wh}(0)\right] + \tfrac{1}{2al}\left[P_{wh}(0) - P_{c0}\right]. \end{aligned} \tag{2.23}$$

Allowing for boundary conditions (2.20) and (2.21) we will look for the solution of equation (2.16) in the form [1]

$$u_r = \sum_{i=1}^{n} \varphi_i(t)\left(1 - \cos\frac{i\pi x}{l}\right). \tag{2.24}$$

Having substituted expression (24) in equation (16), multiplying the both hand sides of the obtained expression by $\left(1 - \cos\frac{i\pi x}{l}\right)$ and integrating it from 0 to l, we get the equation:

$$\begin{aligned} \varphi_i = {} & \tfrac{2}{3l\rho}\left[\int\limits_0^t P_{wh}(\tau)\exp\left[-a(t-\tau)\right]\cos\left[\omega_i(t-\tau)\right]d\tau - \tfrac{a}{\omega_i}\int\limits_0^t P_{wh}(\tau)\times\right. \\ & \times\exp\left[-a(t-\tau)\right]\sin\left[\omega_i(t-\tau)\right]d\tau - \tfrac{P_{wh}(0)}{\omega_i}\exp(-at)\sin(\omega t) - \\ & - \int\limits_0^t P_{wh}(\tau)\exp\left[-a(t-\tau)\right]\cos\left[\omega_i(t-\tau)\right]d\tau + \\ & \left. + \tfrac{a}{\omega_i}\int\limits_0^t P_c(\tau)\exp\left[-a(t-\tau)\right]\sin\left[\omega_i(t-\tau)\right]d\tau + \tfrac{P_c(0)}{\omega_i}\exp(-at)\sin(\omega t)\right]. \end{aligned} \tag{2.25}$$

Having substituted the expression (2.25) in formula (2.24), we get

$$\begin{aligned} u_r = {} & \sum_{i=1}^{n}\left(1 - \cos\tfrac{i\pi x}{l}\right)\left(\tfrac{2}{3l\rho}\left[\int\limits_0^t P_{wh}(\tau)\exp\left[-a(t-\tau)\right]\cosh\left[\omega_i(t-\tau)\right]d\tau - \right.\right. \\ & \tfrac{a}{\omega_i}\int\limits_0^t P_{wh}(\tau)\exp\left[-a(t-\tau)\right]\sinh\left[\omega_i(t-\tau)\right]d\tau - \tfrac{P_{wh}(0)}{\omega_i}\exp(-at)\sinh(\omega t) - \\ & - \int\limits_0^t P_{wh}(\tau)\exp\left[-a(t-\tau)\right]\cosh\left[\omega_i(t-\tau)\right]d\tau + \\ & \left.\left. + \tfrac{a}{\omega_i}\int\limits_0^t P_c(\tau)\exp\left[-a(t-\tau)\right]\sinh\left[\omega_i(t-\tau)\right]d\tau + \tfrac{P_c(0)}{\omega_i}\exp(-at)\sinh(\omega t)\right]\right). \end{aligned} \tag{2.26}$$

Allowing for expression (2.23) and (2.26) from continuity condition expression (2.22) we get

$$\begin{aligned}
&\tfrac{f}{\rho l}\int\limits_0^t P_c(\tau)\exp\left[-2a(t-\tau)\right]d\tau-\tfrac{1}{\rho l}\int\limits_0^t P_{wh}(\tau)\exp\left[-2a(t-\tau)\right]d\tau-\\
&+\tfrac{1}{2al}\exp(-2at)\left[P_c(0)-P_{wh}(0)\right]+\tfrac{1}{2al}\left[P(0)-P_{c0}\right]=\\
&=-2\pi r_c b\tfrac{k}{\mu}\left[\int\limits_0^t (P_k-P_c(\tau))\,(\pi\sum_{\nu=1}^{\infty}\left[A\left(x_\nu\tfrac{R_k}{r_c}\right)\,\left(\tfrac{x_\nu^2\chi}{r_c^2}\right)\right]\times\right.\\
&\left.\times\left[-J_1(x_\nu)\,Y_0\left(x_\nu\tfrac{R_k}{r_c}\right)+Y_1(x_\nu)\,J_0\left(x_\nu\tfrac{R_k}{r_c}\right)\right]\exp\left(-\tfrac{x_\nu^2\chi(t-\tau)}{r_c^2}\right)d\tau\right].
\end{aligned}\tag{2.27}$$

Using the Laplace transform and taking into account convolution and inversion theorem's [5,10], from expression (2.27) we get

$$\begin{aligned}
&P_c=\tfrac{a_1P_k}{(a_1+a_3)}+\left(\tfrac{2a}{\xi}+\tfrac{(\xi-2a)e^{-\xi t}}{\xi}\right)+\\
&+\tfrac{a_3}{(a_1+a_3)}\left(P_{wh}(t)+\left(\int\limits_0^t P_{wh}(\tau)\exp\left[-\xi(t-\tau)\right]d\tau\right)(b_\nu-\xi)\right)+\\
&+\tfrac{2a_2a}{(a_1+a_3)}\left(\tfrac{b_\nu}{\xi}+\tfrac{(\xi-2a)e^{-\xi t}}{\xi}\right),
\end{aligned}\tag{2.28}$$

where

$$\begin{aligned}
&B_\nu=\pi\sum_{\nu=1}^{\infty}\left[A\left(x_\nu\tfrac{R_k}{r_c}\right)\,\left(\tfrac{x_\nu^2\chi}{r_c^2}\right)\right]\left[-J_1(x_\nu)\,Y_0\left(x_\nu\tfrac{R_k}{r_c}\right)+Y_1(x_\nu)\,J_0\left(x_\nu\tfrac{R_k}{r_c}\right)\right],\\
&b_\nu=\left(\tfrac{x_\nu^2\chi}{r_c^2}\right),\quad a_1=-\tfrac{2\pi k}{\mu}r_cbB_\nu,\quad a_2=\tfrac{f}{2al\rho}\left[P_c(0)-P_{wh}(0)\right],\\
&\quad a_3=\tfrac{f}{\rho l}\xi=\tfrac{2aa_1+a_3b_\nu}{a_1+a_3}.
\end{aligned}$$

Now we determine the productivity of the well Q

$$Q=f\left.u\right|_{x=l}.\tag{2.29}$$

Allowing for expressions (2.23), (2.26), from expression (2.29) we get

$$\begin{aligned}
&Q_{x=l}=f\left(\left(\tfrac{4}{3l\rho}\left[\int\limits_0^t P_{wh}(\tau)\exp\left[-a(t-\tau)\right]\cos\left[\omega_i(t-\tau)\right]d\tau-\right.\right.\right.\\
&-\tfrac{a}{\omega_i}\int\limits_0^t P_{wh}(\tau)\exp\left[-a(t-\tau)\right]\sin\left[\omega_i(t-\tau)\right]d\tau-\tfrac{P_{wh}(0)}{\omega_i}\exp(-at)\sin(\omega t)-\\
&-\int\limits_0^t P_{wh}(\tau)\exp\left[-a(t-\tau)\right]\cos\left[\omega_i(t-\tau)\right]d\tau+\\
&\left.\left.+\tfrac{a}{\omega_i}\int\limits_0^t P_c(\tau)\exp\left[-a(t-\tau)\right]\sin\left[\omega_i(t-\tau)\right]d\tau+\tfrac{P_c(0)}{\omega_i}\exp(-at)\sin(\omega t)\right]\right)+\\
&+\tfrac{1}{\rho l}\int\limits_0^t P_c(\tau)\exp\left[-2a(t-\tau)\right]d\tau-\tfrac{1}{\rho l}\int\limits_0^t P_{wh}(\tau)\exp\left[-2a(t-\tau)\right]d\tau-\\
&\left.+\tfrac{1}{2al}\exp(-2at)\left[P_c(0)-P_{wh}(0)\right]+\tfrac{1}{2al}\left[P_{wh}(0)-P_{c0}\right]\right).
\end{aligned}\tag{2.30}$$

Let as determine Q when the well starts-up. Accept that when starting-up a well depending on time the wellhead pressure changes linearly and has the form

$$P_{yc}(t)=P_y(0)-\frac{P_y(0)-P_y(T)}{T_0}t,\tag{2.31}$$

where $P_y(0)$ and $P_y(T)$ are pressure on the wellhead at the initial moment and at the end of oil recovery, T is operational time of the well. Then allowing for formula (2.31) from expression (2.30) we get

$$Q(t) = f\left(\left(\frac{4}{3l\rho}\left[\int_0^t \left(P_y(0) - \frac{P_y(0)-P_y(T)}{T_0}\tau\right)\exp\left[-a(t-\tau)\right]\cos\left[\omega_i(t-\tau)\right]d\tau - \right.\right.\right.$$
$$-\frac{a}{\omega_i}\int_0^t \left(P_y(0) - \frac{P_y(0)-P_y(T)}{T_0}\tau\right)\times\exp\left[-a(t-\tau)\right]\sin\left[\omega_i(t-\tau)\right]d\tau -$$
$$-\frac{P_{wh}(0)}{\omega_i}\exp(-at)\sin(\omega t) - \int_0^t \left(P_y(0) - \frac{P_y(0)-P_y(T)}{T_0}\tau\right)\times$$
$$\times\exp\left[-a(t-\tau)\right]\cos\left[\omega_i(t-\tau)\right]d\tau +$$
$$\left.\left.+\frac{a}{\omega_i}\int_0^t P_{\mathrm{c}}(\tau)\exp\left[-a(t-\tau)\right]\sin\left[\omega_i(t-\tau)\right]d\tau + \frac{P_{\mathrm{c}}(0)}{\omega_i}\exp(-at)\sin(\omega t)\right]\right) +$$
$$\frac{1}{\rho l}\int_0^t P_c(\tau)\exp\left[-2a(t-\tau)\right]d\tau - \frac{1}{\rho l}\int_0^t P_{wh}(\tau)\exp\left[-2a(t-\tau)\right]d\tau -$$
$$\left.+\frac{1}{2al}\exp(-2at)\left[P_c(0) - P(0)\right] + \frac{1}{2al}\left[P_{wh}(0) - P_{c_0}\right]\right). \tag{2.32}$$

By formula (2.32) we make numerical calculations for the following values of parameters

$$c = 1000m\cdot c^{-1};\quad \mu = 10^{-3}Pa\cdot c;\quad h = 10m;\quad k = 10^{-13}m^2;$$
$$\rho = 860kq\cdot m^{-3};\quad l = 2000m, 3000m;\quad P_k = 2.4\cdot 10^7\mathrm{Pa};$$
$$P_0 = 24\cdot 10^6 Pa;\quad P_c(T) = 3\cdot 10^6 Pa;\quad P_c(0) = 24\cdot 10^6 \Pi a;$$
$$P_{atm} = 10^5 \Pi a;\quad T = 180 day;\quad R_k = 200m;\quad \pi = 3,14;$$
$$a = 10^{-1}c^{-1}, 5\cdot 10^{-1}c^{-1};\quad m = 0.2;\quad d = 6\cdot 10^{-2}m;\quad r_c = 7.5\cdot 10^{-2}m.$$

F The results of the obtained calculations were depicted in Fig.1, Fig.2 and Fig. 3. As is seen from Fig. 1, the bottomhole pressure also decreases in the course of time as wellhead pressure and is of almost linear character.

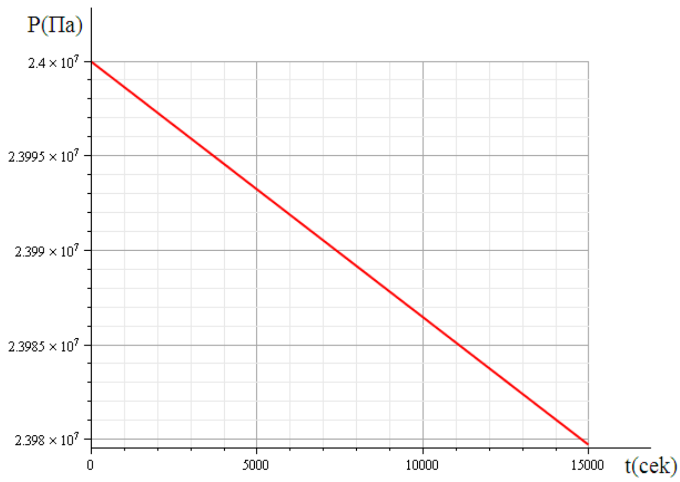

Fig.1. The graph of pressure dynamics at the bottomhole

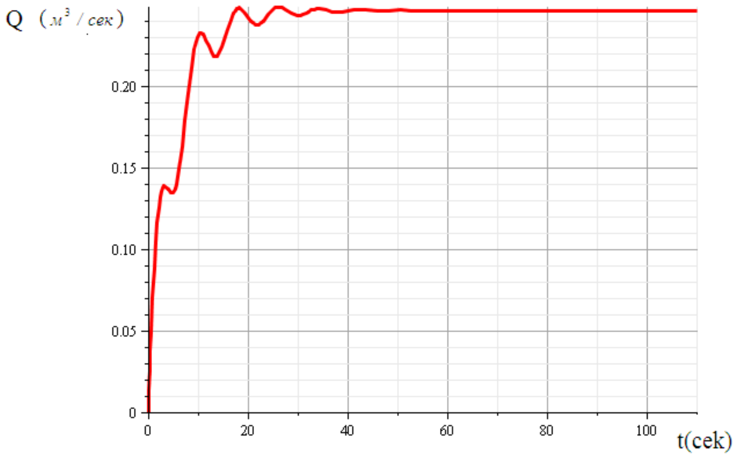

Fig.2. The graph of well productivity dynamics $l = 200m$, $a = 0.1\frac{1}{cek}$.

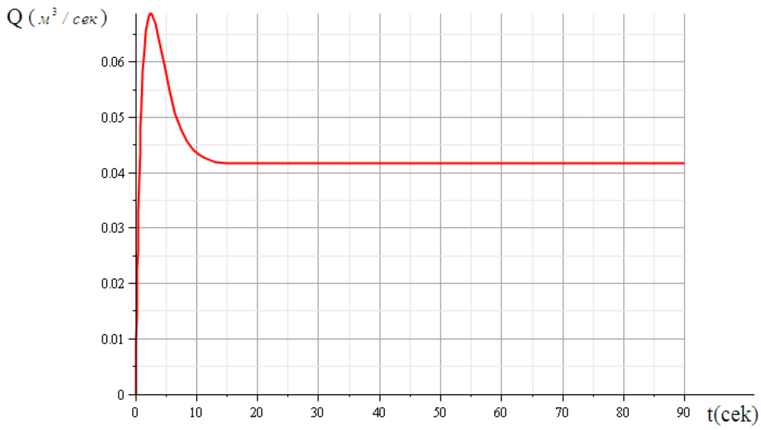

Fig.3. The graph of well productivity dynamics $l = 3000m$, $a = 5 \cdot 0.1\frac{1}{s}$.

It can be seen from Fig.2, and Fig.3 that in the course of time, at first the productivity Q of the well increases and then is stabilized. And with increasing the well depth of resistance factor, the well productivity drops.

The obtained analytic expression allows to determine the time of the well when it is put in the mode into operation and its productivity depending on the parameter of the system, a bed-well.

It should be noted that the obtained formula (2.32) allows to determine the well productivity for any law of pressure change on the wellhead.

3 Conclusion.

An integral model of the process of nonstationary filtration of oil in the conjugate system, a bed-well was constructed. The analytic expressions allowing to determine the well productivity and also bottomhole pressure are obtained.

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